Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/130221

Generalized Thermoelasticity with Diffusion and Voids under Rotation, Gravity and Electromagnetic Field in the Context of Four Theories

S. M. Abo-Dahab[1,2]

[1] Department of Mathematics, Faculty of Science, Taif University, Taif 888, Saudi Arabia.
[2] Department of Mathematics, Faculty of Science, South Valley University, Qena 83523, Egypt

Received: 2 Sep. 2018, Revised: 12 Oct. 2018, Accepted: 20 Oct. 2018
Published online: 1 Mar. 2019

Abstract: In this paper, we investigated a new mathematical model on effect of the diffusion with voids in generalized thermoelastic half-space with electromagnetic field, gravity field, and rotation. The model is formulated in the context of four thermoelastic theories; Classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. The boundary conditions on the surface applied to obtain the enclosed expressions for the displacements, temperature, stresses, concentration of diffusion and volume fraction field in the physical domain using the normal mode method. A comparison will be made for the results obtained in the presence and absence of the new considered variables and displayed graphically. We shall compare the results in the context of the new mathematical model with the previous results obtained by others to ensure the quality of the model and show the physical meaning of the phenomena. Finally, we shall make simulation with Geologists and Petroleum Engineers to show the useful and applications of the new model and generalize the results for the new mathematical model obtained

Keywords: Electromagnetic field, gravity field, rotation; voids, diffusion, normal mode analysis, green-Lindsay, lord-Shulman, dual-phase-lag

1 Introduction

Seismology is the study of mechanical waves that travel on and beneath the surface of the earth. It was first recognized as a scientific discipline in the 1800s with the emergence of the quantitative study of earthquakes, one of the most common natural sources of seismic waves. Although instruments designed to detect earthquakes date back to 132 A.D. (Dewey and Byerly [1]), is the first modern seismometers were developed and installed in observatories around the world in the late 1800s and early 1900s to study the cause of earthquakes and investigate the structure of the earth's interior (Agnew [2]). The first network of seismometers and seismographs to record earthquakes in Kansas was established by the Kansas Geological Survey (KGS) in 1977 to assess the level of seismic activity in the state. As seismic technology and instrumentation improved, active sources were developed to intentionally generate seismic waves for local studies of the earth's subsurface.

During the past few decades, wide spread attentions have been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast, to the conventional theories based on parabolic type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finite of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc.

The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials has been studied since the 19th century. There are a number of theories, which describe mechanical properties of porous materials, and one of them is a Biot consolidation theory of fluid-saturated porous solids (Biot [3]). These theories reduce to classical

* Corresponding author e-mail: sdahb@yahoo.com

elasticity when the pore fluid is absent. In addition, a continuum theory for granular materials, whose matrix material (or skeletal) is elastic and interstices are voids. They formulated this theory from the formal arguments of continuum mechanics and introduced the concept of distributed body, which represents a continuum model for granular materials (sand, grain, powder, etc.) as well as porous materials (rock, soil, sponge, pressed powder, cork, etc.). The basic concept underlying this theory is that the bulk density of the material written as the product of two fields, the density field of the matrix material and the volume fraction field (the ratio of the volume occupied by grains to the bulk volume at a point of the material). This representation of the bulk density of the material introduces an additional kinematic variable in the theory.

The classical and generalized theories of coupled thermoelasticity extensively developed due to their many applications in the advanced structural design problems. Therefore, it is crucial to obtain the deformation and temperature distributions in the structures under thermal shock loads. Recently, the effect of diffusion spread takes a wide range of medical applications, nuclear and engineering, we studied the effect of the diffusion with voids in generalized thermoelastic half-space with an electromagnetic field, gravity field, rotation and initial stress. Lord and Shulman [4] introduced the theory of generalized thermo elasticity with one relaxation time for the special case of an isotropic body. Cowin and Nunziato [5] developed linear elastic materials with voids. Aouadi [6] studied generalized theory of thermoelasic diffusion for an anisotropic media. Aouadi [7] illustrated a problem for an infinite elastic body with a spherical cavity in the theory of generalized thermoelastic diffusion. Aouadi [8] illustrated uniqueness and reciprocity theorem in the theory of generalized thermoelasic diffusion. Singh ([9], [10]) studied reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion. Nowacki ([11]-[13]) illustrated dynamical problems of thermoelastic diffusion in solids. Olesiak and Pyryev [14] studied a coupled quasi-stationary problem of thermodifusion for an elastic cylinder. Sherief and Saleh [15] discussed a half-space problem in the theory of generalized thermoelastic diffusion. Ram et al. [16] studied thermo-mechanical response of generalized thermoelastic diffusion with one relaxation time due to time harmonic sources. Bayones [17] discussed the influence of diffusion on generalized magneto-thermo-viscoelastic problem of a homogenous isotropic material. Abo-Dahab and Singh [18] illustrated influence of magnetic field on wave propagation in generalized thermoelastic solid with diffusion. Xia et al. [19] developed the influence of diffusion on generalized thermoelastic problems of infinite body with a cylindrical cavity. Allam et al. [20] discussed GL model on reflection of P and SV waves from the free surface of thermo-elastic diffusion solid under influence of the electromagnetic field and initial stress. Abouelregal and Abo-Dahab [21] illustrated dual-phase-lag diffusion model for Thomson's phenomenon on electromagneto-thermoelastic an infinite solid cylinder.

Abo-Dahab [22] studied S-waves propagation in a non-homogeneous anisotropic incompressible medium under influences of gravity field, initial stress, electromagnetic field and rotation. Kumar and Kumar [23] illustrated wave propagation and fundamental solution of initially stressed thermoelastic diffusion with voids. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations (Abd-Alla et al.[24], Abd-Alla and Abo-Dahab [25] and Youssef and El-Bary [26]). Kumar and Gupta [27] discussed wave propagation at the boundary surface of inviscid fluid half-space and thermoelastic diffusion solid half-space with dual-phase-lag models. Kumar and Gupta [28] studied dual-phase-lag models of wave propagation at the interface between elastic and thermoelastic diffusion media. Sur and Kanoria [29] developed three-phase-lag elasto-thermodiffusive response in an elastic solid under hydrostatic pressure. Kumar et al. [30] studied axi-symmetric propagation in a thermoelastic diffusion with phase lags. Abouelregal [31] illustrated a problem of a semi-infinite medium subjected to exponential heating using a dual-phase-lag thermoelastic model. Kumar and Kansal [32] discussed propagation of plane waves and fundamental solution in the theories of thermoelastic diffusive materials with voids.

In this paper, we investigated the effect of the diffusion with voids in generalized thermoelastic half-space with an electromagnetic field, gravity field and rotation in the context of Classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. Numerical computation is performed by using a numerical technique and the resulting quantities are shown graphically. Comparisons have been made with the obtained results in the presence and absence of the considered variables. The effect of the diffusion with voids, magnetic field, rotation and gravity field on temperature, displacement and stress in elastic body are studied and indicated that has a perfect influence on the phenomena.

2 Formulation of the problem

Let us consider a homogeneous generalized thermoelastic half-space rotating uniformly with an angular velocity $\overrightarrow{\Omega} = \Omega \overrightarrow{n}$, where, is $\overrightarrow{n}$ a unit vector representing the direction of the axis of rotation. The rectangular Cartesian coordinate system (x, y, z) with y-axis vertically downwards into the medium is introduced. The displacement equation of motion in the rotating frame has two additional terms centripetal acceleration, $\overrightarrow{\Omega} \times \left(\overrightarrow{\Omega} \times \overrightarrow{u}\right)$ due to time varying motion only and Corioli's acceleration $2\overrightarrow{\Omega} \times \overrightarrow{\dot{u}}$, where, $\overrightarrow{u} = (u, 0, w)$ is the dynamic displacement vector, and $\overrightarrow{\Omega} = (0,\Omega,0)$ is the angular velocity .

We consider the normal source acting at the plane surface of generalized thermo-elastic half-space under the influence of gravity and constant primary magnetic field and electric field.

3 Basic equations

The governing equations for a homogeneous generalized thermoelastic half-space with diffusion, voids and Lorentz's body forces in the absence of incremental heat flux at reference temperature T_0 given as follows:

$$\sigma_{ij}= (\lambda e-\gamma\left(1+\tau_1\tfrac{\partial}{\partial t}\right)T+b\Phi_v-\beta_1\left(1+\tau^1\tfrac{\partial}{\partial t}\right)C)\delta_{ij}+2\mu e_{ij}, \tag{1}$$

$$p_c=-\beta_1 e+b_c C-a_c T-b_2^*\Phi_v \tag{2}$$

$$\rho\eta=\gamma e+\alpha T+m\Phi_v+a_c C \tag{3}$$

$$g^*=-be-\xi\Phi_v+mT-\omega_0\dot{\Phi}_v+b_2^*C \tag{4}$$

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{5}$$

$$\omega_{ij}=\frac{1}{2}(u_{j,i}-u_{i,j}) \tag{6}$$

$$S_i=\alpha\Phi_{v,i} \tag{7}$$

The Maxwell's equation is

$$\tau_{ij}=\mu_e\left[H_i h_j+H_j h_i-(\overrightarrow{H}_k.\overrightarrow{h}_k)\delta_{ij}\right],i,j=1,2,3 \tag{8}$$

where, τ_{ij} is Maxwell's stress tensor, which reduces to

$$\tau_{11}=\tau_{33}=\mu_e H^2\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right),\quad \tau_{13}=0$$

Equation of motion is

$$\sigma_{ji,j}+F_i=\rho[\overrightarrow{\ddot{u}}+\overrightarrow{\Omega}\times\left(\overrightarrow{\Omega}\times\overrightarrow{u}\right)+2\overrightarrow{\Omega}\times\overrightarrow{\dot{u}}]_i \tag{9}$$

which tends to

$$\mu u_{i,jj}+(\lambda+\mu)u_{j,ij}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,i}+b\Phi_{v,i}-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)C_{,i}$$
$$+F_i+G_i=\rho[\overrightarrow{\ddot{u}}+\overrightarrow{\Omega}\times\left(\overrightarrow{\Omega}\times\overrightarrow{u}\right)+2\overrightarrow{\Omega}\times\overrightarrow{\dot{u}}]_i \tag{10}$$

where

$$F_i=\left(\overrightarrow{J}\times\overrightarrow{B}\right)_i,\qquad G=\rho g\left(\frac{\partial\omega}{\partial x},0,-\frac{\partial u}{\partial x}\right) \tag{11}$$

The variation of magnetic field and electric field given by Maxwell's equation as the following form:

$$\begin{aligned}
&\overrightarrow{J}+\varepsilon_0\tfrac{\partial\overrightarrow{E}}{\partial t}=\text{curl }\overrightarrow{h}\\
&-\mu_e\tfrac{\partial\overrightarrow{h}}{\partial t}=\text{curl }\overrightarrow{E}\\
&\text{div }\overrightarrow{h}=0\\
&\text{div }\overrightarrow{E}=0\\
&\overrightarrow{E}=-\mu_e\left(\tfrac{\partial\overrightarrow{u}}{\partial t}\times\overrightarrow{H}_0\right)\\
&\overrightarrow{h}=\text{curl}\,(\overrightarrow{u}\times\overrightarrow{H}_0)
\end{aligned} \tag{12}$$

where,

$$\overrightarrow{H}_0=(0,H,0),\ \overrightarrow{H}=\overrightarrow{H}_0+\overrightarrow{h}(x,z,t) \tag{13}$$

Using Eq. (12) we obtain

$$F_x=\mu_e H^2\left[\frac{\partial e}{\partial x}-\varepsilon_0\frac{\partial^2 u}{\partial t^2}\right] \tag{14}$$

$$Fy=0, \tag{15}$$

$$F_z=\mu_e H^2\left[\frac{\partial e}{\partial z}-\varepsilon_0\frac{\partial^2 w}{\partial t^2}\right], \tag{16}$$

The heat conduction equation considering voids and dual-phase-lag model in the form (Allam et al. [20])

$$K\left(1+\tau_\Theta\tfrac{\partial}{\partial t}\right)T_{,ii}=\left(n_1+\tau_q\tfrac{\partial}{\partial t}\right)\left(\rho C_E\tfrac{\partial T}{\partial t}+\alpha_c T_0\tfrac{\partial C}{\partial t}+mT_0\tfrac{\partial\Phi_v}{\partial t}\right)+\gamma T_0\left(n_1+n_0\tau_q\tfrac{\partial}{\partial t}\right)\tfrac{\partial e}{\partial t} \tag{17}$$

The equation of voids is

$$\alpha\Phi_{v,ii}-bu_{i,i}-\zeta\dot{\Phi}_v-\omega_0\dot{\Phi}_v+mT+b_2^*C=\rho\chi\ddot{\Phi}_v \tag{18}$$

The equation of diffusion is

$$\left(1+\tau_q\tfrac{\partial}{\partial t}\right)\left(d\beta_1 e_{,ii}-db_c\left(1+\tau^1\tfrac{\partial}{\partial t}\right)C_{,ii}+da_c\left(1+\tau_1\tfrac{\partial}{\partial t}\right)T_{,ii}\right.$$
$$\left.+db_2^*\Phi_{v,ii}\right)+\left(1+\tau_\eta\tfrac{\partial}{\partial t}\right)\dot{C}=0 \tag{19}$$

We study the above basic equations for the following four different theories:

(i) Classical and Dynamical coupled theory (1956) (CD):

$$n_0=0,\ n_1=1,\ \tau_q=0,\ \ \tau_1=0,\ \tau_\Theta=0,\ \tau^1=0,\ \tau_\eta=0,\ \tau_P=0$$

(ii) Lord and Shulman theory (1967) (LS):

$$n_0=1,\ n_1=1,\ \tau_q>0,\ \tau_1=0,\ \tau_\Theta=0,\ \tau^1=0,\ \tau_\eta<\tau_q,\ \tau_P=0$$

(iii) Green and Linday theory (1972) (GL):

$$n_0=0,\ n_1=1,\ \tau_q>0,\ \ \tau_1\geq\tau_q,\ \tau_\Theta=0,\ \tau^1\geq\tau_q,\ \tau_\eta=0,\ \tau_P=0$$

(iv) Dual-Phase-Lag theory (1956) (DPL):

$$n_0=1,\ n_1=1,\ \tau_q>0,\ \ \tau_1=0,\ \tau_\Theta<\tau_q,\ \tau^1=0,\ \tau_\eta=0,\ \tau_P=0$$

4 Solution of the problem

From equations (10) and (17)-(19) we obtain

$$\mu\nabla^2 u+(\lambda+\mu)\frac{\partial e}{\partial x}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} +b\frac{\partial \Phi_v}{\partial x}-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x}+F_x+\rho g\frac{\partial w}{\partial x} =\rho\left[\frac{\partial^2 u}{\partial t^2}+2\Omega\frac{\partial w}{\partial t}-\Omega^2 u\right] \quad (20)$$

$$\mu\nabla^2 w+(\lambda+\mu)\frac{\partial e}{\partial z}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} +b\frac{\partial \Phi_v}{\partial z}-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial z}+F_z-\rho g\frac{\partial u}{\partial x} =\rho\left[\frac{\partial^2 w}{\partial t^2}-2\Omega\frac{\partial u}{\partial t}-\Omega^2 w\right] \quad (21)$$

$$K\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)T_{,ii}= \left(n_1+\tau_q\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial T}{\partial t}+\alpha_c T_0\frac{\partial C}{\partial t}+mT_0\frac{\partial \Phi_v}{\partial t}\right) +\gamma T_0\left(n_1+n_0\tau_q\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} \quad (22)$$

$$\alpha\left(\frac{\partial^2\Phi_v}{\partial x^2}+\frac{\partial^2\Phi_v}{\partial z^2}\right)-b\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) -\zeta\Phi_v-\omega_0\frac{\partial \Phi_v}{\partial t}+mT+b_2^*C=\rho\chi\frac{\partial^2\Phi_v}{\partial t^2} \quad (23)$$

$$\left(1+\tau_P\frac{\partial}{\partial t}\right)\left[d\beta_1\left(\frac{\partial^2 e}{\partial x^2}+\frac{\partial^2 e}{\partial z^2}\right)-db_c\left(1+\tau^1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2}\right) +da_c\left(1+\tau_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)+db_2^*\left(\frac{\partial^2\Phi_v}{\partial x^2}+\frac{\partial^2\Phi_v}{\partial z^2}\right)\right] +\left(1+\tau_\eta\frac{\partial}{\partial t}\right)C=0 \quad (24)$$

The constitutive relations written as

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T+b\Phi_v-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)C \quad (25)$$

$$\sigma_{yy}=\lambda e-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T+b\Phi_v-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)C \quad (26)$$

$$\sigma_{zz}=(\lambda+2\mu)\frac{\partial w}{\partial z}+\lambda\frac{\partial u}{\partial x}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T+b\Phi_v-\beta_1\left(1+\tau^1\frac{\partial}{\partial t}\right)C \quad (27)$$

$$\sigma_{xz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \quad (28)$$

$$\sigma_{zx}=\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) \quad (29)$$

$$\sigma_{xy}=\sigma_{yx}=0 \quad (30)$$

For simplifications, we shall use the following non-dimensional variables:

$$x_i'=\frac{\omega^*}{c_0}x_i,\ u_i'=\frac{\rho c_0\omega^*}{\gamma T_0}u_i,\ \Omega'=\frac{\Omega}{\omega^*},$$
$$\theta=\frac{T}{T_0},\ \sigma_{ij}'=\frac{\sigma_{ij}}{\gamma T_0},\ \Phi_v'=\frac{\chi}{\gamma T_0}\Phi_v, C'=\frac{\beta_1}{\gamma T_0}C,$$
$$g'=\frac{g}{c_0\omega^*},\ \ (t',\tau',\tau_1',\tau^{1'},\tau_q',$$
$$\tau_\Theta',\tau_p',\tau_\eta')=\omega^*(t,\tau,\tau_1,\tau^1,\tau_q,\tau_\Theta,\tau_p,\tau_\eta),\ \ b^*=\frac{b}{\chi},$$
$$\tau_{ij}'=\frac{\tau_{ij}}{\gamma T_0}. \quad (31)$$

In terms of non-dimensional quantities defined in Eq. (31), the above governing equations (20)-(24) tend to:

$$\left(\frac{\mu}{\rho c_0^2}\right)\nabla^2 u+\left(\frac{\lambda+\mu}{\rho c_0^2}+R_H\right)\frac{\partial e}{\partial x}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial x} +b^*\frac{\partial\Phi_v'}{\partial x}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x}+g\frac{\partial w}{\partial x}=\left[\beta^2\frac{\partial^2 u}{\partial t^2}+2\Omega\frac{\partial w}{\partial t}-\Omega^2 u\right] \quad (32)$$

$$\left(\frac{\mu}{\rho c_0^2}\right)\nabla^2 w+\left(\frac{\lambda+\mu}{\rho c_0^2}+R_H\right)\frac{\partial e}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial z} +b^*\frac{\partial\Phi_v'}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial z}-g\frac{\partial u}{\partial x}=\left[\beta^2\frac{\partial^2 w}{\partial t^2}-2\Omega\frac{\partial u}{\partial t}-\Omega^2 w\right] \quad (33)$$

$$\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)\nabla^2\theta=\left(n_1+\tau_\Theta\frac{\partial}{\partial t}\right)\left(\dot\theta+\zeta_2\dot\Phi_v'+\zeta_3\dot C\right) +\zeta_1\left(n_1+n_0\tau_q\frac{\partial}{\partial t}\right)\dot e \quad (34)$$

$$\nabla^2\Phi_v-a_1\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)-a_2\Phi_v -a_3\frac{\partial\Phi_v}{\partial t}+a_4\theta+a_4''C=a_5\frac{\partial^2\Phi_v}{\partial t^2}. \quad (35)$$

$$\left(1+\tau_p\frac{\partial}{\partial t}\right)\left(\nabla^2 e+a_6\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2\theta-a_8\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C+a_9\nabla^2\Phi_v\right) +a_7\left(\dot C+\tau\eta\ddot C\right)=0. \quad (36)$$

$$a_1=\frac{b\chi}{\rho\alpha\omega^{*2}},\ \ a_2=\frac{\zeta c_0^2}{\alpha\omega^{*2}},\ a_3=\frac{\omega_0 c_0^2}{\alpha\omega^*},$$
$$a_4=\frac{mc_0^2\chi}{\gamma\alpha\omega^{*2}}, a_4'=\frac{b_2^*c_0^2\chi}{\beta_1\alpha\omega^{*2}},$$
$$a_5=\frac{\rho c_0^2\chi}{\alpha},\ \ a_6=\frac{a_c\rho c_0^2}{\beta_1\gamma},$$
$$a_7=\frac{Kc_0^2}{d\beta_1^2 C_E},\ \ a_8=\frac{b_c\rho c_0^2}{\beta_1^2},$$
$$a_9=\frac{b_2^*c_0^2\chi}{\beta_1\chi}, \zeta_1=\frac{\gamma^2 T_0}{\rho K\omega^*},$$
$$\zeta_2=\frac{mT_0\gamma}{\rho C_E\chi},\ \ \zeta_3=\frac{a_cT_0\gamma}{\rho C_E\beta^1}.$$

By Helmholtz theorem, the displacement vector written in the displacement potentials $\Phi(x,z,t)$ and $\Psi(x,z,t)$ form as

$$u=\frac{\partial\Phi}{\partial x}+\frac{\partial\Psi}{\partial z},\quad w=\frac{\partial\Phi}{\partial z}-\frac{\partial\Psi}{\partial x},\quad \vec{\Psi}=(0,-\Psi,0) \tag{37}$$

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2},\qquad \nabla^2\Psi=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}, \tag{38}$$

$$\left(a_{11}\nabla^2-\beta^2\frac{\partial^2}{\partial t^2}+\Omega^2\right)\Phi-\left(g\frac{\partial}{\partial x}-2\Omega\frac{\partial}{\partial t}\right)\Psi-\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta +b^*\Phi_v-\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0 \tag{39}$$

$$\left(g\frac{\partial}{\partial x}-2\Omega\frac{\partial}{\partial t}\right)\Phi+\left(a_{12}\nabla^2-\beta^2\frac{\partial^2}{\partial t^2}+\Omega^2\right)\Psi=0 \tag{40}$$

$$\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)\nabla^2\theta=\left(n_1+\tau_q\frac{\partial}{\partial t}\right)\left(\dot{\theta}+\zeta_2\dot{\Phi}_v'+\zeta_3\dot{C}\right)+\zeta_1\left(n_1+n_0\tau_q\frac{\partial}{\partial t}\right)\dot{e} \tag{41}$$

$$\left(\nabla^2-a_2-a_3\frac{\partial}{\partial t}-a_5\frac{\partial^2}{\partial t^2}\right)\Phi_v-a_1\nabla^2\Phi+a_4\theta+a_4'C=0 \tag{42}$$

where

$$\left(1+\tau_p\frac{\partial}{\partial t}\right)\left(\nabla^2\left(\nabla^2\Phi\right)+a_6\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2\theta-a_8\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C+a_9\nabla^2\Phi_v\right) +\left(a_7\frac{\partial}{\partial t}+a_7\tau_\eta\frac{\partial^2}{\partial t^2}\right)C=0 \tag{43}$$

The constitutive relations written as

$$\sigma_{xx}=b_0\frac{\partial u}{\partial x}+b_1\frac{\partial w}{\partial z}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta+b^*\Phi_v-\left(1+\tau^1\frac{\partial}{\partial t}\right)C \tag{44}$$

$$\sigma_{yy}=b_1\nabla^2\Phi-\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta+b^*\Phi_v-\left(1+\tau^1\frac{\partial}{\partial t}\right)C \tag{45}$$

$$\sigma_{zz}=b_0\frac{\partial w}{\partial z}+b_1\frac{\partial u}{\partial x}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta+b^*\Phi_v-\left(1+\tau^1\frac{\partial}{\partial t}\right)C \tag{46}$$

$$\sigma_{xz}=b_2\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \tag{47}$$

$$\sigma_{zx}=b_2\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) \tag{48}$$

$$\sigma_{xy}=\sigma_{yz}=0 \tag{49}$$

where

$$(b_0,b_1,b_2)=\frac{1}{\rho c_0^2}(\lambda+2\mu,\lambda,\mu),\qquad a_{11}=\frac{\lambda+2\mu}{\rho c_0^2}+R_H,\quad a_{12}=\frac{\mu}{\rho c_0^2}.$$

5 Normal mode analysis

The solution of the considered physical variable decomposed in terms of normal modes and given in the following form:

$$\left[u,w,e,\theta,\Phi,\Psi,h,E,\sigma_{ij},\Phi_v,C\right](x,z,t)= \left[u^*,w^*,e^*,\theta^*,\Phi^*,\Psi^*,h^*,E^*,\sigma_{ij}^*,\Phi_v^*,C^*\right](z)e^{(\omega t+iax)} \tag{50}$$

where, ω and a in the $x-$direction $u^*(z),w^*(z),e^*(z),\theta^*(z),\Phi^*(z),\Psi^*(z),h^*(z),E^*(z),\sigma_{ij}^*(z)$, $\Phi_v^*(z)$ and $C^*(z)$ are the amplitudes of the field quantities.

Substituting from equation (50) into equations (39)-(49) we get:

$$\left(a_{11}D^2-\Lambda_1\right)\Phi^*-\Lambda_2\Psi^*-\Lambda_{10}\theta^*+b^*\Phi_v^*-\Lambda_{11}C^*=0 \tag{51}$$

$$\Lambda_2\Phi^*+\left(a_{12}D^2-\Lambda_3\right)\Psi^*=0 \tag{52}$$

$$-\Lambda_5\left(D^2-\alpha^2\right)\Phi^*+\left(D^2-\Lambda_4\right)\theta^*+\Lambda_6\Phi_v^*+\Lambda_8C^*=0 \tag{53}$$

$$\left(D^2-\Lambda_7\right)\Phi_v^*+a_1\left(-D^2+\alpha^2\right)\Phi^*+a_4\theta^*+a_4'C^*=0 \tag{54}$$

$$\left(D^4-2\alpha^2D^2+\alpha^4\right)\Phi^*+a_6\Lambda_{10}\left(D^2-\alpha^2\right)\theta^* +\left(\Lambda_9-a_8\Lambda_{11}D^2\right)C^*+a_9\left(D^2-\alpha^2\right)\Phi_v^*=0 \tag{55}$$

$$\sigma_{xx}^*=iab_0u^*+b_1Dw^*-\Lambda_{10}\theta^*+b^*\Phi_v^*-\Lambda_{11}C^* \tag{56}$$

$$\sigma_{yy}^*=b_1(D^2-\alpha^2)\Phi^*-\Lambda_{10}\theta^*+b^*\Phi_v^*-\Lambda_{11}C^* \tag{57}$$

$$\sigma_{zz}^*=b_0Dw^*+iab_1u^*-\Lambda_{10}\theta^*+b^*\Phi_v^*-\Lambda_{11}C^* \tag{58}$$

$$\sigma_{xz}^*=b_2Du^*+iab_2w^* \tag{59}$$

$$\sigma_{zx}^*=b_2Du^*+iab_2w^* \tag{60}$$

$$\sigma_{xy}^*=\sigma_{yz}^*=0 \tag{61}$$

where

$$\Lambda_1=a_{11}a^2+\beta^2\omega^2-\Omega^2,$$

$$\Lambda_2=iag-2\Omega\omega,\Lambda_3=a_{12}a^2+\beta^2\omega^2-\Omega^2,$$

$$\Lambda_4=a^2+\frac{\omega\omega_2}{\omega_1},$$

$$\Lambda_5=\frac{\zeta_1\omega\omega_2}{\omega_1},$$

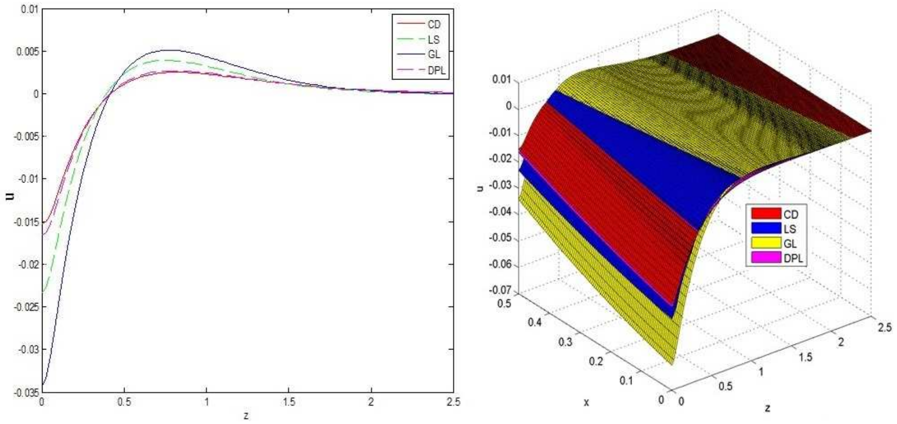


Fig. 1: Horizontal displacement distribution u with electromagnetic field, rotation and gravity field

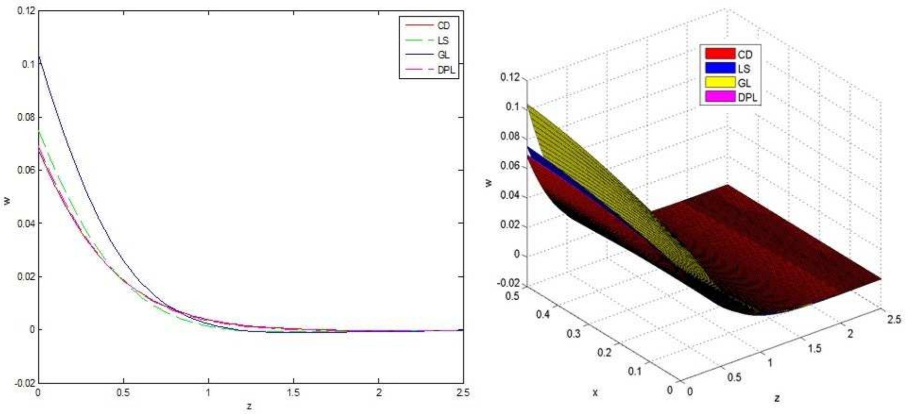


Fig. 2: Vertical displacement distribution w with electromagnetic field, rotation and gravity field

$$\Lambda_6 = \frac{-\zeta_1 \omega \omega_2}{\omega_1},$$
$$\Lambda_7 = a^2 + a_2 + a_3\omega + a_5\omega^2,$$
$$\Lambda_8 = \frac{-\zeta_3 \omega \omega_2}{\omega_1},$$
$$\Lambda_9 = \frac{\omega \omega_2^*}{\omega_1^*} a_7 + \Lambda_{11} a_8 a^2,$$
$$\Lambda_{10} = 1 + \tau_1 \omega,$$
$$\Lambda_{11} = 1 + \tau^1 \omega,$$
$$\Lambda_{12} = n_1 + n_0 \tau_q \omega,$$
$$\omega_1 = 1 + \tau_\Theta \omega,$$
$$\omega_2 = 1 + \tau_q \omega,$$
$$\omega_1^* = 1 + \tau_P \omega,$$
$$\omega_2^* = 1 + \tau_\eta \omega.$$

Eliminating $\Psi^*(z)$, $\Phi^*(z), C^*(z)$ and $\theta^*(z)$ in Equations (51)-(55), we get the differential equation for $\Phi^*(z)$:

$$[D^{10} - AD^8 + BD^6 - CD^4 + ED^2 - L]\{\Phi^*(z)\} = 0. \quad (62)$$

In a similar manner we arrive at

$$[D^{10} - AD^8 + BD^6 - CD^4 + ED^2 - L]\{\Psi^*(z), \theta^*(z), \Phi_v^*(z), C^*(z)\} = 0 \quad (63)$$

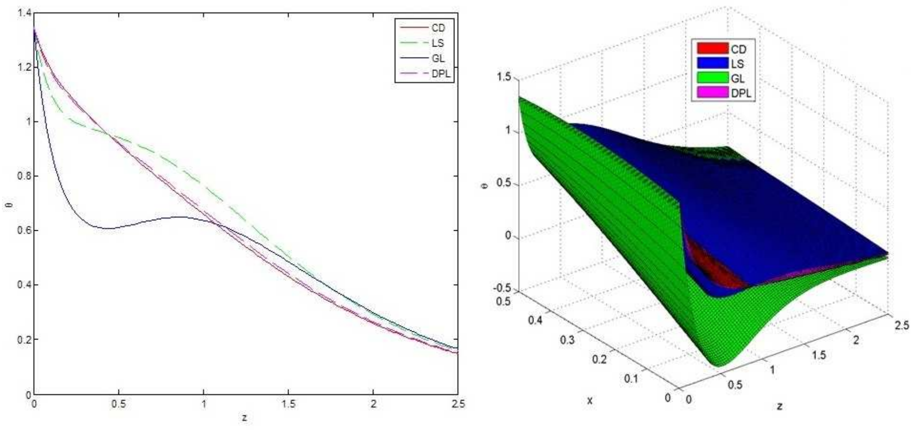


Fig. 3: The distribution of the temperature θ with electromagnetic field, rotation and gravity field

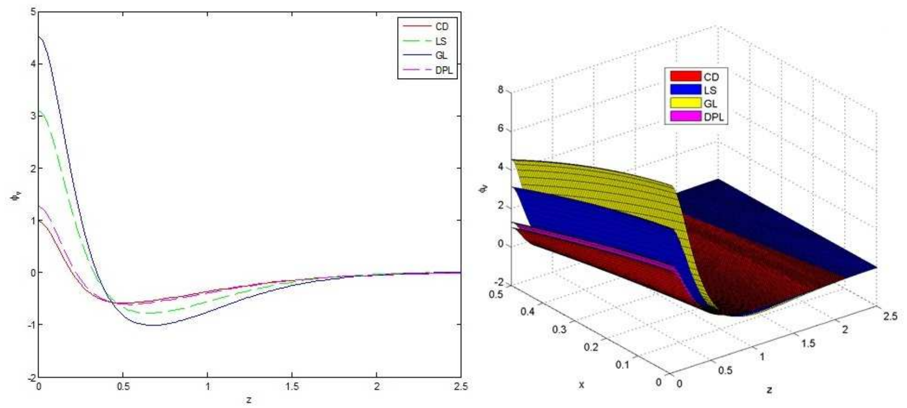


Fig. 4: The change in fraction field distribution Φ_v with electromagnetic field, rotation and gravity field

where, A, B, C, E and L to Eq. (63) are given in the Appendix A.

Equation (62) written in the following form:

$$(D^2-k_1^2)(D^2-k_2^2)(D^2-k_3^2)(D^2-k_4^2)(D^2-k_5^2)\{\Phi^*(z)\}=0 \tag{64}$$

where, k_j^2 are the roots of the characteristic equation of equation (64), which is bounded as is given by

$$\Phi^*(z)=\sum_{j=1}^{5} R_j e^{-k_j z}, \tag{65}$$

$$\Psi^*(z)=\sum_{j=1}^{5} H_{1j} R_j e^{-k_j z}, \tag{66}$$

$$\Phi_v^*(z)=\sum_{j=1}^{5} H_{2j} R_j e^{-k_j z}, \tag{67}$$

$$C^*(z)=\sum_{j=1}^{5} H_{3j} R_j e^{-k_j z}, \tag{68}$$

$$\theta^*(z)=\sum_{j=1}^{5} H_{4j} R_j e^{-k_j z}, \tag{69}$$

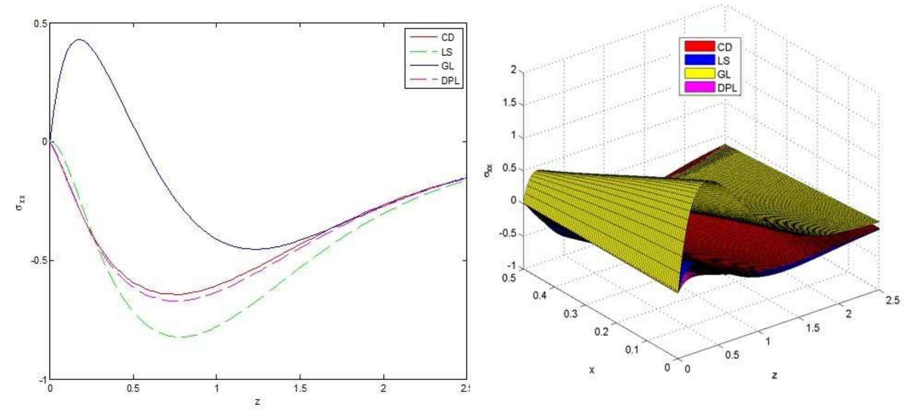


Fig. 5: Distribution of normal stress component σ_{xx} with electromagnetic field, rotation and gravity field

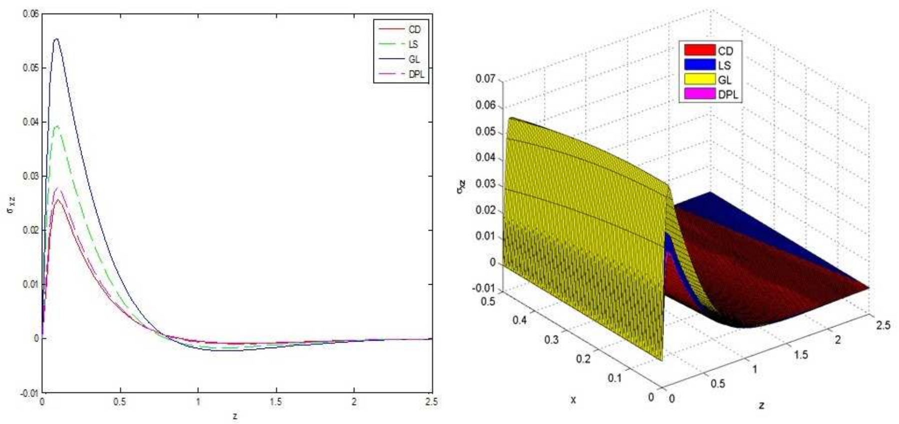


Fig. 6: Distribution of shear stress component σ_{xz} with electromagnetic field, rotation and gravity field

$$u^*(z) = \sum_{j=1}^{5} M_{1j} R_j e^{-k_j z}, \tag{70}$$

$$w^*(z) = \sum_{j=1}^{5} M_{2j} R_j e^{-k_j z}, \tag{71}$$

$$\sigma_{xx}^*(z) = \sum_{j=1}^{5} M_{3j} R_j e^{-k_j z}, \tag{72}$$

$$\sigma_{yy}^*(z) = \sum_{j=1}^{5} M_{4j} R_j e^{-k_j z}, \tag{73}$$

$$\sigma_{zz}^*(z) = \sum_{j=1}^{5} M_{5j} R_j e^{-k_j z}, \tag{74}$$

$$\sigma_{xz}^*(z) = -\sum_{j=1}^{5} M_{6j} R_j e^{-k_j z}, \tag{75}$$

where,
H_{1j}, H_{2j}, H_{3j}, H_{4j}, M_{1j}, M_{2j}, M_{3j}, M_{4j}, M_{5j} and M_{6j} in Eqs. (66)-(75) are given in the Appendix B.

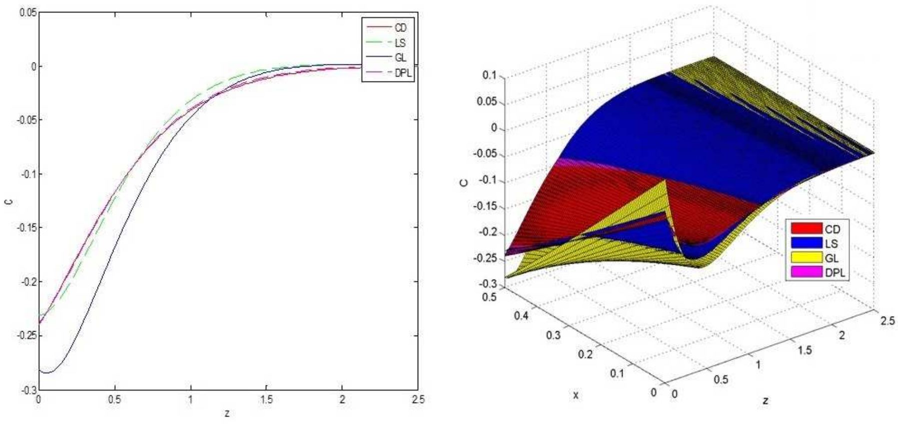


Fig. 7: Distribution of concentration of diffusion C with electromagnetic field, rotation and gravity field

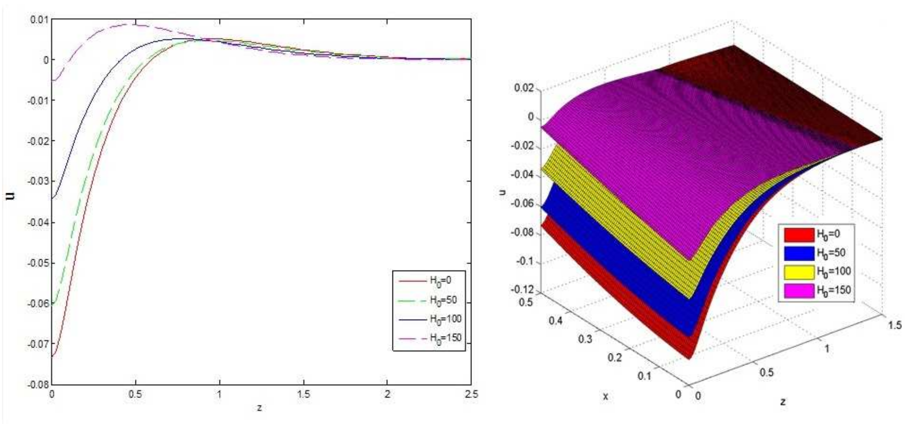


Fig. 8: Variation of the horizontal displacement u with magnetic field for Green and Lindsay's (G-L)

5.1 Applications

We consider that the boundary conditions at $z = 0$ take the form in order to determine the parameters $R_1,\ R_2,\ R_3,\ R_4$, and R_5, are

$$\begin{aligned}
&\theta(x,0,t) = f(x,0,t) = f^* e^{(\omega t + iax)},\\
&[\sigma_{xx} + \tau_{xx}](x,0,t) = [\sigma_{xz} + \tau_{xz}](x,0,t) = 0,\\
&\frac{\partial C}{\partial z} = 0,\\
&\frac{\partial \Phi_v}{\partial z} = 0
\end{aligned} \tag{76}$$

where, $f(x,t)$ an arbitrary is a function of x,t and f^* is a constant.

Using the expressions of the variables considered into the above boundary conditions (76), we can obtain the following equations satisfied by the parameters:

$$\sum_{j=1}^{5} H_{4j} R_j = f^* \tag{77}$$

$$\sum_{j=1}^{5} (M_{3j} + R_H \Gamma_1) R_j = 0 \tag{78}$$

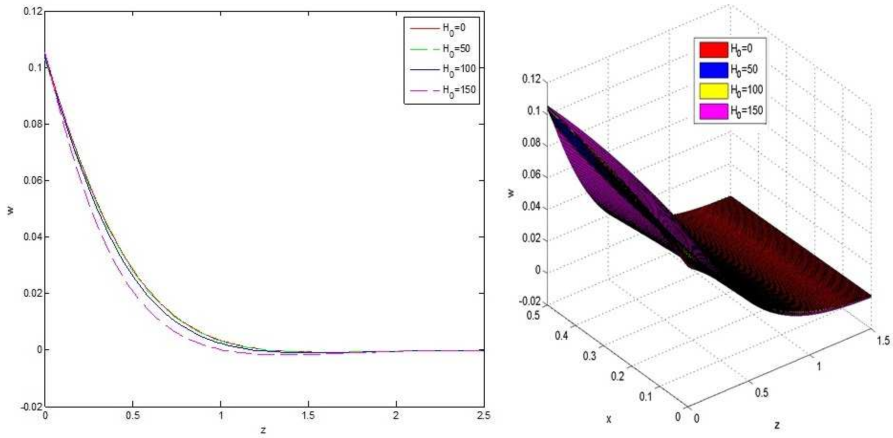


Fig. 9: Variation of the vertical displacement w with magnetic field for Green and Lindsay's (G-L)

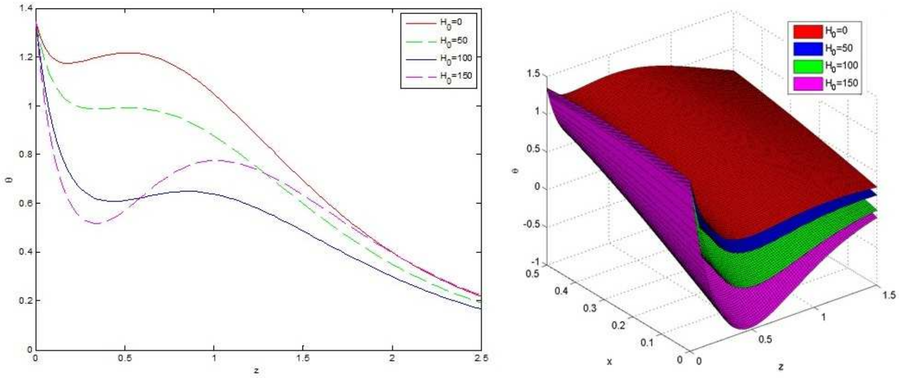


Fig. 10: Variation of the temperature θ with magnetic field for Green and Lindsay's (G-L)

$$\sum_{j=1}^{5} M_{6j}R_j = 0 \tag{79}$$

$$\sum_{j=1}^{5} K_j H_{2j} R_j = 0 \tag{80}$$

$$\sum_{j=1}^{5} K_j H_{3j} R_j = 0 \tag{81}$$

Solving the above system of equations in (77)-(81) using the inverse of matrix method, we get the parameters $(R_i,\ j = 1,2...,5)$.

We obtain the expressions of the displacement components, force stress, temperature distribution, volume fraction field and concentration of the diffusion.

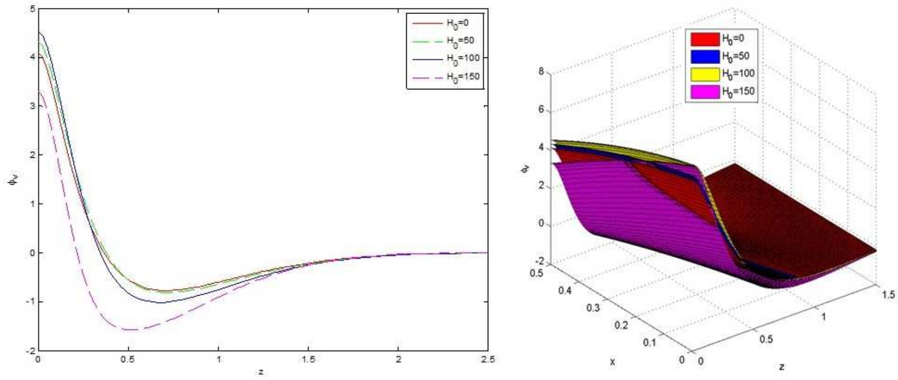


Fig. 11: Variation of fraction field distribution Φ_v with magnetic field for Green and Lindsay's (G-L)

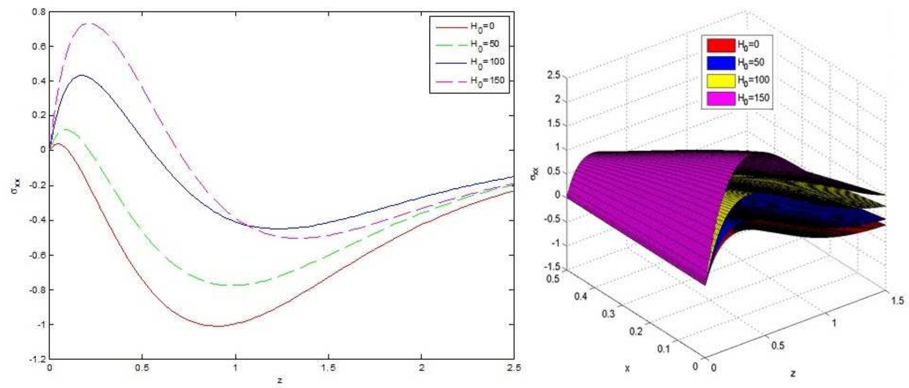


Fig. 12: Distribution of normal stress component σ_{xx} with magnetic field for Green and Lindsay's (G-L)

6 Numerical results and discussion

We take the values of parameters for copper material, the physical data given below (Nowacki [11]):

$$\lambda = 7.76\times 10^{10}\,N.m^{-2}, \mu = 3.86\times 10^{10}\,Kg.m^{-1}.S^{-2},$$
$$C_E = 383.1J.Kg^{-1}.K^{-1}, K = 386\,W.m^{-1}.K^{-1},$$
$$\alpha_t = 1.78\times 10^{-5}K^{-1}, \rho = 8954\,Kg.m^{-3},\ T_0 = 293^{o}k,$$
$$f^* = 1, \omega = \omega_0 + i\xi,\quad \omega_0 = 3.5,\ \ \xi = -4.5,$$
$$a = 1.2,\ \ \tau_q = 0.1,\ \ \tau_\Theta = 0.08,\ \tau_1 = 0.6,\ \tau^1 = 0.1,$$
$$t = 0.09, x = 0.5, 0 \le x \le 2.5.$$

For voids parameters are
$b = 1.13849\times 10^6,\quad \omega_0 = 0.078\times 10^{-3},\quad \varkappa = 1.756\times 10^{-15}, \alpha = 3.688\times 10^{-5},\ \ m = 2\times 10^6,\ \ \zeta = 1.475\times 10^{10}$.

For diffusion parameters are
$b_c = 0.9\times 10^6,\ \ \alpha_c = 1.2\times 10^4,\ \ \tau_P = 0.3,\quad \tau_\eta = 0.09,\quad \alpha_c = 1.98, \times 10^{-4},\quad d = 0.85, \times 10^{-8}, b_2^* = 2.9, \times 10^{12}$.

A Matlab program is used to make the calculations.

Fig. 1 shows the variation of the horizontal displacement distribution with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL)

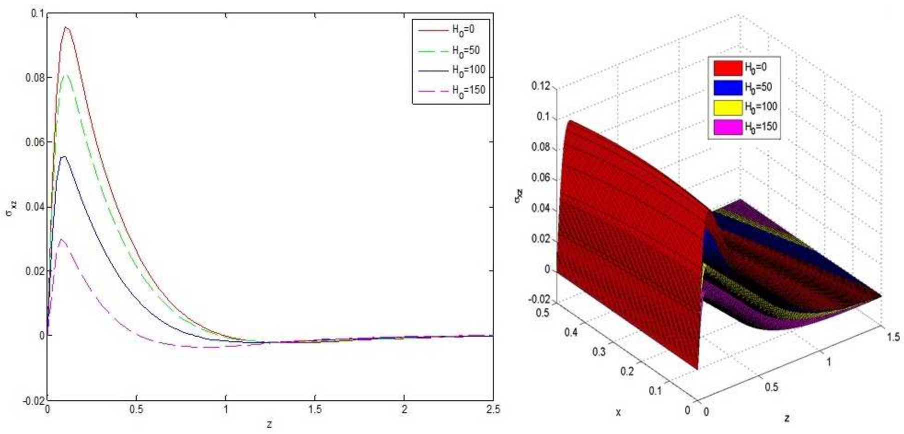


Fig. 13: Distribution of shear stress component σ_{xz} with magnetic field for Green and Lindsay's (G-L)

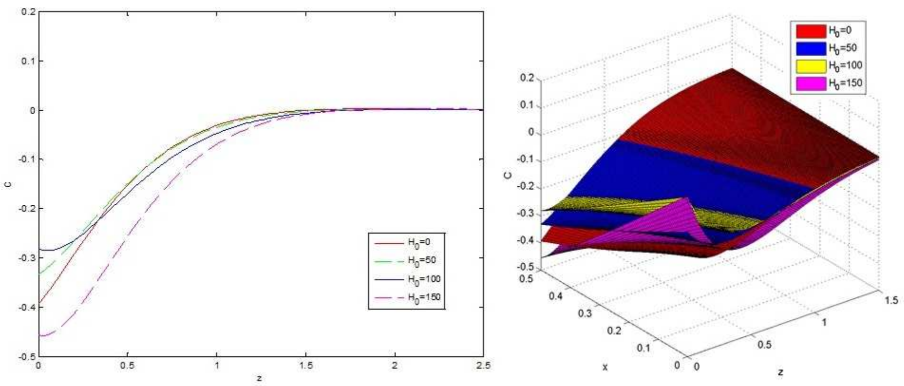


Fig. 14: Distribution of concentration of diffusion C with magnetic field for Green and Lindsay's (G-L)

models. It is observed that the horizontal displacement distribution u in (CD) greater than in (DPL), while in (DPL) greater than in (LS), as well as in (LS) greater than in (GL) in the interval [0,0.4], while in the interval [0.4,1.6] the horizontal displacement distribution in (GL) greater than in (LS), while in (LS) greater than in (DPL), as well as in (DPL) greater than in (CT), as well as it coincides in the interval [1.6,2.5] with electromagnetic field, rotation and gravity field. Fig. 2 displays the variation of the axial displacement distribution w with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the vertical displacement distribution in (LS) greater than in (DPL), while in (DPL) greater than in (CT), as well in greater than in (CT) in the interval [0,0.7], while in the interval [0.7,1.8], the vertical displacement distribution in (CT) greater than in (DPL), while in (DPL) greater than in (LS), as well in (LS) greater than in (GL), as well it coincides in the interval [1.8,2.5] with electromagnetic field, rotation and gravity field. Fig. 3 shows the variation of the distribution of the temperature θ with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the distribution of the temperature in (CT) greater than in (DPL), while in

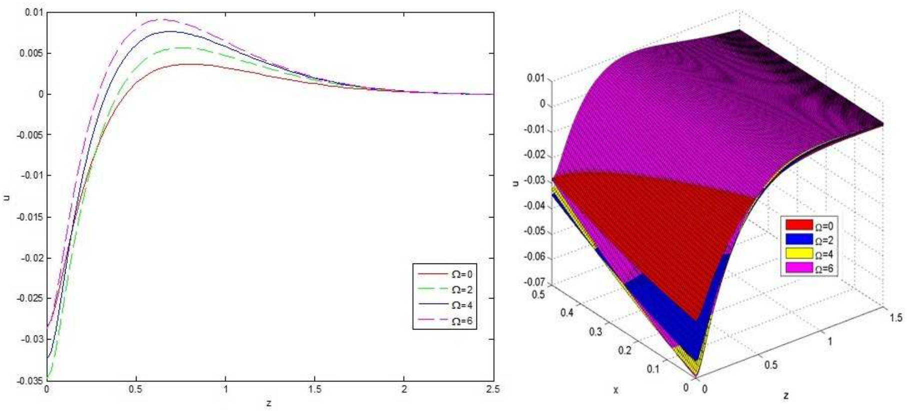


Fig. 15: Variations of the horizontal displacement u with rotation for Green and Lindsay's (G-L)

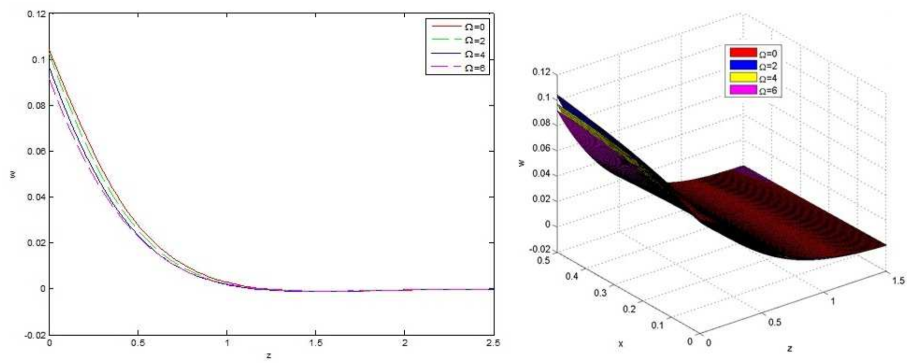


Fig. 16: Variations of the vertical displacement w with rotation for Green and Lindsay's (G-L)

(DPL) greater than in (LS), as well in (LS) greater than in (GL) in the interval [0,0.5], while in the interval [0.5,1.5] the distribution of the temperature in (LS) greater than in (GL), while in (GL) greater than in (DPL), as well as in (DPL) greater than in (CT), as well in the interval [1.5,2.5] it coincides in the theories (LS) and (GL), while it coincides in the theories (CT) and (DPL) with electromagnetic field, rotation, gravity field. Fig. 4shows the variation of the change in fraction field distribution Φ_v with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the change in fraction field distribution in (GL) greater than in (LS), while in (LS) greater than in (DPL), as well as in (DPL) greater than in (CT) in the interval [0,0.4], while in the interval [0.4,2] the change in fraction field distribution in (CT) greater than in (DPL), while in (DPL) greater than in (LS), as well as in (LS) greater than in (GL), as well it coincides in the interval [2,2.5] with electromagnetic field, rotation, gravity field. Fig. 5appears the variation of the distribution of normal stress component with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the distribution of stress component in (GL) greater than in (CT), while in (CT) greater than in (DPL), as well as in

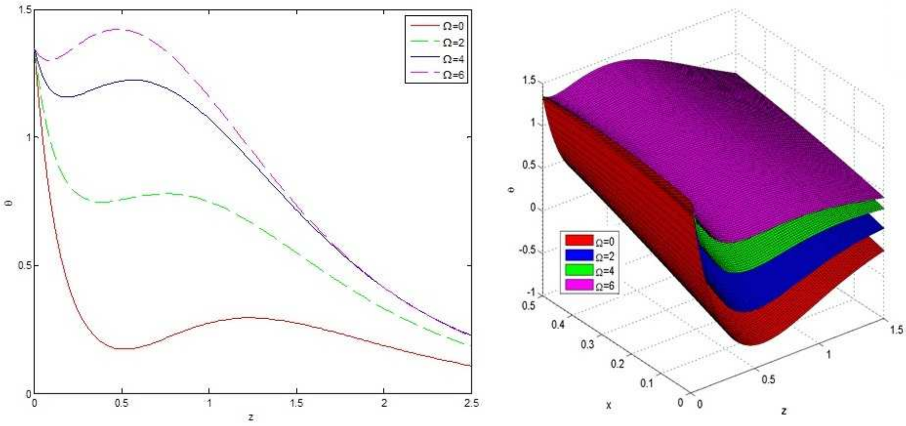


Fig. 17: Variations of the temperature θ with rotation for Green and Lindsay's (G-L)

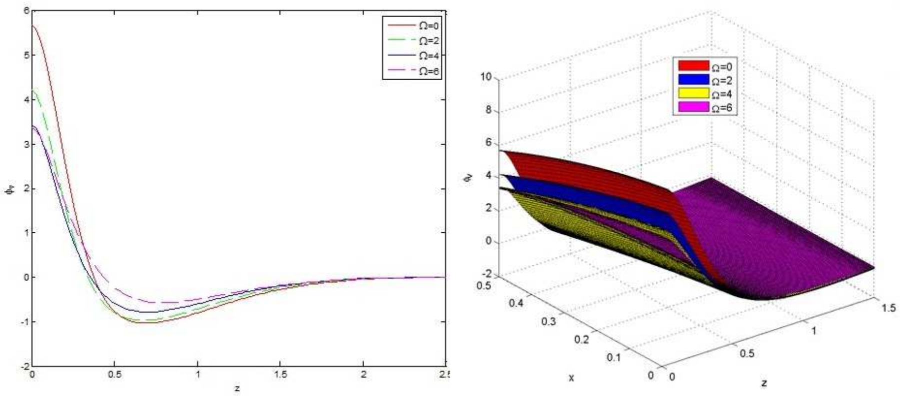


Fig. 18: Variation of fraction field distribution Φ_v with rotation for Green and Lindsay's (G-L)

(DPL) greater than in (LS) in the interval [0,0.4], while in the interval [0.4,2.5] the distribution of stress component in (GL) greater than in (LS), while in (LS) greater than in (CT), as well as in (CT) greater than in (DPL).

Fig. 6 displays the variation of the distribution of tangential stress component σ_{xx} with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the distribution of stress component in (GL) greater than in (LS), while in (LS) greater than in (DPL), as well as in (DPL) greater than in (CT) in the interval [0,0.7], while in the interval [0.7,2.5] the distribution of stress component in (CT) greater than in (DPL), while in (DPL) greater than in (LS), as well as in (LS) greater than in (GL), as well it coincides in the interval [2,2.5] with electromagnetic field, rotation and gravity field. Fig. 7shows the variation of the distribution of concentration of diffusion C with respect to the axial z for different theories of the classical (CT), Lord Shulman (LS), Green Lindsay (GL) and Dual-Phase-Lag (DPL) models. It is observed that the distribution of concentration of diffusion in (CT) greater than in (DPL),

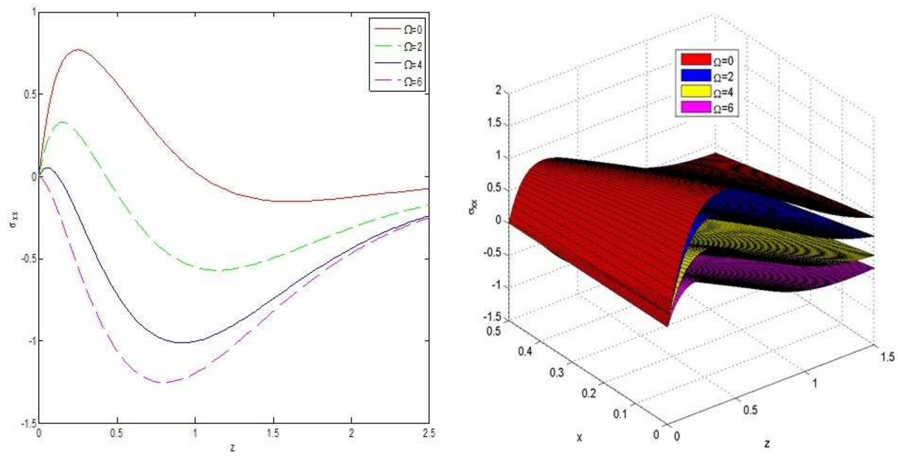


Fig. 19: stress component $\sigma_x x$ with rotation for Green and Lindsay's (G-L)

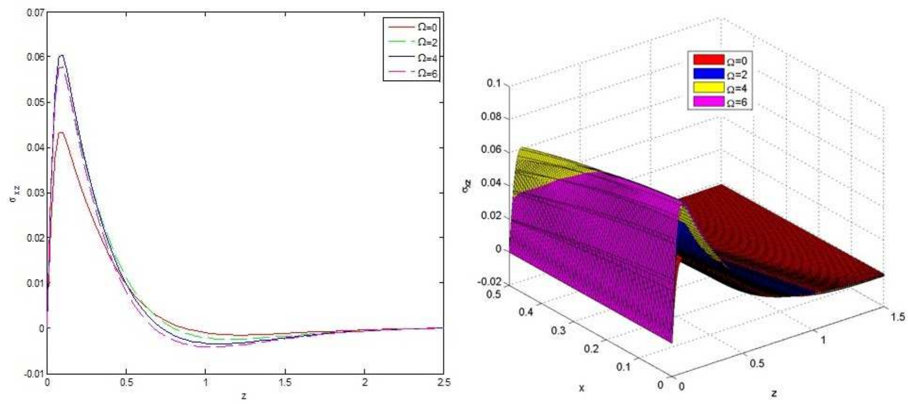


Fig. 20: Variations of stress component $\sigma_x z$ with rotation for Green and Lindsay's (G-L)

while in (DPL) greater than in (LS), as well as in (LS) greater than in (GL) in the interval [0,0.5], while in the interval [0.5,2.5], the distribution of concentration of diffusion in (LS) greater than in (CT), while in (CT) greater than in (DPL), as well as in (DPL) greater than in (GL), as well increases with increasing of axial z. Fig. 8illustrates the horizontal displacement u with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0.

In Fig. 8, an overview shows that the horizontal displacement always increases with the increase of axial z, while increases with increasing of magnetic field in the interval [0,0.8], as well it decreases with increasing of magnetic field in the interval [0.8,2] and it coincides in the interval [2,2.5]. The horizontal displacement has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,2]. From Fig. 9, it appears that the variation of the axial displacement w with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The axial displacement w decreases with increasing of magnetic field and axial z, while it coincides in the interval [2,2.5]. Fig. 10 shows that the variation of the temperature θ with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The temperature increases with the decreasing of magnetic field, while it decreases with the

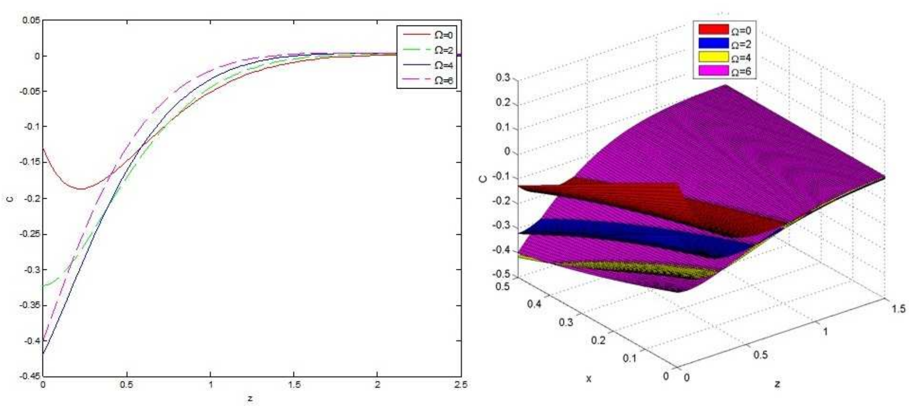


Fig. 21: Variations of concentration of diffusion C with rotation for Green and Lindsay's (G-L)

increasing of axial z, as well the temperature has an oscillatory behavior for thermoelastic diffusion and voids in the whole range of the axial z. Fig. 11shows that the variation of the fraction field distribution Φ_v with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The fraction field distribution has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,1.5], while it coincides in the interval [1.5,2.5]. Fig. 12displays that the variation of the normal stress component σ_{xx} with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The stress normal component increases with increasing of magnetic field, while it has an oscillatory behavior for thermoelastic diffusion and voids in the whole range of the axial z. Fig. 13shows that the variation of the tangential stress component with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The tangential stress component coincides in the interval [1.5,2.5], while it has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,1.5]. Fig. 14clears that the variation of the concentration of diffusion C with respect to z-axis in Green and Lindsay's theory for different values of magnetic field H_0. The concentration of diffusion has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,1.5] and it coincides in the interval [1.5,2.5].

Fig. 15 illustrates the horizontal displacement u with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. In Fig. 15, an overview shows that the horizontal displacement always increases with the increase of rotation in the interval [0.2,1.7], while it coincides in the interval [1.7,2.5]. The horizontal displacement has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,1.7]. Fig. 16 shows that the variation of the axial displacement w with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The axial displacement decreases with increasing of rotation in the interval [0,1.3] and axial z, while it coincides in the interval [2,2.5]. Fig. 17shows that the variation of the temperature θ with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The temperature increases with the decreasing of rotation, while the temperature has an oscillatory behavior for thermoelastic diffusion and voids in the whole range of the axial z. Fig. 18shows that the variation of the fraction field distribution Φ_v with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The fraction field distribution has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,1.7], while it coincides in the interval [1.7,2.5]. Fig. 19 shows that the variation of the normal stress component σ_{xx} with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The stress normal component decreases with increasing of rotation, while it has an oscillatory behavior for thermoelastic diffusion and voids in the whole range of the axial . Fig. 20 shows that the variation of the tangential stress component with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The tangential stress increases with increasing of rotation in the interval [0,0.5], while it coincides in the interval [1.8,2.5], as well as it decreases in the interval and tangential stress component has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0.5,1.8]. Fig. 21 shows that the variation of the concentration of diffusion C with respect to z-axis in Green and Lindsay's theory for different values of rotation Ω. The concentration of diffusion has an oscillatory behavior for thermoelastic diffusion and voids in the interval [0,2] and it coincides in the interval [2,2.5].

7 Conclusion

In this paper, we observed from graphical results that, effect in electromagnetic field, gravity field and rotation with diffusion and voids generalized thermoelastic half-space in the context of Classical and Dynamical (CT), Green and Lindsay's (G-L), Lord-Shulman (L-S) and the dual-phase-lag (DPL) theories play important role in thermoelasticity field.

The analysis of graphs permits us some concluding remarks:

1. The medium deforms due to the application of rotation with magnetic field which indicates the magnetothermoelastic coupled effects with vacuum on physical quantities.

2. The rotation, electric field, magnetic field and gravity field play a significant role in the distribution of all the physical quantities. The physical quantities vary (increase or decrease) as rotation, gravity field increase. Presence of rotation and gravity field restrict the quantities to oscillate.

3. The displacement components and stress components show an oscillatory nature with the decreasing of rotation, magnetic field and gravity field. These trends obey elastic and thermoelastic properties of a solid under investigation.

4. The temperature has a significant effect on the resulting quantities. The theory of Green and Lindsay of magneto-thermoelasticity describes the behavior of the particles of elastic body more real than the theory of classical thermoelasticity.

5. The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric solids. The results presented in this paper should prove useful for researchers in material science, designers of new materials, physicists as well as for those working on the development of magneto-thermoelasticity and in practical situations as in geophysics, optics, acoustics, geomagnetic and oil prospecting etc. The used methods in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.

Applications: The results obtained in this paper indicate to measure sonic vibrations on urban buildings due to aircraft traffic, in building stone quarrying or in mining operations for an estimation of vibration influences on the mine or other nearby residential buildings.

References

[1] J. Dewey and P. Byerly, The early history of seismometry (to 1900), Bulletin of the Seismological Society of America , Vol. 59, pp. 183-227 (1969).

[2] D.C. Agnew, History of Seismology; in, International handbook of earthquake and engineering seismology, Part A, W. H. Lee, H. Kanamori, P.C. Jennings, and C. Kisslinger, eds.: San Diego, California, Academic Press, pp. 3-11 (2002).

[3] M.A. Biot, Thermoelasticity and irreversible thermodynamics, Journal of Applied Physics, Vol. 27, pp. 240-253 (1956).

[4] H.W. Lord and Y. Shulman, A generalized dynamical theory of thermoelasticity, Journal of the Mechanics and Physics of Solids, Vol. 15, pp. 299-309 (1967).

[5] S.C. Cowin and J.W. Nunziato, Linear elastic materials with voids, Journal of Elasticity, Vol. 13, pp. 125-147 (1983).

[6] M. Aouadi, Generalized theory of thermoelasic diffusion for an anisotropic media, Journal of Thermal Stresses, Vol. 31, pp. 270-285 (2008).

[7] M. Aouadi, A problem for an infinite elastic body with a spherical Cavity in the theory of generalized thermoelastic diffusion, Int. J. Solids and Structures, Vol. 44, pp. 5711-5722 (2007).

[8] M. Aouadi, Uniqueness and reciprocity theorem in the theory of generalized thermoelasic diffusion, Journal of Thermal Stresses, Vol. 30, pp. 665-678 (2007).

[9] B. Singh, Reflection of SV waves from the free surface of an elastic solid in generalized thermoelastic diffusion, Journal of Sound and Vibration, Vol. 291, pp. 764-778 (2006).

[10] B. Singh, Reflection of P and SV waves from free surface of an elastic soild with generalized thermodiffusion, Journal of Earth System and Science, Vol. 114, pp. 764-778 (2006).

[11] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids I, Bull. Acad. Pol. Sci. Ser. Sci. Tech., Vol. 22, pp. 55-64 (1974).

[12] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids II, Bull. Acad. Pol. Sci. Ser. Sci. Tech., Vol. 22, pp. 129-135 (1974).

[13] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids III, Bull. Acad. Pol. Sci. Ser. Sci. Tech., Vol. 22, pp. 275-266 (1974).

[14] Z.S. Olesiak and Y.A. Pyryev, A coupled quasi-Stationary problem of thermodifusion for an elastic cylinder, International Journal of Engineering Science, Vol. 33, pp. 773-780 (1995).

[15] H.H .Sherief and H. Saleh, A half-space problem in the theory of generalized thermoelastic diffusion, International Journal of Solids and Structures, Vol. 42, pp. 4484-4493 (2005).

[16] R.N. Ram Sharma and R. Kumar, Thermomechanical response of generalized thermoelastic diffusion with one relaxation time due to time harmonic sources, International Journal of Thermal Science, Vol. 47, pp. 315-323 (2008).

[17] F.S. Bayones, The influence of diffusion on generalized magneto-thermo-viscoelastic problem of a homogenyous isotropic material, Advanced in Theoretical and Applied Mechanics, Vol. 5, pp. 69-92 (2012).

[18] S.M. Abo-Dahab and B. Singh, ”Influences of magnetic field on wave propagation in generalized

thermoelastic solid with diffusion, Archive of Mechanics, Vol. 61, pp. 121-136 (2009).
[19] R. Xia, X.Tian and Y. Shen, The influence of diffusion on generalized thrmoelastic problems of infinite body with a cylindrical cavity, International Journal of Engineering Science, Vol. 47, pp. 669-679 (2009).
[20] M.N.M. Allam, S.Z. Rida, S.M. Abo-Dahab, R.A. Mohamed and A.A. Kilany, GL model on reflection of P and SV waves from the free surface of thermo-elastic diffusion solid under influence of the electromagnetic field and initial stress, Journal of Thermal Stresses, Vol. 37, pp. 471-487 (2014).
[21] A.E. Abouelregal and S.M. Abo-Dahab, Dual-phase-lag diffusion model for Thomson's phenomenon on electromagneto-thermoelastic an infinitely solid cylinder, Journal of Computational and Theoretical Nanoscience, Vol. 11, pp. 1-9 (2014).
[22] S.M. Abo-Dahab, S-waves propagation in a non-homogeneous anisotropic incompressible medium under influences of gravity field, initial stress, electromagnetic field and rotation, Applied Mathematical and Information Sciences, Vol. 1, pp. 363-376 (2016).
[23] S.R. Kumar and R. Kumar, Wave propagation and fundamental solution of initially stressed thermoelastic diffusion with voids, Journal of Solid Mechanics, Vol. 3, pp. 298-314 (2011).
[24] A.M. Abd-Alla, A.N. Abd-Alla and N.A. Zeidan, Thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder, Journal of Thermal Stresses, 23, 313-428 (2000).
[25] A.M. Abd-Alla, and S.M. Abo-Dahab, Effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity, Journal of Thermal Stresses, Vol. 35, pp. 892-912 (2012).
[26] H.M. Youssef and A.A. El-Bary, Thermoelastic Material Response Due to Laser Pulse Heating in Context of Four Theorems of Thermoelasticity, Journal of Thermal Stresses, Vol. 37, pp. 1379-1389 (2014).
[27] R. Kumar and V. Gupta, Wave Propagation at the Boundary Surface of Inviscid Fluid Half-Space and Thermoelastic Diffusion Solid Half-Space with Dual-Phase-Lag Models, Journal of Solids Mechanics, Vol. 7, pp. 312-32 (2015).
[28] R. Kumar and V. Gupta, Dual-Phase-Lag Models of wave propagation at the interface between elastic and thermoelastic diffusion media, Journal of Engineering Physics and Thermophysics, Vol. 88, pp. 247-259 (2015),.
[29] A. Sur and M .Kanoria, Three-phase-lag elasto-thermodiffusive response in an elastic solid under hydrostatic pressure, International Journal of Advanced Applied Mathematics and Mechanics, Vol. 3, pp. 121-137 (2015).
[30] R. Kumar, R.L. Singh Reen and S.K. Garg, Axi-symmetric propagation in a thermoelastic diffusion with phase lags, American Journal of Sciences and Technology, Vol. 3, pp. 82-96 (2016).
[31] A.E. Abouelregal, A problem of a semi-infinite medium subjected to exponential heating using a dual-phase-lag Thermoelastic Model, AppliedMathematics, Vol. 2, pp. 619-624 (2011).
[32] R. Kumar and T. Kansal, Propagation of plane waves and fundamental solution in the theories of thermoelastic diffusive materials with voids, International Journal of Applied Mathematics and Mechanics, Vol. 8, pp. 84-103 (2012).

8 Appendices

8.1 Appendix A

$$
\begin{aligned}
A^* = \Lambda_{11}\,(&-\Lambda_3-\Lambda_4a_{12}-\Lambda_7a_{12}+\Lambda_1a_8a_{12}+\Lambda_3a_8a_{11}+\Lambda_7\Lambda_{11}a_8a_{11}a_{12}+a_1a_9a_{12}+\Lambda_5\Lambda_{10}a_6a_{12}-a_1a_8a_{12}b^*\\
&+\Lambda_5\Lambda_{10}a_8a_{12}+\Lambda_4a_8a_{11}a_{12}-2a^2a_{12})+a_{12}\left(\Lambda_9a_{11}+a_4''b^*-\Lambda_8\Lambda_{10}-a_9a_{11}a_4''-\Lambda_8\Lambda_{10}a_6a_{11}\right)
\end{aligned}
$$

$$
\begin{aligned}
B^* = \Lambda_{11}\Big(&\Lambda_3\left[\Lambda_4+\Lambda_7-a_1a_9\right]-\Lambda_6a_4a_{12}-\Lambda_1\Lambda_3a_8-2a^2a_1a_9a_{12}-\Lambda_6\Lambda_{10}a_1a_8a_{12}-\Lambda_6\Lambda_{10}a_1a_6a_{12}+2\Lambda_3a^2+\Lambda_4\Lambda_7a_{12}\\
&+2\Lambda_7a^2a_{12}-\Lambda_5a_4a_9a_{12}-\Lambda_1\Lambda_7a_8a_{12}-\Lambda_3\Lambda_7a_8a_{11}-2\Lambda_4a^2a_{12}-\Lambda_3\Lambda_4a_8a_{11}-\Lambda_2^2a_8-\Lambda_5\Lambda_7\Lambda_{10}a_6a_{12}+a^2a_{12}\\
&-\Lambda_3\Lambda_5\Lambda_{10}a_6-\Lambda_1\Lambda_4a_8a_{12}+\Lambda_4a_1a_8a_{12}b^*+\Lambda_6a_4a_8a_{11}a_{12}-\Lambda_4a_1a_9a_{12}-\Lambda_3\Lambda_5\Lambda_{10}a_8-\Lambda_7\Lambda_5\Lambda_{10}a_8a_{12}+\Lambda_5a_4a_8a_{12}b^*a_8a_{12}b^*)\\
&+\Lambda_3\Lambda_{11}a_1a_8b^*-\Lambda_5\Lambda_{10}a^2a_{12}(2\Lambda_{11}a_6+a_8)-\Lambda_4\Lambda_7a_8a_{11}a_{12}+\Lambda_{11}a^2a_1\\
&+\Lambda_{10}(\Lambda_5a_6a_{12}a_4''b^*+\Lambda_8a^2a_6a_{11}a_{12}+\Lambda_3\Lambda_8+\Lambda_6a_{12}a_4''\\
&+\Lambda_7\Lambda_8a_{12}-\Lambda_5\Lambda_9a_{12}+2\Lambda_8a^2a_{12}-\Lambda_8a_1a_9a_{12}-\Lambda_8a_1a_6a_{12}b^*\\
&+\Lambda_7\Lambda_8a_6a_{11}a_{12}+\Lambda_5a_9a_{12}a_4''+\Lambda_6a_6a_{11}a_{12}a_4''\\
&+\Lambda_1\Lambda_8a_6a_{12}+\Lambda_3\Lambda_8a_6a_{11})+a_{12}(\Lambda_9a_1b^*-\Lambda_1\Lambda_9-\Lambda_8a_4b^*+\Lambda_1a_9a_4''-\Lambda_4\Lambda_9a_{11}-\Lambda_4\Lambda_7\Lambda_9a_{11}a_4''b^*-2a^2a_4''b^*\\
&+\Lambda_4a_9a_{11}a_4''+a^2a_9a_{11}a_4''+\Lambda_8a_4a_9a_{11})-\Lambda_3\Lambda_9a_{11}+\Lambda_3a_9a_{11}a_4''b^*
\end{aligned}
$$

$$
\begin{aligned}
C^* = \Lambda_{11}\Big(&\Lambda_3\Big(\Lambda_1\Lambda_4a_8-2\Lambda_7a^2+2a^2a_1a_9-\Lambda_4a_1a_8b^*-a^4-2\Lambda_4a^2+\Lambda_6\Lambda_{10}a_1a_6+\Lambda_5\Lambda_7\Lambda_{10}a_8+\Lambda_5\Lambda_7\Lambda_{10}a_6-\Lambda_5a_4a_8b^*\\
&+2\Lambda_5\Lambda_{10}a^2a_6+\Lambda_6a_4+\Lambda_4a_1a_9+\Lambda_4\Lambda_7a_8a_{11}\\
&-\Lambda_6a_4a_8a_{11}+\Lambda_1\Lambda_7a_8+\Lambda_6\Lambda_{10}a_1a_8-a^2a_1a_8b^*+\Lambda_5a_4a_9+\Lambda_5\Lambda_{10}a^2a_8)\\
&-2\Lambda_4\Lambda_7a^2a_{12}+\Lambda_2^2\Lambda_4a_8-\Lambda_3\Lambda_4\Lambda_7-\Lambda_7a^4a_{12}+\Lambda_2^2\Lambda_6a_8-\Lambda_3\Lambda_4\Lambda_7\\
&-\Lambda_7a^4a_{12}+\Lambda_2^2\Lambda_7a_8+2\Lambda_6a^2a_4a_{12}+a^4a_1a_9a_{12}\\
&+a^4a_1a_9a_{12}+\Lambda_1\Lambda_4\Lambda_7a_8a_{12}+\Lambda_5\Lambda_{10}a^4a_6a_{12}+2\Lambda_4\Lambda_{11}a^2a_1a_9a_{12}\\
&+\Lambda_5\Lambda_7\Lambda_{10}a^2a_8a_{12}+2\Lambda_6\Lambda_{10}a^2a_1a_6a_{12}+\Lambda_6\Lambda_{10}a^2a_1a_8a_{12}\\
&-\Lambda_5a^2a_4a_8a_{12}b^*-\Lambda_1\Lambda_6a_4a_8a_{12}-\Lambda_1a^2a_9a_{12}a_4''-\Lambda_4a^2a_1a_8a_{12}b^*-\Lambda_4a^2a_{12}\\
&+2\Lambda_5a^2a_4a_9a_{12})+\Lambda_{10}(-\Lambda_3\Lambda_7\Lambda_8+\Lambda_3\Lambda_8a_1a_9\\
&+\Lambda_5\Lambda_9a^2a_{12}-\Lambda_2^2\Lambda_8a_6-2\Lambda_3\Lambda_8a^2-\Lambda_1\Lambda_8a^2a_6a_{12}-\Lambda_3\Lambda_8a^2a_6a_{11}\\
&-\Lambda_1\Lambda_7\Lambda_8a_6a_{12}-\Lambda_3\Lambda_8a^2a_6a_{11}a_{12}-\Lambda_3\Lambda_6a_4''-\Lambda_3\Lambda_5a_6a_4''b^*\\
&+\Lambda_3\Lambda_8a_1a_6b^*-\Lambda_1\Lambda_6a_6a_{12}a_4''+2\Lambda_5\Lambda_7\Lambda_{11}a^2a_6a_{12}+\Lambda_6\Lambda_9a_1a_{12}\\
&-2\Lambda_6a^2a_{12}a_4''+\Lambda_5\Lambda_7\Lambda_9a_{12}-2\Lambda_5a^2a_9a_{12}a_4''-\Lambda_8a^4a_{12}\\
&-\Lambda_3\Lambda_5a_9a_4''-2\Lambda_5a^2a_6a_{12}a_4''b^*-\Lambda_3\Lambda_7\Lambda_8a_6a_{11}\\
&-\Lambda_1\Lambda_3\Lambda_8a_6+\Lambda_3\Lambda_5\Lambda_9-\Lambda_3\Lambda_6a_6a_{11}a_4''+2\Lambda_8a^2a_1a_6a_{12}b^*+2\Lambda_8a^2a_1a_9a_{12}-\\
&\Lambda_6a^2a_6a_{11}a_{12}a_4''-2\Lambda_5a^2a_6a_{12}a_4''b^*-\Lambda_3\Lambda_7\Lambda_8a_6a_{11}-\Lambda_1\Lambda_3\Lambda_8a_6+\Lambda_3\Lambda_5\Lambda_9-\Lambda_3\Lambda_6a_6a_{11}a_4''+2\Lambda_8a^2a_1a_6a_{12}b^*+2\Lambda_8a^2a_1a_9a_{12}-\\
&\Lambda_6a^2a_6a_{11}a_{12}a_4''-2\Lambda_7\Lambda_8\Lambda_{10}a^2a_{12})+a_{12}(-\Lambda_9a^2a_1b^*+a^4a_4''b^*-\Lambda_4\Lambda_9a_1b^*-\Lambda_1\Lambda_8a_4a_9+\Lambda_1\Lambda_4\Lambda_9+2\Lambda_8a^4a_4b^*-\Lambda_5\Lambda_9a_4b^*\\
&-\Lambda_8a^4a_4a_9a_{11}-\Lambda_8a^2a_9a_{11}a_4''+\Lambda_4\Lambda_7\Lambda_9a_{11}+2\Lambda_4a^2a_4''b^*\\
&-\Lambda_6\Lambda_9a_4a_{11}-\Lambda_1\Lambda_4a_9a_4''+\Lambda_1\Lambda_7\Lambda_9)+\Lambda_3(\Lambda_8a_4b^*-\Lambda_1\Lambda_9a_4''+\Lambda_4a_4''b^*\\
&-a^2a_9a_{11}a_4''-\Lambda_9a_1b^*+2a^2a_4''b^*-\Lambda_8a_4a_9a_{11}+\Lambda_7\Lambda_9a_{11}-\Lambda_4a_9a_{11}a_4''+\Lambda_1\Lambda_9+\Lambda_4\Lambda_9a_{11})+\Lambda_2^2(-a_9a_4''+\Lambda_9)
\end{aligned}
$$

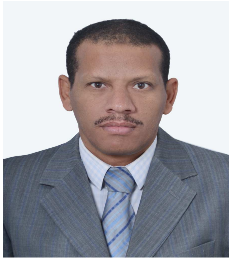
$$
\begin{aligned}
E^* =\ & \Lambda_{11}(\Lambda_3\Lambda_4a^2a_1a_8b^* - \Lambda_3a^4a_1a_9 - \Lambda_6a^4a_4a_{12} + \Lambda_3\Lambda_4\Lambda_{11}a^4 + \Lambda_3\Lambda_5a^2a_4a_8b^* + \Lambda_3\Lambda_7a^4 \\
& -\Lambda_2^2\Lambda_4\Lambda_7a_8 - \Lambda_6\Lambda_{10}a^4a_1a_6a_{12} + \Lambda_1\Lambda_3\Lambda_9a_4a_8 \\
& +\Lambda_5\Lambda_{10}a^2a_6a_{12}a_4''b^* + 2\Lambda_3\Lambda_5\Lambda_{10}a^2a_6a_4''b^* - 2\Lambda_3\Lambda_8\Lambda_{10}a^2a_1a_6b^* - \Lambda_5a^4a_4a_9a_{12} - \Lambda_1\Lambda_3\Lambda_4\Lambda_7\Lambda_8 - \Lambda_3\Lambda_5\Lambda_7\Lambda_{10}a^2(a_8-a_6) \\
& -\Lambda_5\Lambda_7\Lambda_{10}a^2a_6a_{12} - \Lambda_3\Lambda_5\Lambda_{10}a^2a_6 + \Lambda_2^2\Lambda_6a_4a_8 + \Lambda_4\Lambda_7a^2a_{12} - 2\Lambda_3\Lambda_6a^2a_4 - 2\Lambda_3\Lambda_4a^2a_1a_9 - 2\Lambda_3\Lambda_6\Lambda_{10}a^2a_1a_6 - \Lambda_3\Lambda_6\Lambda_{10}a^2a_1a_8 \\
& -2\Lambda_3\Lambda_5\Lambda_{11}a^2a_4a_9 + 2\Lambda_3\Lambda_4\Lambda_7\Lambda_{11}a^2 - \Lambda_4a^2a_1a_9a_{12}) + \Lambda_{10}(\Lambda_6a^2a_{12}a_4'' + \Lambda_3\Lambda_8a^4 \\
& +\Lambda_2^2\Lambda_8a^2a_6 - \Lambda_3\Lambda_5\Lambda_7\Lambda_9 - \Lambda_3\Lambda_6\Lambda_9a_1 + \Lambda_1\Lambda_3\Lambda_6a_6a_4'' \\
& +2\Lambda_3\Lambda_6a^2a_4'' + \Lambda_2^2\Lambda_6a_6a_4'' + 2\Lambda_3\Lambda_5a^2a_9a_4'' + \Lambda_1\Lambda_7\Lambda_8a^2a_6a_{12} \\
& +\Lambda_3\Lambda_7\Lambda_8a^2a_6a_{11} + 2\Lambda_3\Lambda_7\Lambda_8a^2 + \Lambda_1\Lambda_6a^2a_6a_{12}a_4'' + \Lambda_3\Lambda_6a^2a_6a_{11}a_4'' \\
& -\Lambda_8a^4a_1a_6a_{12}b^* + \Lambda_1\Lambda_3\Lambda_8a^2a_6 + \Lambda_2^2\Lambda_7\Lambda_8a_6 - \Lambda_3\Lambda_5\Lambda_9a^2 \\
& -\Lambda_6\Lambda_9a^2a_1a_{12} - 2\Lambda_3\Lambda_8a^2a_1a_9a_{12} + \Lambda_5a^4a_9a_{12}a_4'' + \Lambda_1\Lambda_3\Lambda_7\Lambda_8\Lambda_{10}a_6 \\
& +\Lambda_7\Lambda_8a^4a_{12} - \Lambda_5\Lambda_7\Lambda_9a^2a_{12}) + \Lambda_3(-\Lambda_1\Lambda_4\Lambda_9 - \Lambda_1\Lambda_7\Lambda_9 - 2\Lambda_8a^2a_4b^* - 2\Lambda_4a^2a_4''b^* + \Lambda_9a^2a_1b^* \\
& +\Lambda_1a^2a_9a_4'' - \Lambda_4\Lambda_7\Lambda_9a_{11} + \Lambda_4\Lambda_9a_1b^* \\
& +\Lambda_1\Lambda_8a_4a_9 + \Lambda_6\Lambda_9a_4a_{11} + \Lambda_4a^2a_9a_{11}a_4'' - a^4a_4''b^* + \Lambda_1\Lambda_4a_9a_4'' + \Lambda_5\Lambda_9a_4b^* \\
& +\Lambda_8a^2a_4a_9a_{11}) + a_{12}(-\Lambda_1\Lambda_4\Lambda_7\Lambda_9 + \Lambda_1\Lambda_6\Lambda_9a_4 - \Lambda_8a^2a_4b^* \\
& -\Lambda_4a^2a_4''b^* + \Lambda_4\Lambda_9a^2a_1b^* + \Lambda_5\Lambda_9a^2a_4b^* + \Lambda_1\Lambda_8a^2a_4a_9 \\
& +\Lambda_1\Lambda_4a^2a_9a_4'') + \Lambda_2^2(\Lambda_4a_9a_4'' + \Lambda_8a_4a_9 + a^2a_9a_4'' - \Lambda_4\Lambda_9 - \Lambda_7\Lambda_9)
\end{aligned}
$$

$$
\begin{aligned}
L^* =\ & \Lambda_3(\Lambda_{10}(\Lambda_6\Lambda_9a^2a_1 + \Lambda_8a^4a_1a_6b^* - \Lambda_1\Lambda_7\Lambda_8a^2a_6 + \Lambda_6\Lambda_{11}a^4a_1a_6 + \Lambda_5\Lambda_7\Lambda_9a^2 \\
& -\Lambda_6a^4a_4'' + \Lambda_8a^4a_1a_9 - \Lambda_1\Lambda_6a^2a_6a_4'' - \Lambda_5a^4a_6a_4''b^* \\
& +\Lambda_5\Lambda_7\Lambda_{11}a^2a_6 - \Lambda_5a^4a_9a_4'') + \Lambda_{11}a^4(\Lambda_5a_4a_9 - \Lambda_4\Lambda_7 + \Lambda_6a_4 + \Lambda_4a_1a_9) + \Lambda_8a^2a_4b^* \\
& +\Lambda_4a^4a_4''b^* - \Lambda_1\Lambda_4a^2a_9a_4'' - \Lambda_7\Lambda_8\Lambda_{10}a^4 \\
& -\Lambda_5\Lambda_9a^2a_4b^* - \Lambda_1\Lambda_8a^2a_4a_9 + \Lambda_1\Lambda_4\Lambda_7\Lambda_9 - \Lambda_1\Lambda_6\Lambda_9a_4 - \Lambda_4\Lambda_9a^2a_1b^*) + \Lambda_2^2(-\Lambda_6\Lambda_9a_4 + \Lambda_4\Lambda_7\Lambda_9 - \Lambda_8a^2a_4a_9 - \Lambda_6\Lambda_{10}a^2a_6a_4'' \\
& -\Lambda_7\Lambda_8\Lambda_{10}a^2a_6 - \Lambda_4a^2a_9a_4'')
\end{aligned}
$$

$$
F = \Lambda_{11}a_{12}(1-a_8a_{11}), \qquad A = \frac{A^*}{F}, \qquad B = \frac{B^*}{F}, \qquad C = \frac{C^*}{F}, \qquad E = \frac{E^*}{F}, \qquad L = \frac{L^*}{F}
$$

8.2 Appendix B

$$
\begin{aligned}
A_1 &= \Gamma_4\Gamma_2 - \Lambda_5\Lambda_{10}\Gamma_1, \qquad A_2 = \Gamma_5a_6\Lambda_{10} - a_4\Gamma_7, \qquad B_1 = b^*\Gamma_2 + \Lambda_6\Lambda_{10}, \qquad B_2 = \Gamma_1(\Gamma_3a_6\Lambda_{10} - a_4a_9), \\
C_1 &= \Lambda_8\Lambda_{10} - \Lambda_{11}\Gamma_2, \qquad C_2 = a_4''a_6\Lambda_{10}\Gamma_1 - a_4\Gamma_6, \quad D = -\Lambda_2\Gamma_2, \quad \Gamma_1 = k_j^2 - a^2, \quad \Gamma_2 = k_j^2 - \Lambda_4, \\
\Gamma_3 &= k_j^2 - \Lambda_7, \quad \Gamma_4 = a_{11}k_j^2 - \Lambda_1, \quad \Gamma_5 = -a_1(k_j^2 - a^2), \quad \Gamma_6 = \Lambda_9 - k_j^2a_8\Lambda_{11}, \quad \Gamma_7 = k_j^2(k_j^2 - 2a^2) + a^4, \\
H_{1j} &= \frac{\Lambda_2}{\Lambda_3 - a_{12}k_j^2}, \quad H_{2j} = \frac{A_1C_2 - A_2C_1 + DC_2H_{1j}}{B_2C_1 - B_1C_2}, \quad H_{3j} = \frac{A_1B_2 - A_2B_1 + DB_2H_{1j}}{B_1C_2 - B_2C_1}, \quad H_{4j} = -\frac{\Gamma_7 + a_9\Gamma_1H_{2j} + \Gamma_6H_{2j}}{a_6\Lambda_{10}\Gamma_1}, \\
M_{1j} &= ia - k_jH_{1j}, \qquad M_{2j} = k_j + iaH_{1j}, \qquad M_{3j} = iab_0M_{1j} + b_1k_jM_{2j} - \Lambda_{10}H_{4j} - \frac{P}{\gamma T_0} + b^*H_{2j} - \Lambda_{11}H_{3j}, \\
M_{4j} &= b_1\Gamma_1 - \Lambda_{10}H_{4j} - \frac{P}{\gamma T_0} + b^*H_{2j} - \Lambda_{11}H_{3j}, \qquad M_{5j} = iab_1M_{1j} + b_0k_jM_{2j} - \Lambda_{10}H_{4j} - \frac{P}{\gamma T_0} + b^*H_{2j} - \Lambda_{11}H_{3j}, \\
M_{6j} &= (b_2+b_3)k_jM_{1j} + iaM_{2j}(b_2-b_3).
\end{aligned}
$$



El-Sayed Mohamed Abo-Dahab, Professor in Applied Mathematics (Continuum Mechanics), he Born in Egypt-Sohag-El-maragha-Ezbet Bani-Helal in 1973. He has got Master in Applied Mathematics in 2001 from SVU, Egypt. He has got PHD in 2005 from Assiut University, Egypt. In 2012 he has got Assistant Professor Degree in Applied Mathematics. In 2017 he has got Professor Degree in Applied Mathematics. He works in elasticity, thermoelasticity, fluid mechanics, fibre-reinforced, magnetic field. He is the author or co-author of over 175 scientific publications in Science, Engineering, Biology, Geology, Acoustics, Physics, Plasma, ..., etc. He is a reviewer of 89 an international Journals in Solid Mechanics and Applied Mathematics. His research papers have been cited in many articles and textbooks. He authored many books in mathematics. He obtained a lot of local and international prizes in Science and Technology.