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Bayesian Estimation of Generalized Half-logistic Distribution under Accelerated Life tests with Type-I Censoring

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Abstract: In this paper, we consider partially step-stress ALT model when the lifetime of units under normal condition follows the generalized half-logistic lifetime distribution based on Type-1 censored scheme. The likelihood functions of the parameters are derived and solved to present the Maximum Likelihood Estimators (MLEs) of the model parameters. The approximate confidence intervals are also proposed. Bayesian point and credible intervals are developed and results are discussed through two numerical examples.

Keywords: Accelerate life tests; generalized half-logistic distribution; ML estimation; Bayes estimation; MCMC.

1 Introduction

The one shape parameter generalized half-logistic distribution is presented as a special case of shape and scale parameters generalized half-logistic with distribution presented by

$$F(x)=1-2^{\alpha}\left(1+\exp\left\{\frac{x}{\lambda}\right\}\right)^{-\alpha}, \tag{1}$$

with $\lambda = 1$. The random variable X is called generalized half-logistic distributed random variable with shape parameter α, if X has the probability density function (pdf) presented by

$$f(x) = \alpha 2^{\alpha} \exp\{x\} (1+\exp\{x\})^{-(\alpha+1)},\ x > 0,\ \alpha > 0. \tag{2}$$

And the cumulative distribution function (cdf), survival function $S(t)$ and hazard rate function $h(t)$, are respectively given by

$$F(x)=1-2^{\alpha}(1+\exp\{x\})^{-\alpha}, \tag{3}$$

$$S(t) = 2^{\alpha}(1+\exp\{t\})^{-\alpha}, \tag{4}$$

and

$$h(t) = \alpha \exp\{t\}(1+\exp\{t\})^{-1}. \tag{5}$$

The half logistic distribution is considered by Balakrishnan [1] of the absolute standard logistic variate. For more detail about generalized half-logistic distribution see Balakrishnan and Hossain [2], Ramakrishnan [3] and Arora et al. [4]. Bayesian viewpoint of generalized half-logistic distribution is discussed in Kim et al. [5]. The reliability functions of the reliability functions generalized half-logistic distribution is discussed recently by Chaturvedi et all [6] and Awodutire et al. [7].

Accelerate Life Tests (ALTs) is applied in different area in life testing experiments with different type of acceleration according to [**?**]. The type in which stress is kept in a constant stress level is called constant stress ALTs but the type in which stress is keep in increasing form in time as given in [13] called progressive stress ALTs. In the final one, stress is changed for a given specified prior time or number of failures, called step stress ALTs. Different work is presented in the literature of ALTs see [14, 15] and recently see [16]. In several applications of ALTs, partially ALTs is more common in the life test experiments, in which the test is run at normal and stress conditions. The common partially ALTs is called partially constant-stress ALTs, the experiment runs simultaneously at normal and stress condition. Also, for

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partially step-stress ALTs, the experiment is run at normal condition and stress change at a prefixed time or number through the experiment.

Censoring in the life test experiments, is more common because of reducing the time and cost of the experiment. The simple and famous ones in life testing experiments are Type-I and Type-II censoring schemes. In Type-I censoring terminate the experiment is at a prior pre-fixed time point, but in Type-II censoring is at a prior pre-fixed number of failures. The general censoring scheme which allows to remove the units at any point of the test, is called progressive Type-II right censoring, for important reviews of the literature on progressive censoring, see Balakrishnan and Aggarwala [17].

Our objective in this paper is to estimate the generalized half-logistic distribution under partially step-stress ALTs in the presence of type-I censoring scheme. The model parameter and accelerated factor are estimated by the maximum likelihood and Bayes methods. Also, interval estimations are obtained with the two methods. Our results are illustrative through discussing the two numerical examples.

This paper is organized as follows, the model described is exposed in Section 2. The point and approximate interval maximum likelihood estimation are discussed in Section 3. Bayes point and credible intervals are developed in Section 4. Numerical studies of two numerical examples are presented in Section 5. Finally, some comments are exposed in Section 6.

2 Model description

Let n independent items are tested under normal condition and the prefixed times, τ^* and τ are given, where $\tau < \tau^*$. The test switches to the higher stress level at prefixed time τ, but τ^* is the final time of the test. Thus the lifetime of a test item W, exposed to different two stages, includes normal and accelerated conditions. Then, in partially step-stress ALTs the lifetime of the items is presented by

$$W = \begin{cases} T, & T < \tau \\ \tau + \rho^{-1}(T-\tau), & T > \tau, \end{cases} \quad (6)$$

where the lifetime T of the items is computed at normal condition and ρ is the accelerated factor. The pdf under partially step-stress ALTs of random variable W is presented by

$$f(w) = \begin{cases} 0, & w < 0, \\ f_1(w), & 0 < w \le \tau \\ f_2(w), & w > \tau. \end{cases} \quad (7)$$

Under consideration, the one-shape parameter for generalized half-logistic, the pdf $f_1(w)$ is presented by (2) and $f_2(w)$ is defined by

$$f_2(w) = \alpha\rho 2^{\alpha} \exp\{\tau + \rho(w-\tau)\} \times (1+\exp\{\tau+\rho(w-\tau)\})^{-(\alpha+1)}, \quad (8)$$

The cdf, $S_2(w)$, and hazard rate function $h_2(w)$, is given by

$$F_2(w) = 1 - 2^{\alpha}(1+\exp\{\tau+\rho(w-\tau)\})^{-\alpha}, \quad (9)$$

$$S_2(w) = 2^{\alpha}(1+\exp\{\tau+\rho(w-\tau)\})^{-\alpha}, \quad (10)$$

and

$$h_2(w) = \alpha\rho \frac{\exp\{\tau+\rho(w-\tau)\}}{1+\exp\{\tau+\rho(w-\tau)\}}. \quad (11)$$

Under Type-I censoring scheme and partially step-stress ALTs, the experiment is terminated at W_n when $W_n < \tau^*$ and terminated at τ^* when $W_n > \tau^*$. Hence, the random sample of the total lifetime W is defined as, $W_1 < W_2 < \ldots < W_J < \tau < W_{J+1} < \ldots < W_r$ where J is the number of items failed under normal stress level and r is the total number failed at all test.

The likelihood function of observed values $W_1 < W_2 < \ldots < W_J < \tau < W_{J+1} < \ldots < W_r$ in the case of $\tau* < W_n$, is described as

$$L(\varepsilon|\underline{w}) = C\prod_{i=1}^{J} f_1(w_i) \prod_{i=J+1}^{r} f_2(w_i)(S_2(\tau^*))^{n-r}, \quad (12)$$

where $C = \frac{n!}{(n-r)!}$, ε is the model parameters vector (α, ρ).

3 Estimations with Maximum Likelihood Method

3.1 Point estimation

From the likelihood function in (12) and lifetime data $W_1 < W_2 < \ldots < W_J < \tau < W_{J+1} < \ldots < W_r$ from generalized half-logistic distributions (2) and (8), the likelihood function $L(\alpha,\rho|\underline{w})$ without normalized constant is then given by

$$\begin{aligned} L(\alpha,\rho|\underline{w}) = \alpha^r \rho^{r-J} \exp\Bigg\{ & n\alpha\log 2 + \sum_{i=1}^{J} w_i - (\alpha+1) \\ & \times \sum_{i=1}^{J} \log(1+\exp\{w_i\}) + \sum_{i=J+1}^{r} (\tau+\rho(w_i-\tau)) \\ & - \alpha(n-r)\log(1+\exp\{\tau+\rho(\tau^*-\tau)\}) \\ & - (\alpha+1)\sum_{i=J+1}^{r} \log(1+\exp\{\tau+\rho(w_i-\tau)\}) \Bigg\}. \end{aligned} \quad (13)$$

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Hence the log-likelihood function $\ell(\alpha,\rho|\underline{w}) = \log L(\alpha,\rho|\underline{w})$ is presented by

$$\begin{aligned}\ell(\alpha,\rho|\underline{w}) =& r\log\alpha + (r-J)\log\rho + n\alpha\log 2\\ &+ \sum_{i=1}^{J} w_i - (\alpha+1)\sum_{i=1}^{J}\log\left(1+\exp\{w_i\}\right)\\ &+ \sum_{i=J+1}^{r}(\tau+\rho(w_i-\tau)) - (\alpha+1)\\ &\times \sum_{i=J+1}^{r}\log\left(1+\exp\{\tau+\rho(w_i-\tau)\}\right)\\ &- \alpha(n-r)\log\left(1+\exp\{\tau+\rho(\tau^*-\tau)\}\right).\end{aligned} \quad (14)$$

Then the likelihood equations of parameters α and ρ can be presented after taking the first partial derivatives of equation (14) and equating each to zero, as follows

$$\begin{aligned}\frac{\partial\ell(\alpha,\rho|\underline{w})}{\partial\alpha} =& \frac{r}{\alpha} + n\log 2 - \sum_{i=1}^{J}\log\left(1+\exp\{w_i\}\right)\\ &- \sum_{i=J+1}^{r}\log\left(1+\exp\{\tau+\rho(w_i-\tau)\}\right)\\ &- (n-r)\log\left(1+\exp\{\tau+\rho(\tau^*-\tau)\}\right) = 0\end{aligned} \quad (15)$$

Then

$$\begin{aligned}\alpha(\rho) = r\Bigg[&-n\log 2 + \sum_{i=1}^{J}\log\left[1+\exp\{w_i\}\right]\\ &+ (n-r)\log\left[1+\exp\{\tau+\rho(\tau^*-\tau)\}\right]\\ &+ \sum_{i=J+1}^{r}\log\left[1+\exp\{\tau+\rho(w_i-\tau)\}\right]\Bigg]^{-1},\end{aligned} \quad (16)$$

As well as

$$\begin{aligned}\frac{\partial\ell(\alpha,\rho|\underline{w})}{\partial\rho} =& \frac{(r-J)}{\rho} + \sum_{i=J+1}^{r}(w_i-\tau) - (\alpha+1)\\ &\times \sum_{i=J+1}^{r}\frac{(w_i-\tau)\exp\{\tau+\rho(w_i-\tau)\}}{1+\exp\{\tau+\rho(w_i-\tau)\}}\\ &- \frac{\alpha(n-r)(\tau^*-\tau)\exp\{\tau+\rho(\tau^*-\tau)\}}{1+\exp\{\tau+\rho(\tau^*-\tau)\}}.\end{aligned} \quad (17)$$

Which is reduced to

$$\begin{aligned}&\frac{(r-J)}{\rho} + \sum_{i=J+1}^{r}(w_i-\tau)\\ &- (\alpha+1)\sum_{i=J+1}^{r}\frac{(w_i-\tau)\exp\{\tau+\rho(w_i-\tau)\}}{1+\exp\{\tau+\rho(w_i-\tau)\}}\\ &- \frac{\alpha(n-r)(\tau^*-\tau)\exp\{\tau+\rho(\tau^*-\tau)\}}{1+\exp\{\tau+\rho(\tau^*-\tau)\}} = 0.\end{aligned} \quad (18)$$

The profile log-likelihood function can be obtained from (14) by replaced the parameter α with the value presented by (16) to obtain

$$h(\rho) = \ell(\alpha(\rho),\rho|\underline{w}), \quad (19)$$

then the initial value of parameters ρ is obtained to present the maximum likelihood estimates $\hat{\rho}$ of ρ with any iteration method such as fixed-point method or quasi-Newton Raphson. The maximum likelihood estimates $\hat{\alpha}$ of α is obtained from (16) after replace ρ with $\hat{\rho}$.

3.2 Interval estimation

The Fisher information matrix $\omega(\alpha,\rho)$ is the negative expectation of second derivatives of log the likelihood function. Practice, $\omega^{-1}(\alpha,\rho)$ is estimated by $\omega^{-1}(\hat{\alpha},\hat{\rho})$. Hence, the normal approximation is used as follows

$$(\hat{\alpha},\hat{\rho}) \to N\left((\alpha,\rho),\omega_0^{-1}(\hat{\alpha},\hat{\rho})\right), \quad (20)$$

where $\omega_0^{-1}(\alpha,\rho)$ is observed information matrix, presented by the second partial derivatives of (14) with respect to α and ρ presented by

$$\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\alpha^2} = \frac{-r}{\alpha^2}, \quad (21)$$

$$\begin{aligned}\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\alpha\partial\rho} =& \frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\rho\partial\alpha} =\\ &- \sum_{i=J+1}^{r}\frac{(w_i-\tau)\exp\{\tau+\rho(w_i-\tau)\}}{1+\exp\{\tau+\rho(w_i-\tau)\}}\\ &- \frac{(n-r)(\tau^*-\tau)\exp\{\tau+\rho(\tau^*-\tau)\}}{1+\exp\{\tau+\rho(\tau^*-\tau)\}}.\end{aligned} \quad (22)$$

$$\begin{aligned}\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\rho^2} =& \frac{-(r-J)}{\rho^2} - (\alpha+1)\\ &\times \sum_{i=J+1}^{r}\frac{(w_i-\tau)^2\exp\{\tau+\rho(w_i-\tau)\}}{(1+\exp\{\tau+\rho(w_i-\tau)\})^2}\\ &- \frac{\alpha(n-r)(\tau^*-\tau)^2\exp\{\tau+\rho(\tau^*-\tau)\}}{(1+\exp\{\tau+\rho(\tau^*-\tau)\})^2}.\end{aligned} \quad (23)$$

Then observed information matrix is presented by

$$\omega^{-1}(\hat{\alpha},\hat{\rho}) = \begin{bmatrix} -\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\alpha^2} & -\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\alpha\partial\rho}\\ -\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\rho\partial\alpha} & -\frac{\partial^2\ell(\alpha,\rho|\underline{w})}{\partial\rho^2}\end{bmatrix}^{-1}_{\hat{\alpha},\hat{\rho}}. \quad (24)$$

From the normal distribution of $(\hat{\alpha},\hat{\rho})$. Hence, the 100(1-2$\theta$)% approximate confidence intervals of α and ρ is presented by

$$\left(\hat{\alpha}\mp z_\theta\sqrt{V_{11}}\right) \text{ and } \left(\hat{\rho}\mp z_\theta\sqrt{V_{22}}\right). \quad (25)$$

Where V_{11} and V_{22} are the elements of the diagonal of $\omega^{-1}(\hat{\alpha},\hat{\rho})$ and z_θ is the percentile right-tail with probable of θ standard normal distribution.

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4 Estimations with Bayes Method

The Bayesian estimation of model parameters considers the assumption of independent gamma prior distributions of the model parameters α and ρ with prior densities functions as follows

$$\pi_1^*(\alpha) \propto \alpha^{a-1} \exp\{-b\alpha\},\ (a,\ b > 0) \tag{26}$$

and

$$\pi_2^*(\rho) \propto \rho^{c-1} \exp\{-d\rho\},\ (c,\ \mathrm{d} > 0). \tag{27}$$

Hence the joint prior density $\pi_1^*(\alpha,\rho)$ of parameters α and ρ can be written as

$$\pi_1^*(\alpha,\rho) \propto \alpha^{a-1}\rho^{c-1} \exp\{-b\alpha - d\rho\}. \tag{28}$$

From the likelihood function (13) and the joint prior density $\pi^*(\alpha,\rho)$, the joint posterior density given the data $\pi(\alpha,\rho|\underline{w})$, can be written as

$$\begin{aligned}\pi(\alpha,\rho|\underline{w}) \propto\ & \alpha^{r+a-1}\rho^{r+c-J-1} \exp\Bigg\{\sum_{i=1}^{J} w_i \\ & - (\alpha+1)\sum_{i=1}^{J} \log(1+\exp\{w_i\}) - \alpha(b - n\log 2) \\ & - d\rho - \alpha(n-r)\log(1+\exp\{\tau+\rho(\tau^*-\tau)\}) \\ & + \sum_{i=J+1}^{r} (\tau+\rho(w_i-\tau)) \\ & - (\alpha+1)\sum_{i=J+1}^{r} \log(1+\exp\{\tau+\rho(w_i-\tau)\})\Bigg\}.\end{aligned} \tag{29}$$

Different loss function can be applied, but we consider squared error loss function to present Bayes estimate of any function of parameters $g(\alpha,\rho)$, as follows

$$\hat{g}_B(\alpha,\rho) = E(g(\alpha,\rho)) = \int_\alpha\int_\rho g(\alpha,\rho)\times\pi(\alpha,\rho|\underline{w})d\alpha d\rho. \tag{30}$$

MCMC Approach

Integral (30) can not be obtained in a closed form, but different approximate methods can be used. In this paper, we adopt the important one, called the MCMC method as follows. The Bayes point and interval estimation of the model parameters with the help of MCMC method is considered for different classes of MCMC method. The Gibbs with Metropolis algorithms is more general technique called by Metropolis within-Gibbs is applied here, for more detail, see Soliman et all [18] and Abd-Elmougod et al [19].

From the joint posterior density function in (29), the conditional posterior pdf's of parameter α is Gamma(A,B) distribution with A=$r+a$ and B, presented by

$$\begin{aligned}B =\ & (b - n\log 2) + \sum_{i=1}^{J}\log(1+\exp\{w_i\}) \\ & + \sum_{i=J+1}^{r}\log(1+\exp\{\tau+\rho(w_i-\tau)\}) \\ & + (n-r)\log(1+\exp\{\tau+\rho(\tau^*-\tau)\})\end{aligned} \tag{31}$$

and the pdf's of parameter ρ is given

$$\begin{aligned}\pi_1(\rho|\alpha,\underline{w}) \propto\ & \rho^{r+c-J-1}\exp\Bigg\{\rho\sum_{i=J+1}^{r}(w_i-\tau) \\ & - d\rho - \alpha(n-r)\log(1+\exp\{\tau+\rho(\tau^*-\tau)\}) \\ & - (\alpha+1)\sum_{i=J+1}^{r}\log(1+\exp\{\tau+\rho(w_i-\tau)\})\Bigg\}\end{aligned} \tag{32}$$

The plots of (32) show that the similarity of normal distribution that is used to generate from these distributions, the MH method is used, see Metropolis et al. [20] with normal proposal distribution as the following algorithm.

MCMC algorithm (MH under Gibbs sampling):

1: With initial vector $(\alpha^0,\rho^0) = (\hat{\alpha},\hat{\rho})$, set $k = 1$.
2: Generate α^k from Gamma(A,B).
3: Generate ρ^k from (32) with MH under the $N(\rho^{k-1},\sigma)$ proposed distribution, where σ is obtained from variances-covariances matrix.
4: After obtaining the parameters vector (α^k,ρ^k), set $k = k+1$.
5: Steps from 2 to 4 is repeated N times.
6: The Bayes estimate of α under the MCMC methods is given by

$$\hat{\alpha}_B = E(\alpha|\underline{w}) = \frac{1}{N-M}\sum_{i=M+1}^{N}\alpha^i \tag{33}$$

where M is the number of iterations, we need to get into stationary distribution. The posterior variance of α is given by

$$V(\alpha|\underline{w}) = \frac{1}{N-M}\sum_{i=M+1}^{N}\left(\alpha^i - \hat{\alpha}_B\right)^2. \tag{34}$$

Also, the credible intervals of α, by ordering the value α^{M+1}, α^{M+2},. . . ,α^N as $\alpha^{(1)}$, $\alpha^{(2)}$,. . . ,$\alpha^{(N-M)}$. Then the $100(1-2\theta)\%$ symmetric credible interval is

$$\left(\alpha^{\theta(N-M)},\ \alpha^{(1-\theta)(N-M)}\right). \tag{35}$$

7: The Bayes estimate of ρ under the MCMC methods is given by

$$\hat{\rho}_B = E(\rho|\underline{w}) = \frac{1}{N-M}\sum_{i=M+1}^{N}\rho^i, \tag{36}$$

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and the posterior variance of ρ is given by

$$V(\rho|\underline{w}) = \frac{1}{N-M}\sum_{i=M+1}^{N}\left(\rho^{i}-\hat{\rho}_{B}\right)^{2}. \quad (37)$$

Also, the $100(1-2\theta)\%$ symmetric credible interval is

$$\left(\rho^{\theta(N-M)},\ \rho^{(1-\theta)(N-M)}\right) \quad (38)$$

4.1 Numerical Computations

The theoretical results presented in this paper are discussed and illustrated in this section through the two numerical examples. The quality measure of estimation method can be discussed as follows.

4.2 Example 1

An estimation procedure discussed in this paper is illustrated through the numerical example as follows. The simulated data sample is generated from generalized half-logistic distribution with parameters $(\alpha,\rho)=(0.2, 2)$, $n=30$ and independent two time $(\tau,\ \tau^{*})$=(3, 6). The simulated data is presented in Table 1 below. The point maximum likelihood estimates and related Bayes MCMC estimates are presented in Table 2. Also, the 95% approximate confidence intervals as well as credible intervals are presented in Table 2. The plan of MCMC method is described by running the chain for 11, 000 times and discarding the first 1000 values as burn-in. Fig. (1-4) show simulation number of α and ρ generated by MCMC method and the corresponding histogram. We observe that the credible intervals are narrower than the approximate intervals and always include the population parameter values.

4.3 Example 2

The simulated data sample is generated from generalized half-logistic distribution with parameters $(\alpha,\rho)=(1.5,,1.5)$, $n=30$ and independent two time $(\tau,\ \tau^{*})$=(1.0, 1.5). The simulated data are presented in Table 3 below. The results as the last example is presented in Table 4. Also, Fig. (5-8) show simulation number of α and ρ generated by MCMC method and the corresponding histogram.

5 Perspective

In this section, we give some comments observed from the two discussed examples about the type-I censoring generalized half-logistic data under the step-stress ALTs model. The MLEs and asymptotic confidence intervals are compared with the Bayes estimators and credible intervals.

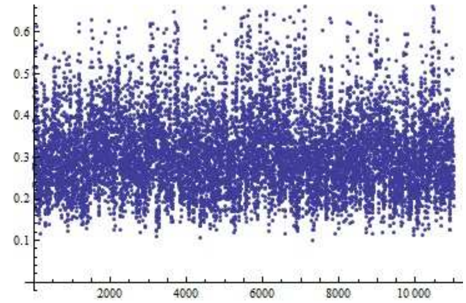

Fig. 1: Simulation number of α obtained by MCMC method.

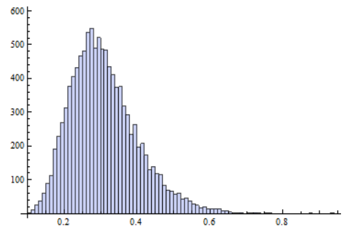

Fig. 2: Histogram of α obtained by MCMC method.

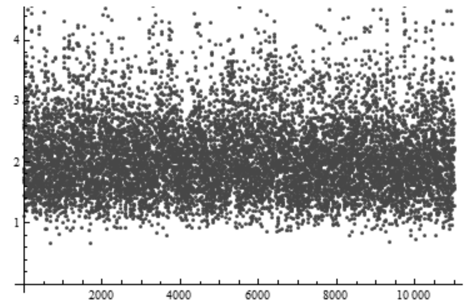

Fig. 3: Simulation number of ρ obtained by MCMC method.

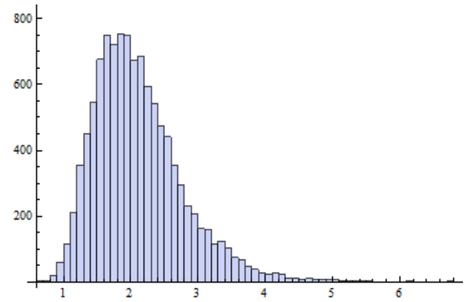

Fig. 4: Histogram of ρ obtained by MCMC method.

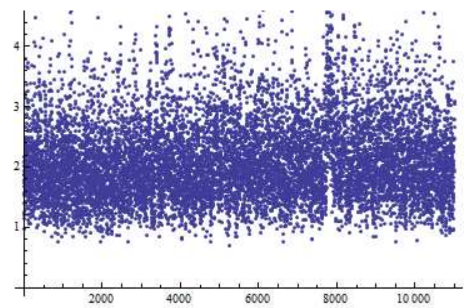

Fig. 5: Simulation number of α obtained by MCMC method.

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Table 1: The simulated samples (Example 1).

0.2475	0.3226	0.3538	0.4732	0.6495	1.0717	1.6324
2.8161	2.8343	2.9326	3.03	3.2768	3.2804	3.6353
3.7106	3.8711,	4.1109	4.2032	4.2514	4.2693	4.8369
4.938	5.039	5.5608				

Table 2: MLEs and Bayes estimate with 95% approximate and credibale confidence intervales

Pa.s	$(.)_{ML}$	$(.)_{Baye}$	95% MLE	Length	95%Bayes	Length
$\alpha =$	0.176401	0.3118	(0.0672038, ,0.285598)	0.2184	(0.1652, 0.5330)	0.3678
$\rho =$	2.35727	2.1211	(0.464876, ,4.24966)	3.7848	(1.1366, 3.7244)	2.5878

Table 3: The simulated samples (Example 2).

0.0183	0.0222	0.0474	0.1524	0.1567	0.1924	0.3399
0.3652	0.3740	0.4085	0.4675	0.4943	0.5752	0.5941
0.7097	0.7416	0.9177	0.9595	1.0667	1.0863	1.2491
1.2719	1.3197	1.327	1.3389			

Table 4: MLEs and Bayes estimate with 95% approximate and credibale confidence intervales

Pa.s	$(.)_{ML}$	$(.)_{Baye}$	95% MLE	Length	95%Bayes	Length
$\alpha =$	1.5245	2.1366	(0.818694, 2.23046)	1.4118	(1.1164, 3.7669)	2.6505
$\rho =$	1.4201	2.1048	(0.248406, 2.59174)	2.3433	(1.1161, 3.6782)	2.5621

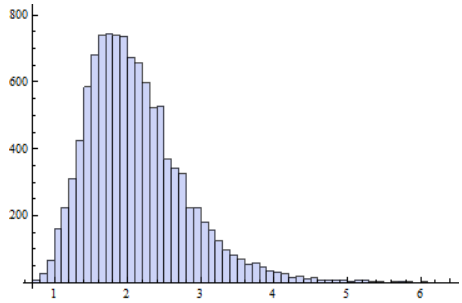

Fig. 6: Histogram of α obtained by MCMC method.

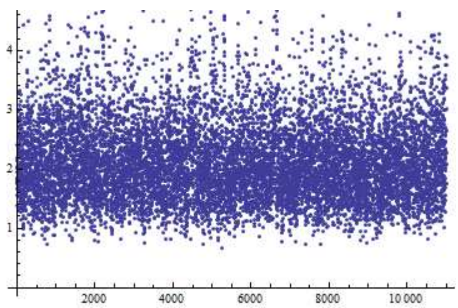

Fig. 7: Simulation number of ρ obtained by MCMC method.

1. The credible intervals are narrower than the approximate intervals and always include the population parameter values.
2. For different sample sizes, different schemes and different censoring parameters (τ, τ^{*}) results are more acceptable .

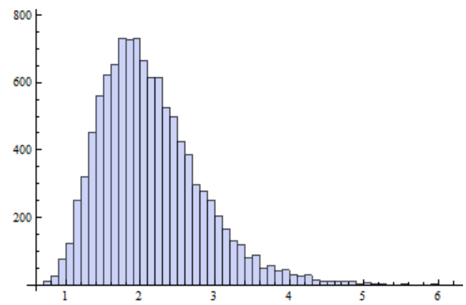

Fig. 8: Histogram of ρ obtained by MCMC method.

3. Different figures in the approximation with the help of MCMC method are expressed the normal distribution of the simulated estimates around the mean estimate shows the convergence in estimations procedure .

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