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Bipolar Intuitionistic Fuzzy Graphs and its Matrices

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Abstract: In this paper the notions of subdivisions intuitionistic bipolar fuzzy graph, matrix of bipolar intuitionistic fuzzy graph, and line intuitionistic bipolar fuzzy graph are introduced and studied.

Keywords: Intuitionistic bipolar fuzzy set, Intuitionistic bipolar fuzzy graph, Incidence intuitionistic bipolar fuzzy matrix, Line intuitionistic bipolar fuzzy graph.

1 Introduction

Zadeh [1] in 1965 introduced the concept of a fuzzy subset. Fuzzy sets theory becomes a vigorous research area in different disciplines including mathematics, engineering, computer science, signal processing and graph theory. Atanassove [2] studied intuitionistic fuzzy set concept as a generalization of fuzzy sets. Various authors have studied and concluded some results for intuitionistic fuzzy graph [3-6]. In 1994, Zhang [7] introduced bipolar fuzzy set's notion as a generalization of fuzzy sets, Massa'deh et al [8-20] introduced many new concepts, including fuzzy, bipolar fuzzy and intuitionistic fuzzy groups, rings and graphs. In 2011, Akram [21] studied a bipolar fuzzy set in graph theory and among the subjects he studied regular, irregular, totally neighborly irregular graph. Many authors introduced and discussed some properties of graph theory in bipolar fuzzy sets [22-25]. In 2015, Ezhilmaran and Sankar [26, 27] defined intuitionistic bipolar fuzzy graph. They introduced some concepts such as neighborly irregular intuitionistic fuzzy graph, totally irregular intuitionistic fuzzy graph, highly irregular intuitionistic fuzzy graph. In this paper, we study more results of intuitionistic bipolar fuzzy graph, incident matrix for intuitionistic bipolar fuzzy graph, line intuitionistic bipolar fuzzy graph, subdivision intuitionistic bipolar fuzzy graph. These concepts are useful in information analysis of computer networks.

2 Preliminaries

The following definitions will be used in the sequel.

Definition2.1: [22**]** A graph is an ordered pair $G = (V, E)$, where V is the set of vertices of G^* and E is the set of edges of G^*. Two vertices **x**, **y** in an unordered graph G^* are said to be adjacent in the graph if (x, y) is an edge in G^*. A simple graph is an unordered graph that has no loops and no more than one edge between any two different vertices.

Definition2.2: [26**]** A subgraph of a graph $G^* = (V, E)$ is a graph $G' = (V', E')$, where $V' \subseteq V$ and $E' \subseteq E$.

Definition2.3: [26**]** A crisp graph $G^* = (V, E)$ is an unordered triple $(V(G^*), E(G^*), \Psi_{G^*})$ consisting of a non-empty $V(G^*)$ of vertices, a set of edges $E(G^*)$ disjoint from $V(G^*)$, and an incidence function Ψ_{G^*} that associates with each edge of G^* an unordered pair of vertices of G^*.

Definition2.4: [26**]** let $G^* = (V, E)$ be a crisp graph, the length of a path $P = V_1V_2 \ldots V_{n+1}$ in G is n.

Definition2.5: [25**]** let $G^* = (V, E)$ be a crisp graph, the distance between any two vertices V_i and V_j in G^* is denoted by $d_{G^*}(V_i, V_j)$ and is defined as the minimum length of the path connecting the vertices V_i, V_j.

Definition2.6: [25**]** An $m\times n$ matrix is a rectangular array of $m \times n$ numbers arranged in n rows and m columns, $m\times n$ is called the size on the order of the matrix.

Definition2.7: Two matrices A and B are said to be equal if they have the same order and the corresponding entries in both matrices are equal.

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3 Main Results

Definition 3.1:[27] Let X be a non-empty set, the intuitionistic bipolar fuzzy set δ = {x, $\mu^+(x)$, $\mu^-(x)$, $\lambda^+(x)$, $\lambda^-(x)$: x ∈X} Where: $\mu^+(x)$: x → [0, 1] and $\mu^-(x)$: x → [-1, 0], $\lambda^+(x)$: x→ [0, 1], $\lambda^-(x)$: x → [-1, 0], are the mappings such that: $0 \le \mu^+(x) + \lambda^+(x) \le 1$, $-1 \le \mu^-(x) + \lambda^-(x) \le 0$. We use the position membership degree $\mu^+(x)$ to denote the satisfaction degree to an intuitionistic bipolar fuzzy set δ, and the negative of an element x some implicit counter property corresponding to an intuitionistic bipolar fuzzy set. Similarly, we use the positive non-membership degree ($\lambda^+(x)$) to denote the satisfaction degree of an element **x** to the property corresponding to an intuitionistic bipolar fuzzy set and the negative non-membership degree ($\lambda^-(x)$) to denote the satisfaction degree of an element x to some implicit counter property corresponding to an intuitionistic bipolar fuzzy set. If $\mu^+(x) \neq 0$, $\mu^-(x) = 0$, $\lambda^+(x) = 0$ and $\lambda^-(x) = 0$ then we have a situation that x is regarded as having only the positive membership property of an intuitionistic bipolar fuzzy set. If $\mu^+(x) = 0$, $\mu^-(x) \neq 0$, $\lambda^+(x) = 0$ and $\lambda^-(x) = 0$ then we have a situation that x is regarded as having only the negative membership property of an intuitionistic bipolar fuzzy set. If $\mu^+(x) = 0$, $\mu^-(x) = 0$, $\lambda^+(x) \neq 0$ and $\lambda^-(x) = 0$ then we have a situation that x regarded as having only the positive non-membership property of an intuitionistic bipolar fuzzy set. It is possible for an element x to be such that $\mu^+(x) \neq 0$, $\mu^-(x) \neq 0$, $\lambda^+(x) \neq 0$ and $\lambda^-(x) \neq 0$ when the membership and non-membership function of the property overlaps with its counter properties over some portion of x.

Definition 3.2: if δ and γ are two intuitionistic bipolar fuzzy sets, such that:

$\delta = (\mu^+_\delta(x), \mu^-_\delta(x), \lambda^+_\delta(x)$ and $\lambda^-_\delta(x))$

$\gamma = (\mu^+_\gamma(x), \mu^-_\gamma(x), \lambda^+_\gamma(x)$ and $\lambda^-_\gamma(x))$, then:

$(\delta \cap \gamma)(x) = (\mu^+_\delta(x) \wedge \mu^+_\gamma(x),\ \mu^-_\delta(x) \wedge \mu^-_\gamma(x))$

$(\delta \cup \gamma)(x) = (\mu^+_\delta(x) \vee \mu^+_\gamma(x),\ \mu^-_\delta(x) \vee \mu^-_\gamma(x))$

$(\delta \cap \gamma)(x) = (\lambda^+_\delta(x) \wedge \lambda^+_\gamma(x),\ \lambda^-_\delta(x) \wedge \lambda^-_\gamma(x))$

$(\delta \cup \gamma)(x) = (\lambda^+_\delta(x) \vee \lambda^+_\gamma(x),\ \lambda^-_\delta(x) \vee \lambda^-_\gamma(x))$

Definition3.3: An intuitionistic bipolar fuzzy graph $G = (V, E)$ is said to be a strong intuitionistic bipolar fuzzy graph if

$\mu^+_{\delta ij} = \min(\mu^+_{\delta i}, \mu^+_{\delta j})$ and $\lambda^+_{\delta ij} = \max(a_i, a_j)$

$\mu^-_{\delta ij} = \max(\mu^-_{\delta i}, \mu^-_{\delta j})$ and $\lambda^+_{\delta ij} = \min(a_i, a_j)$

$\forall (a_i, a_j) \in E$

Definition3.4: An intuitionistic bipolar fuzzy graph $G = (V, E)$ is said to be a complete intuitionistic bipolar fuzzy graph if

$\mu^+_{\gamma ij} = \min(\mu^+_{\gamma i}, \mu^+_{\gamma j})$ and $\lambda^+_{\delta ij} = \max(a_i, a_j)$

$\mu^-_{\gamma ij} = \max(\mu^-_{\gamma i}, \mu^-_{\gamma j})$ and $\lambda^+_{\delta ij} = \min(a_i, a_j)$

For all $(a_i, a_j) \in V$.

Definition3.5: If $G = (V, E)$ is an intuitionistic bipolar fuzzy graph, then the order of G is defined and denoted by $O(G) = \left(O_\mu(G), O_\lambda(G)\right)$ where $O_{\mu\delta}(G) = \left(\sum_{a_i \epsilon v} \mu^+_{\delta i}, \sum_{a_i \epsilon v} \mu^-_{\delta i}\right)$ and $O_{\lambda\delta}(G) = \left(\sum_{a_i \epsilon v} \lambda^+_{\delta i}, \sum_{a_i \epsilon v} \lambda^-_{\delta i}\right)$

Definition3.6: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph, then the size is defined as $S(G) = \left(S_\mu(G), S_\lambda(G)\right)$

$S_\mu(G) = \sum_{a_{ij} \epsilon E} \mu^+_{\gamma ij}, \sum_{a_{ij} \epsilon E} \mu^-_{\gamma ij}$ and

$S_\lambda(G) = \sum_{a_{ij} \epsilon E} \lambda^+_{\gamma ij}, \sum_{a_{ij} \epsilon E} \lambda^-_{\gamma ij}$

Definition3.7: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph. A path P in G is a sequence of distinct vertices $a_1, a_2, \cdots\cdots, a_n$ such that either one of the following conditions is held

1) $\mu^+_{\delta ij} > 0, |\mu^-_{\delta ij}| > 0$

   And $\lambda^+_{\delta ij} = 1$ $\lambda^-_{\delta ij} = 0$ for some i and j

2) $\mu^+_{\delta ij} > 0, |\mu^-_{\delta ij}| > 0$

   $\lambda^+_{\delta ij} > 0, |\lambda^-_{\delta ij}| > 0$

Definition3.8: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph. The length of a path $P = a_1, a_2, \cdots\cdots, a_{n+1}$ $(n > 0)$ in $G$is n

Definition3.9: An intuitionistic bipolar fuzzy graph. $G = (V, E)$ is connected if any two vertices are joined by a path.

Definition3.10: If G is an intuitionistic bipolar fuzzy graph. The $\mu -$ strength $(\lambda - strength)$ of a path $P = a_1, a_2, \cdots\cdots, a_n$ in an intuitionistic bipolar fuzzy graph G is denoted by.

$SP_{\mu(G)}$ $\left(SP_{\lambda(G)}\right)$& is defined as

$max\left(min\{\mu^+_{\delta ij}\}, max\{|\mu^-_{\delta ij}|\}\right)$,

$max\left(max\{\lambda^+_{\delta ij}\}, min\{|\lambda^-_{\delta ij}|\}\right)$

Respectively for all $i, j = 1, 2, \cdots\cdots, n$

Definition3.11: If $a_i, a_j \in V \le G$, the connectedness $\mu -$ strength $(\lambda - strength)$ between the vertices a_i and a_j in G is

$Co\ NN_{\mu(G)}\ (a_i, a_j) = \max\{SP_{\mu(c)}\}$

$Co\ NN_{\lambda(G)}\ (a_i, a_j) = \min\{SP_{\lambda(c)}\}$

For all possible path betweena_i and a_j.

Definition3.12: Let G be an intuitionistic bipolar fuzzy graph. A walk is a sequence of vertices and edge. Where the end points of each edge are the preceding and following vertices in the sequence such that either one of the following conditions is held

1) $\mu^+_{\delta ij} > 0, |\mu^-_{\delta ij}| > 0$

   $\lambda^+_{\delta ij} = 0$, $\lambda^-_{\delta ij} = 0$ for some i and j

2) $\mu^+_{\delta ij} > 0, |\mu^-_{\delta ij}| > 0$

   $\lambda^+_{\delta ij} > 0, |\lambda^-_{\delta ij}| > 0$

If a walk begins at a_i and ends at a_j.then it is an $a_i - a_j$ walk.

A walk is closed if it begins and ends at the same vertex.

Theorem3.13: if $G = (V, E)$ is an intuitionistic bipolar fuzzy graph. G Contains a x – y walk of length r, then G contains a x – y path of length r.

Definition3.14: An intuitionistic bipolar fuzzy graph G*= (V, E) is a pair (δ, γ) where $\delta = <\mu^+_\delta, \mu^-_\delta, \lambda^+_\delta, \lambda^-_\delta>$ is an intuitionistic bipolar fuzzy set in V and $\gamma = <\mu^+_\gamma, \mu^-_\gamma, \lambda^+_\gamma, \lambda^-_\gamma>$ is an intuitionistic bipolar fuzzy set in V × V such that (V, E) is an intuitionistic bipolar fuzzy graph. The neighborhood of a vertex **a** in G is defined by $N(a) = (N_\mu^+(a), N_\mu^-(a), N_\lambda^+(a), N_\lambda^-(a))$, where:

1) $N_\mu^+(a) = \{b \in V: \mu^+_\gamma(ab) \le \min(\mu^+_\delta(a), \mu^+_\delta(b))\}$

2) $N_\mu^-(a) = \{b \in V: \mu^-_\gamma(ab) \ge \max(\mu^-_\delta(a), \mu^-_\delta(b))\}$

3) $N_\lambda^+(a) = \{b \in V: \lambda^+_\gamma(ab) \ge \max(\lambda^+_\delta(a), \lambda^+_\delta(b))\}$

4) $N_\lambda^-(a) = \{b \in V: \lambda^+_\gamma(ab) \le \min(\lambda^-_\delta(a), \lambda^-_\delta(b))\}$

Example 3.15:

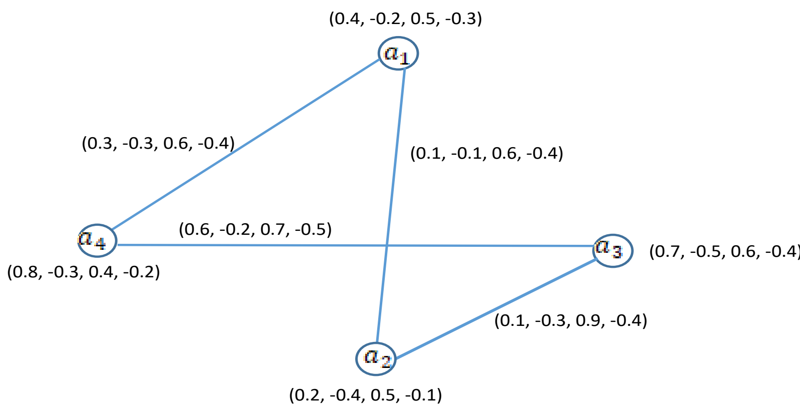

Fig.1: Intuitionistic bipolar fuzzy graph

Fig.1: A display of an intuitionistic bipolar fuzzy graph G = (V, E), such that: V = {a_1, a_2, a_3, a_4} and E = {e_1, e_2, e_3, e_4}.

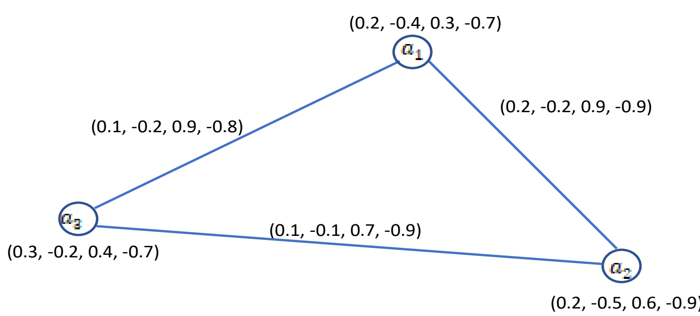

Fig.2: Intuitionistic bipolar fuzzy graph

Definition 3.16: The dimension or the order of an intuitionistic bipolar fuzzy matrix A is the number of rows and columns, the dimension of intuitionistic bipolar fuzzy matrix with m–rows and n-columns is m × n

Definition 3.17:

Let $A = \{< a\mu^+_{ij}, a\mu^-_{ij}, a\lambda^+_{ij}, a\lambda^-_{ij} >\} m\ x\ n$ be an intuitionistic fuzzy matrix. The transpose of the matrix A is denoted by A^T and is defined as $A^T = \{< a\mu^+_{\delta ij}, a\mu^-_{\delta ij}, a\lambda^+_{\delta ij}, a\lambda^-_{\delta ij} >\}$

Definition 3.18: Two intuitionistic bipolar fuzzy matrices $A = \{< a\mu^+_{\delta ij}, a\mu^-_{\delta ij}, a\lambda^+_{\delta ij}, a\lambda^-_{\delta ij} >\}$, $B = \{< b\mu^+_{ij}, b\mu^-_{ij}, b\lambda^+_{ij}, b\lambda^-_{ij} >\}$ are said to be equal if they have the same order m x n and corresponding entries in both matrices are equal , this means that

$a\mu^+_{\delta ij} = b\mu^+_{\delta ij}, a\lambda^+_{\delta ij} = b\lambda^+_{\delta ij}, a\lambda^-_{\delta ij} = b\lambda^-_{\delta ij}$ for $i = 1, \cdots\cdots, m$ and $j = 1, \cdots\cdots, n$ and it's denoted by A=B

Definition 3.19:

If $A = \{< a\mu^+_{\delta ij}, a\mu^-_{\delta ij}, a\lambda^+_{\delta ij}, a\lambda^-_{\delta ij} >\} m\ x\ n$, $B = \{< b\mu^+_{\delta ij}, b\mu^-_{\delta ij}, b\lambda^+_{\delta ij}, b\lambda^-_{\delta ij} >\} m\ x\ n$ are two intuitionistic bipolar fuzzy matrices, then the sum of A and B which is denoted by $A^{+}_{max-min}B$, is defined as

$$A^{+}_{max-min}B = \{< C\mu^{+}_{\delta ij}, C\mu^{-}_{\delta ij}, C\lambda^{+}_{\delta ij}, C\lambda^{-}_{\delta ij}\} =$$
$$[< max < a\mu^{+}_{\delta ij}, b\mu^{+}_{\delta ij} >, min < a\mu^{-}_{\delta ij}, b\mu^{-}_{\delta ij} >,$$
$$\max < a\lambda^{+}_{\delta ij}, b\lambda^{+}_{\delta ij} >, min < a\lambda^{-}_{\delta ij}, b\lambda^{-}_{\delta ij}]$$

$1 \le i \le m, 1 \le j \le n$

Example3.20: Consider an intuitionistic bipolar fuzzy graph $G = (V, E)$ such that

$$V = \{a_1, a_2, a_3, a_4, a_5\},$$
$$E = \left\{\begin{matrix}(a_1, a_2), (a_1, a_3), (a_1, a_4), (a_2, a_3), \\ (a_2, a_4), (a_3, a_4), (a_4, a_5)\end{matrix}\right\}$$

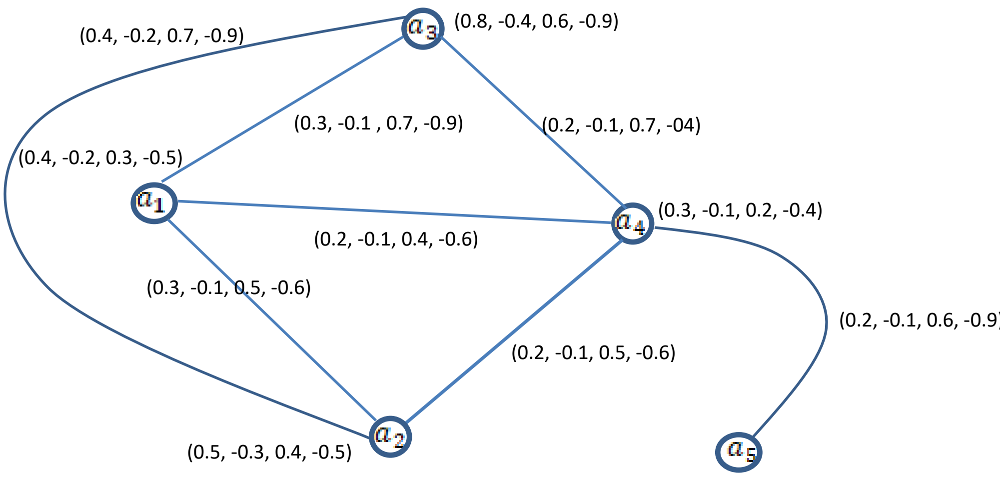

Fig.3: Intuitionistic bipolar fuzzy graph

The intuitionistic bipolar fuzzy matrix for G_1 is given by

$$\begin{array}{cc}
a_1 & \begin{bmatrix} (0,0,0,0)\ (0.3,-0.1,0.5,-0.6)(0.3,-0.1,0.7,-0.9) \\ (0.2,-0.1,0.4,-0.6)(\ 0,0,0,0) \end{bmatrix} \\
a_2 & \begin{bmatrix} (0.3,-0.1,0.5,-0.6)(0,0,0,0)\ (0.4,-0.2,0.3,-0.5) \\ (0.2,-0.1,0.5,-0.6)(\ 0,0,0,0) \end{bmatrix} \\
a_3 & \begin{bmatrix} (0.3,-0.1,0.7,-0.9)(0.4,-0.2,0.3,-0.5)(0,0,0,0) \\ (0.2,-0.1,0.7,-0.9)(\ 0,0,0,0) \end{bmatrix} \\
a_4 & \begin{bmatrix} (0.2,-0.1,0.7,-0.6)(0.2,-0.1,0.5,-0.6) \\ (0.2,-0.1,0.7,-0.9)(0,0,0,0)\ (0.2,-0.1,0.6,-0.9) \end{bmatrix} \\
a_5 & \begin{bmatrix} (0,0,0,0)(0,0,0,0)(0,0,0,0) \\ (0.2,-0.1,0.6,-0.9)(\ 0,0,0,0) \end{bmatrix}
\end{array}$$

Definition 3.21:
Let $A = \{< a\mu^{+}_{\delta ij}, a\mu^{-}_{\delta ij}, a\lambda^{+}_{\delta ij}, a\lambda^{-}_{\delta ij} >\} m\ x\ n$ be an intuitionistic bipolar fuzzy matrix and r be a positive integer than the r^{th} power of an intuitionistic bipolar fuzzy matrix is denoted by A^{r} and is defined as max-min product of r-copies of an intuitionistic bipolar fuzzy matrix A.

Definition 3.22:
Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph. The index of matrix representation of intuitionistic bipolar fuzzy graph is of the form $V_1 \in V \times V$ where $V = \{a_1, a_2, \dots \dots, a_n\}$ and

$$E = \{< \mu^{+}_{\delta ij}, \mu^{-}_{\delta ij}, \lambda^{+}_{\delta ij}, \lambda^{-}_{\delta ij} >\} m\ x\ n =$$

$$\begin{bmatrix}
< \mu^{+}_{\delta 11}, \mu^{-}_{\delta 11}, \lambda^{+}_{\delta 11}, \lambda^{-}_{\delta 11} > \cdots\cdots < \mu^{+}_{\delta 1n}, \mu^{-}_{\delta 1n}, \lambda^{+}_{\delta 1n}, \lambda^{-}_{\delta 1n} > \\
< \mu^{+}_{\delta 21}, \mu^{-}_{\delta 21}, \lambda^{+}_{\delta 21}, \lambda^{-}_{\delta 21} > \cdots\cdots < \mu^{+}_{\delta 2n}, \mu^{-}_{\delta 2n}, \lambda^{+}_{\delta 2n}, \lambda^{-}_{\delta 2n} > \\
\vdots \qquad\qquad \vdots \\
< \mu^{+}_{\delta n1}, \mu^{-}_{\delta n1}, \lambda^{+}_{\delta n1}, \lambda^{-}_{\delta n1} > \cdots\cdots < \mu^{+}_{\delta nn}, \mu^{-}_{\delta nn}, \lambda^{+}_{\delta nn}, \lambda^{-}_{\delta nn} >
\end{bmatrix}$$

where $< \mu^{+}_{\delta ij}, \mu^{-}_{\delta ij}, \lambda^{+}_{\delta ij}, \lambda^{-}_{\delta ij} >\in [0,1]X[0,1]$ $(1 \le i, j \ge n)$, the edge between two vertices a_i *and* a_j is indexed by $< \mu^{+}_{\delta ij}, \mu^{-}_{\delta ij}, \lambda^{+}_{\delta ij}, \lambda^{-}_{\delta ij} >$.

Remark 3.23: For any intuitionistic bipolar fuzzy graph, the index matrix representation is an intuitionistic bipolar fuzzy graph.

Definition3.24: If $G = (V, E)$ is an intuitionistic bipolar fuzzy graph where $V = \{a_1, a_2, \dots \dots, a_n\}$ The incident G matrix of m intuitionistic bipolar fuzzy graph G is $B = \{< b\mu^{+}_{\delta ij}, b\mu^{-}_{\delta ij}, b\lambda^{+}_{\delta ij}, b\lambda^{-}_{\delta ij} >\} n\ x\ m$ where n and m represent the number of vertices and number of edges of G respectively, whose entries of B are as following

$$B = \{< b\mu^{+}_{\delta ij}, b\mu^{-}_{\delta ij}, b\lambda^{+}_{\delta ij}, b\lambda^{-}_{\delta ij} >\} n\ x\ m =$$

$$\left\{\begin{matrix} < \mu^{+}_{\delta}(e_j), \mu^{-}_{\delta}(e_j), \lambda^{+}_{\delta}(e_j), \lambda^{-}_{\delta}(e_j) > \\ \\ if\ an\ edge\ e_j\ is\ incident\ on\ the\ vertex\ a_i \\ \\ < -1, 0, 1 >, 0 \quad otherwise \end{matrix}\right\}$$

It can also be represented in the matrix form

$$B = \{< b\mu^{+}_{\delta ij}, b\mu^{-}_{\delta ij}, b\lambda^{+}_{\delta ij}, b\lambda^{-}_{\delta ij} >\} n\ x\ m =$$

$$\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{bmatrix} < \mu^{+}_{\delta}(e_1), \mu^{-}_{\delta}(e_1), \lambda^{+}_{\delta}(e_1), \lambda^{-}_{\delta}(e_1) > < \mu^{+}_{\delta}(e_2), \mu^{-}_{\delta}(e_2), \lambda^{+}_{\delta}(e_2), \lambda^{-}_{\delta}(e_2) > < \mu^{+}_{\delta}(e_n), \mu^{-}_{\delta}(e_n), \lambda^{+}_{\delta}(e_n), \lambda^{-}_{\delta}(e_n) > \\ < \mu^{+}_{\delta}(e_1), \mu^{-}_{\delta}(e_1), \lambda^{+}_{\delta}(e_1), \lambda^{-}_{\delta}(e_1) > < \mu^{+}_{\delta}(e_2), \mu^{-}_{\delta}(e_2), \lambda^{+}_{\delta}(e_2), \lambda^{-}_{\delta}(e_2) > < \mu^{+}_{\delta}(e_n), \mu^{-}_{\delta}(e_n), \lambda^{+}_{\delta}(e_n), \lambda^{-}_{\delta}(e_n) > \\ \vdots \\ < \mu^{+}_{\delta}(e_1), \mu^{-}_{\delta}(e_1), \lambda^{+}_{\delta}(e_1), \lambda^{-}_{\delta}(e_1) > < \mu^{+}_{\delta}(e_2), \mu^{-}_{\delta}(e_2), \lambda^{+}_{\delta}(e_2), \lambda^{-}_{\delta}(e_2) > < \mu^{+}_{\delta}(e_n), \mu^{-}_{\delta}(e_n), \lambda^{+}_{\delta}(e_n), \lambda^{-}_{\delta}(e_n) > \end{bmatrix}$$

Where $< \mu^{+}_{\delta}(e_i), \mu^{-}_{\delta}(e_i), \lambda^{+}_{\delta}(e_i), \lambda^{-}_{\delta}(e_i) > \ \epsilon\ [0,1]x[-1,0]x[0,1]x[-1,0]$

Definition3.25: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph, where $V = \{a_1, a_2, \dots\dots, a_n\}$ and $E = \{e_1, e_2, \dots\dots, e_k\}$ Then the line intuitionistic bipolar fuzzy graph is denoted by $G_l = (V_l, E_l)$ when the vertices of G_l are in abjection with the edges of G and there exits an edge between the vertices of G_l if the corresponding edges of G are adjacent. The membership and non membership value of V_l *and* E_l are defined as following

$$\mu^{+}_{\delta}(a_i) = \mu^{+}_{\delta}(e_i) \quad \text{and} \quad \lambda^{+}_{\delta L}(a_i) = \lambda^{+}_{\delta}(e_i)$$

$$\forall e_i \in E$$

$$\mu^{-}_{\delta}(a_i) = \mu^{-}_{\delta}(e_i) \quad \text{and} \quad \lambda^{-}_{\delta L}(a_i) = \lambda^{-}_{\delta}(e_i)$$

$$\mu^{+}_{\delta l}(a_i, a_j) = \begin{cases} \min\left(\mu^{+}_{\delta l}(a_i), \mu^{+}_{\delta l}(a_j)\right), if\ e_i\ and\ e_j\ are\ adjacent\ in\ G \\ max\left(\mu^{-}_{\delta l}(a_i), \mu^{-}_{\delta l}(a_j)\right), if\ e_i\ and\ e_j\ areadjacent\ in\ G \\ (-1,0,1)\ otherwise \end{cases}$$

$$\lambda^{+}_{\delta l}(a_i, a_j) = \begin{cases} \max\left(\lambda^{+}_{\delta l}(a_i), \lambda^{+}_{\delta l}(a_j)\right), if\ e_i\ and\ e_j\ are\ adjacent\ in\ G \\ \min\left(\lambda^{-}_{\delta l}(a_i), \lambda^{-}_{\delta l}(a_j)\right), if\ e_i\ and\ e_j\ are\ adjacent\ in\ G \\ (-1,0,1)\ otherwise \end{cases}$$

Example3.26: Consider an intuitionistic bipolar fuzzy graph$G = < V, E >$, such that $V = \{a_1, a_2, a_3, a_4\}$, $E = \{a_1a_2, a_1a_3, a_1a_4, a_2a_3, a_2a_4, a_3a_4\}$ such that

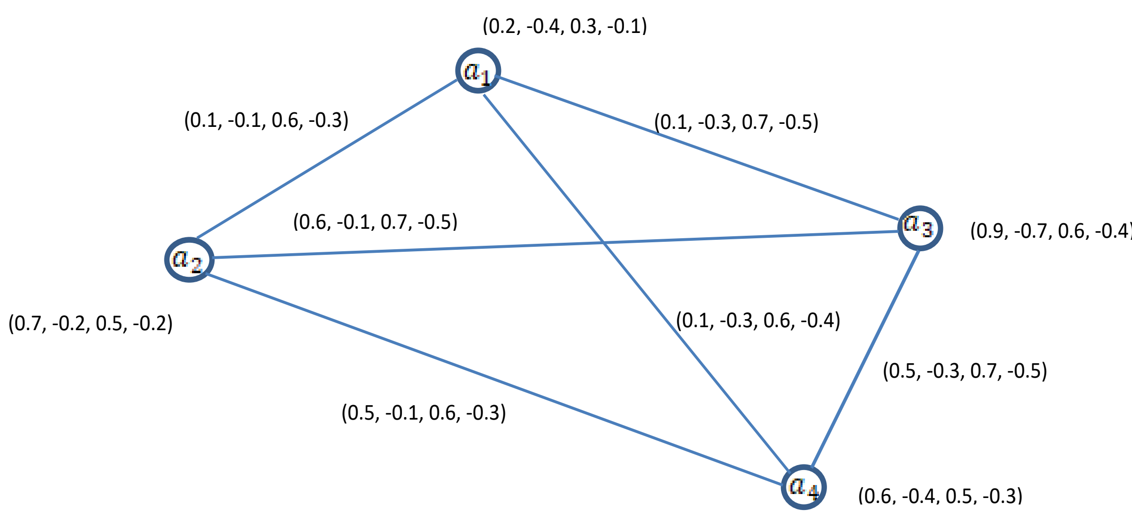

Fig. 4: Intuitionistic bipolar fuzzy graph

Then the line intuitionistic bipolar fuzzy graph G is given as

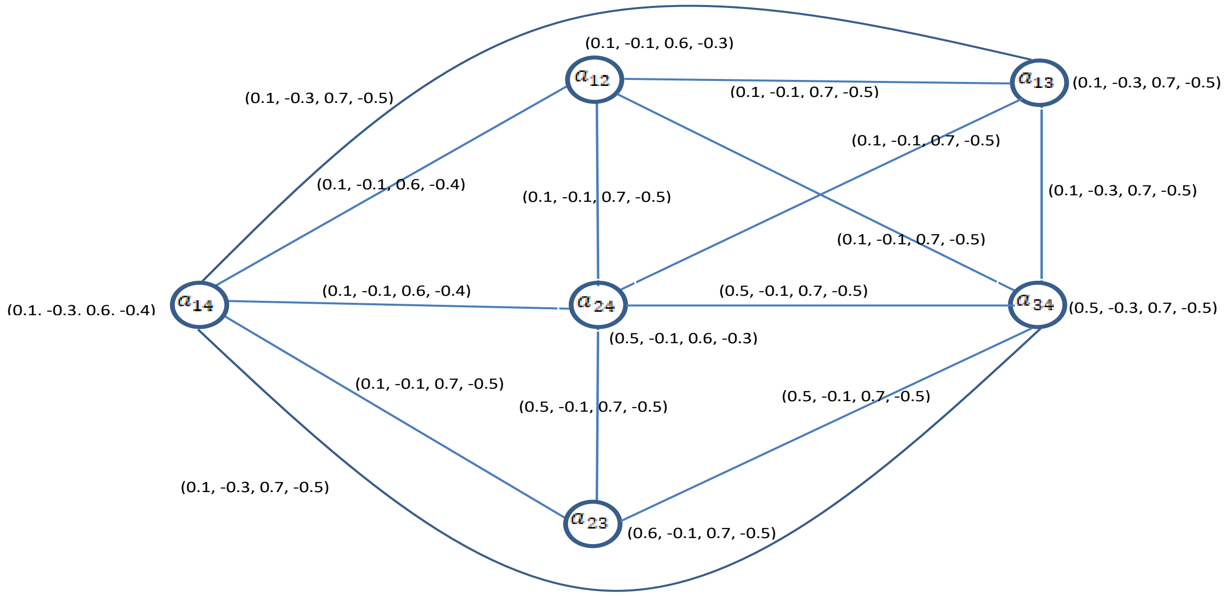

Fig. 5: Line intuitionistic bipolar fuzzy graph

$V_l = \{a_{12}, a_{13}, a_{24}, a_{14}, a_{34}, a_{23}\}$ And

$$E_l = \left\{ \begin{array}{l} (a_{12}a_{13}), (a_{13}a_{34}), (a_{34}a_{24}), (a_{12}a_{24}), \\ (a_{12}a_{34}), (a_{13}a_{24}), (a_{34}a_{23}), (a_{24}a_{23}), \\ (a_{23}a_{14}), (a_{34}a_{14}), (a_{12}a_{14}), (a_{14}a_{24}), \\ (a_{13}a_{14}) \end{array} \right\}$$

Definition 3.27: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph with underlying crisp graph $G = (V, E)$ then the subdivision of an intuitionistic bipolar fuzzy graph G is denoted by $SD_G = (SD_v, SD_e)$ and it is obtained by adding a vertex C_r into every edge $e_{ij} = (a_i, a_j) \epsilon\, E$ of G such that the membership and the non-membership of vertex a_r and the edges $c_r\, a_j$ and $a_r\, a_j$ are defined as following

$SD\mu_\delta^+(C_r) = \mu_{\delta ij}^+$ & $SD\lambda_\delta^+(C_r) = \lambda_{\delta ij}^+$

$SD\mu_\delta^-(C_r) = \mu_{\delta ij}^+$ & $SD\lambda_\delta^-(C_r) = \lambda_{\delta ij}^-$

For all $C_r \,\epsilon\, SD_v$

1) $SD\mu_\delta^+(a_i, C_r) \le \min\{SD\mu_\delta^+(a_i), SD\mu_\delta^+(C_r)\}$
   $SD\mu_\delta^-(a_i, C_r) \ge \max\{SD\mu_\delta^-(a_i), SD\mu_\delta^-(C_r)\}$
2) $SD\lambda_\delta^+(a_i, C_r) \ge \max\{SD\lambda_\delta^+(a_i), SD\lambda_\delta^+(C_r)\}$
   $SD\lambda_\delta^-(a_i, C_r) \le \min\{SD\lambda_\delta^-(a_i), SD\lambda_\delta^-(C_r)\}$
3) $SD\mu_\delta^+(C_r, a_j) \le \min\{SD\mu_\delta^+(C_r), SD\mu_\delta^+(a_j)\}$
   $SD\mu_\delta^-(C_r, a_j) \ge \max\{SD\mu_\delta^-(C_r), SD\mu_\delta^-(a_j)\}$
4) $SD\lambda_\delta^+(C_r, a_j) \ge \max\{SD\lambda_\delta^+(C_r), SD\lambda_\delta^+(a_j)\}$
   $SD\lambda_\delta^-(C_r, a_j) \le \min\{SD\lambda_\delta^-(C_r), SD\lambda_\delta^-(a_j)\}$

$\forall\, a_i, a_{j\,,} C_r \in SD_r$

Example 3.28: Consider an intuitionistic bipolar fuzzy graph$G = (V, E)$, such that

$v = \{a_1, a_2, a_3, a_4\}$ *and* $E = \{a_1a_2, a_1a_4, a_2a_3, a_3a_4\}$ Such as

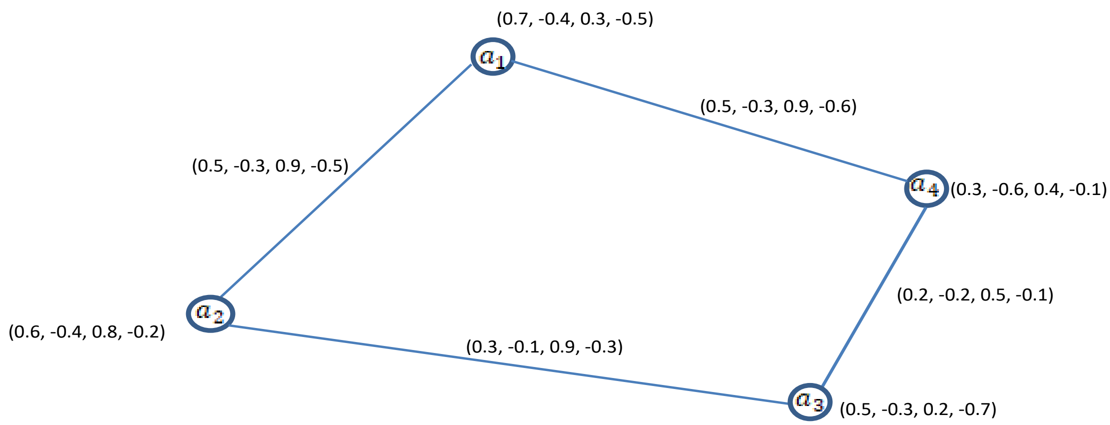

Fig 6.a: Intuitionistic bipolar fuzzy graph

Then the subdivision intuitionistic bipolar fuzzy graph G is

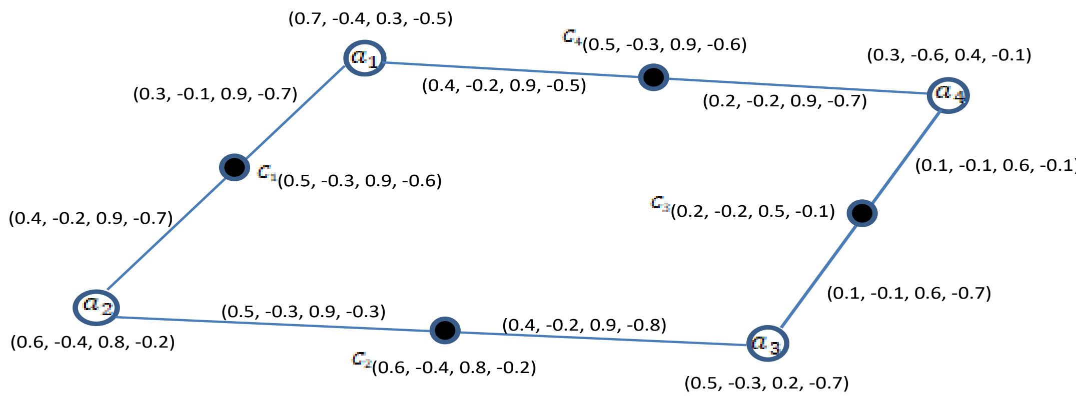

Fig 6.b: Subdivision intuitionistic bipolar fuzzy graph

With $SD_v = \{a_1, a_2, a_3, a_4, C_1, C_2, C_3, C_4\}$ and $SD_E = \{a_1c_4, a_1c_1, c_1a_2, a_2c_2, c_2a_3, a_3c_3, c_3a_4, a_4c_4\}$

Definition3.29: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph with the underlying crisp graph$G^r = (V, E)$. Thus the power of an intuitionistic bipolar fuzzy graph G is denoted by $G^r = (V^r, E^r)$ where $V^r = V$ and the vertices a_i *and* a_j are adjacent in G^r iff $d_G{}^*(a_i, a_j) \le r$. The membership values of the edges of G^r are defined as follows

$$\left(\mu_\delta^{+k}(a_i,a_j), \mu_\delta^{-k}(a_i,a_j), \lambda_\delta^{+k}(a_i,a_j), \lambda_\delta^{-k}(a_i,a_j)\right) = \begin{cases} \left(min(\mu^+_{\delta i},\mu^+_{\delta j}), max(\mu^-_{\delta i},\mu^-_{\delta j}), max(\lambda^+_{\delta i},\lambda^+_{\delta j}), min(\lambda^-_{\delta i},\lambda^-_{\delta j})\right) \text{ if } dG^*(v_i,v_j)\le r \\ (-1,0,1), otherwise \end{cases}$$

Example 3.30: Let $G = (V, E)$ be an intuitionistic bipolar fuzzy graph such $v = \{a_1, a_2, a_3, a_4, a_5\}$ *and* $E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_5\}$ such that

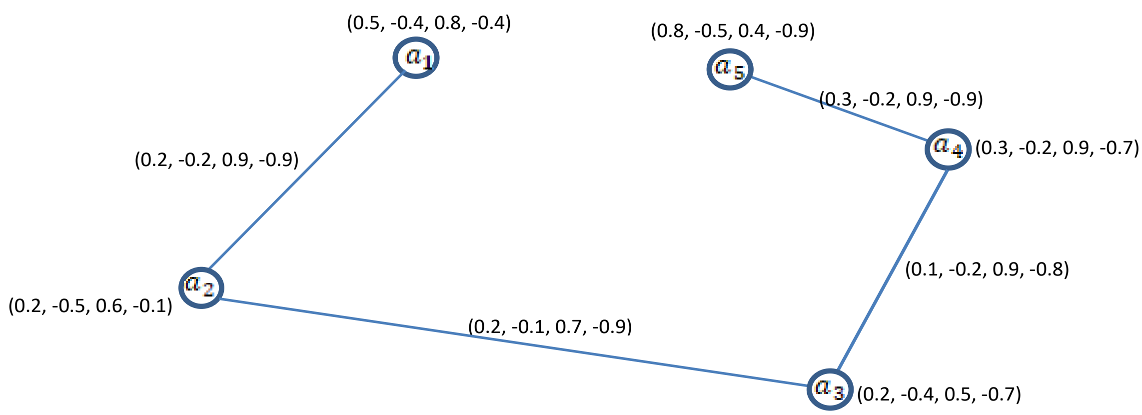

Fig.7.a: Intuitionistic bipolar fuzzy graph

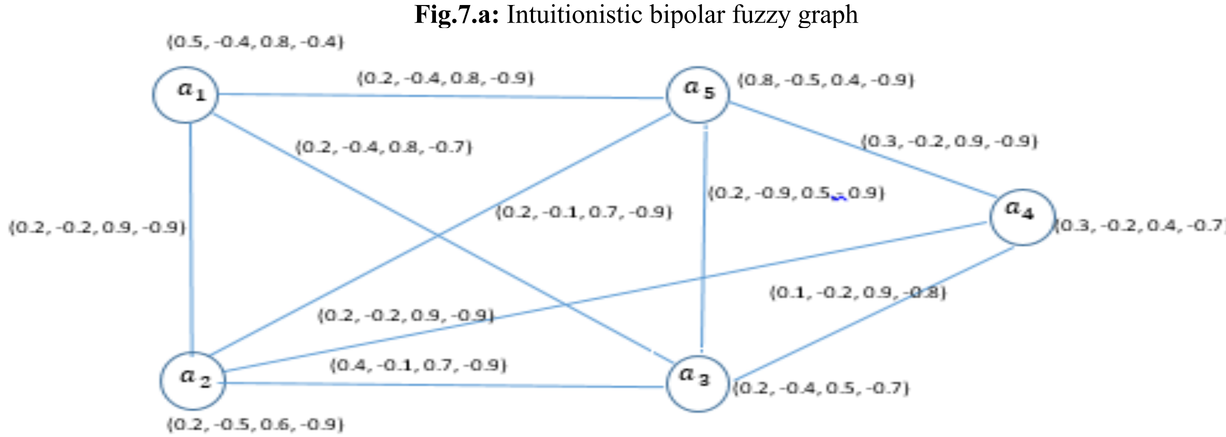

Fig.7.b: Intuitionistic bipolar fuzzy graph of G^2

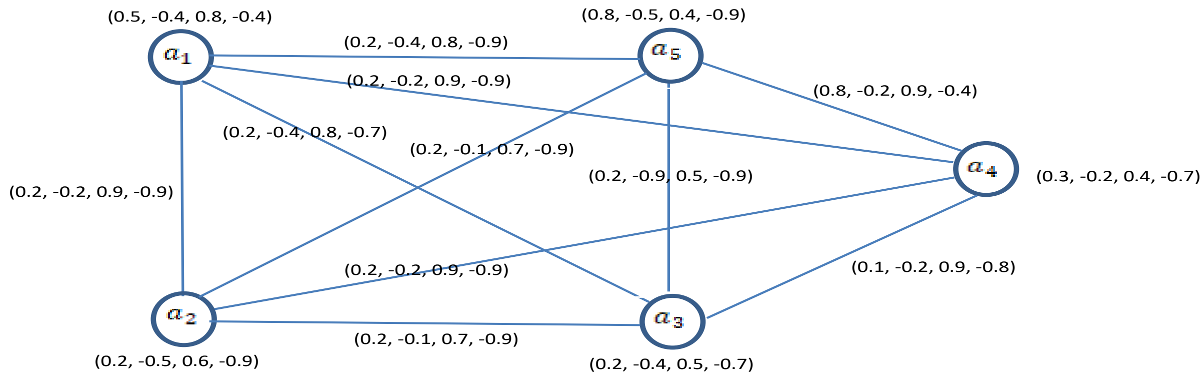

Fig.7.c: Intuitionistic bipolar fuzzy graph of G^3

And intuitionistic bipolar fuzzy matrices representing G^2 and G^3 are

$$A^2 = \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{bmatrix} (0,0,0,0) & (0.2,-0.2,0.9,-0.9) & (0.2,-0.4,0.8,-0.7) & (0,0,0,0) & (0,0,0,0) \\ (0.2,-0.2,0.9,-0.9) & (0,0,0,0) & (0.2,-0.1,0.7,-0.9) & (0.2,-0.3,0.6,-0.9) & (0,0,0,0) \\ (0.2,-0.4,0.8,-0.7) & (0.2,-0.1,0.7,-0.9) & (0,0,0,0) & (0.1,-0.2,0.9,-0.8) & (0.2,-0.1,0.5,-0.9) \\ (0,0,0,0) & (0.2,-0.2,0.6,-0.9) & (0.1,-0.4,0.5,-0.7) & (0,0,0,0) & (0.8,-0.2,0.9,-0.9) \\ (0,0,0,0) & (0,0,0,0) & (0.2,-0.9,0.5,-0.9) & (0.3,-0.2,0.9,-0.9) & (0,0,0,0) \end{bmatrix}$$

$$A^3 = \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{bmatrix} (0,0,0,0) & (0.2,-0.2,0.9,-0.9) & (0.2,-0.4,0.8,-0.7) & (0,0,0,0) & (0.2,-0.4,0.8,-0.9) \\ (0.2,-0.2,0.9,-0.9) & (0,0,0,0) & (0.2,-0.1,0.7,-0.9) & (0.2,-0.2,0.9,-0.9) & (0.2,-0.1,0.7,-0.9) \\ (0.2,-0.9,0.8,-0.7) & (0.2,-0.1,0.7,-0.9) & (0,0,0,0) & (0.1,-0.2,0.9,-0.8) & (0.2,-0.9,0.5,-0.9) \\ (0,0,0,0) & (0.2,-0.2,0.6,-0.9) & (0.1,-0.4,0.5,-0.7) & (0,0,0,0) & (0.8,-0.2,0.9,-0.9) \\ (0.2,-0.4,0.8,-0.9) & (0.2,-0.1,0.7,-0.9) & (0.2,-0.9,0.5,-0.9) & (0.8,-0.2,0.9,-0.9) & (0,0,0,0) \end{bmatrix}$$

Theorem 3.31: if $G = (V, E)$ strong intuitionistic bipolar fuzzy graph and $SD_G = (SD_V, SD_E)$ are the subdivision and an intuitionistic bipolar fuzzy graph G. Then $S_\mu(SD_G) \leq 2\, S_\mu(G)$ and $S_\lambda(SD_G) \leq 2\, S_\lambda(G)$

Proof: Consider

$$S_\mu(SD_G) = \sum_{e_{ik}, e_{kj} \in SD_E} \left(SD_{\mu_\gamma^+}(e_{ik}) + SD_{\mu_\gamma^+}(e_{kj}) \right)$$

$$\leq \sum SD_{\mu_\delta^+}(a_k) + \sum SD_{\mu_\delta^+}(a_k)$$

$$\leq \sum \mu_{\gamma ij}^+ + \sum \mu_{\gamma ij}^+$$

$$\leq 2 \sum \mu_{\gamma ij}^+$$

$$\leq 2 S_\mu(G)$$

Also

$$S_\mu(SD_G) = \sum_{e_{ik}, e_{kj} \in SD_E} \left| SD_{\mu_\gamma^-}(e_{ik}) + SD_{\mu_\gamma^-}(e_{kj}) \right|$$

$$\leq \sum \left| SD_{\mu_\delta^-}(a_k) \right| \sum SD_{\mu_\delta^+}(a_k)$$

$$\leq \sum \left| \mu_{\gamma ij}^- \right| + \sum \left| \mu_{\gamma ij}^- \right|$$

$$\leq 2 \sum \left| \mu_{\gamma ij}^- \right|$$

$$\leq 2 S_\mu(G)$$

&

$$S_\lambda(SD_G) = \sum_{e_{ik}, e_{kj} \in SD_E} \left| SD_{\lambda_\gamma^-}(e_{ik}) + SD_{\lambda_\gamma^-}(e_{kj}) \right|$$

$$\leq \sum SD_{\lambda_\delta^+}(a_k) + \sum SD_{\lambda_\delta^+}(a_k)$$

$$\leq \sum \lambda_{\gamma ij}^+ + \sum \lambda_{\gamma ij}^+$$

$$\leq 2 \sum \lambda_{\gamma ij}^+$$

$$\leq 2 S_\lambda(G)$$

Also

$$S_\lambda(SD_G) = \sum_{e_{ik}, e_{kj} \in SD_E} \left| SD_{\lambda_\gamma^-}(e_{ik}) + SD_{\lambda_\gamma^-}(e_{kj}) \right|$$

$$\leq \sum \left| SD_{\lambda_\delta^-}(a_k) \right| + \sum \left| SD_{\lambda_\delta^+}(a_k) \right|$$

$$\leq \sum \left| \lambda_{\gamma ij}^- \right| + \sum \left| \lambda_{\gamma ij}^- \right|$$

$$\leq 2 \sum \left| \lambda_{\gamma ij}^- \right|$$

$$\leq 2 S_\lambda(G)$$

Theorem3.32 Let G = (V, E) be an intuitionistic bipolar fuzzy graph and A = [<$\mu^+_{\delta ij}$, $\mu^-_{\delta ij}$, $\lambda^+_{\delta ij}$, $\lambda^-_{\delta ij}$>] be the index matrix of G. Then for each positive integer r then

A^r = Connectedness strength of a_j - a_j is a walk of length r….*

Proof: Suppose that A^r = [<$a^+_{\mu\delta ij}$, $a^-_{\mu\delta ij}$, $a^+_{\lambda\delta ij}$, $a^-_{\lambda\delta ij}$>] is the r^{th} power of the intuitionistic bipolar fuzzy matrix A, we prove (*) by mathematical induction such that:

Step 1**:** for r = 1 then A^r = A. Then A = [<$\boldsymbol{\mu^+_{\delta ij}}$, $\boldsymbol{\mu^-_{\delta ij}}$, $\boldsymbol{\lambda^+_{\delta ij}}$, $\boldsymbol{\lambda^-_{\delta ij}}$>], where $\boldsymbol{\mu^+_{\delta ij}}$ is the connectedness strength of ($\boldsymbol{a_i}$, $\boldsymbol{a_j}$) where the walk's length is one and λ^+_{ij} is the connectedness strength of ($\boldsymbol{a_i}$, $\boldsymbol{a_j}$) length walk is one = ($\boldsymbol{Conn_{\mu^+_\delta}(G)}$ $(\boldsymbol{a_i, a_j})$, $\boldsymbol{Conn_{\mu^-_\delta}(G)}$ $(\boldsymbol{a_i, a_j})$, $\boldsymbol{Conn_{\lambda^+_\delta}(G)}$ $(\boldsymbol{a_i, a_j})$, $\boldsymbol{Conn_{\lambda^-_\delta}(G)}$ $(\boldsymbol{a_i, a_j})$) such that: $(\boldsymbol{a_i, a_j})$ is $\boldsymbol{a_i}$ - $\boldsymbol{a_j}$ length of walk is one.

Step 2: suppose that it is true for r = k, where **k** is any integer.

By the inductive hypothesis, $(i,j)^{th}$ entry of A^r = ($Conn_{\mu^+_\delta}(G)$, $Conn_{\mu^-_\delta}(G)$, $Conn_{\lambda^+_\delta}(G)$, $Conn_{\lambda^-_\delta}(G)$), where (a_i, a_j) is a_i - a_j length of walk is k. Now we need to show that it is true for r = k + 1. $(i,j)^{th}$ entry of A^{k+1} = $(i,j)^{th}$ entry of A^k max – min (i,j) entry of A = ($Sup_{L=1\ldots n}$(min {$a^+_{\mu\delta iL}$, $b^+_{\mu\delta Lj}$}, $Inf_{L=1\ldots n}$(max {$a^-_{\mu\delta iL}$, $b^-_{\mu\delta Lj}$}, $Inf_{L=1\ldots n}$(max {$a^+_{\lambda\delta iL}$, $b^+_{\delta Lj}$}, $Sup_{L=1\ldots n}$(min {$a^-_{\lambda\delta iL}$, $b^-_{\lambda\delta Lj}$}) = (max (min ($Conn_{\mu^+_\delta}(G)$ (a_i, a_L), min (max ($Conn_{\mu^-_\delta}(G)$ (a_L, a_i), min (max ($Conn_{\lambda^+_\delta}(G)$ (a_i, a_L), max (min ($Conn_{\lambda^-_\delta}(G)$ (a_i, a_L))), where (a_i, a_L) is a_i - a_L length walk is k and (a_L, a_i) is a_L - a_i length walk is one = $Conn_{\mu^+_\delta}(G)(a_i, a_j)$, $Conn_{\mu^-_\delta}(G)(a_i, a_j)$, $Conn_{\lambda^+_\delta}(G)(a_i, a_j)$, $Conn_{\lambda^-_\delta}(G)(a_i, a_j)$) where (a_i, a_j) is a_i - a_j length walk is ***k+1*** ∀ a_i ∊ **V**. Therefore, it is true for every r that is $(i,j)^{th}$ entry of A^r = connectedness strength of $a_i - a_j$ length walk in r.

4 Conclusions

The concept of bipolar intuitionistic fuzzy sets introduced by Sankar and Ezhilmaran can be used in many problems of real life, Where we have applied this concept to the topics listed in [28- 30], on the other hand we will apply a bipolar intuitionistic fuzzy sets to generalize the idea of these papers [31-33]. In this paper, we have studied notions of subdivisions intuitionistic bipolar fuzzy graph, matrix of bipolar intuitionistic fuzzy graph, and line intuitionistic bipolar fuzzy graph.

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