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13-Brauer Trees of the Symmetric Group S_{22}

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Abstract: In this paper, all the blocks of defect one and the decomposition numbers are all zero or one, then the Brauer tree is a graph connecting the irreducible ordinary characters. The method (r,r')-inducing (restricting) is used to compute the Brauer trees of the symmetric group S_{22} *modulo* $P = 13$, which gives the irreducible modular spin characters *modulo* $P = 13$. Also, the 13-decomposition matrices of the spin characters of S_{22} are found.

Keywords: Modular representations and characters, Brauer trees, Decomposition matrix for the spin characters.

1 Introduction

The Symmetric group S_n has a representation group $\bar{S}_n$ of order $2(n!)$ and it has a central subgroup $Z = \{-1,1\}$ such that $\bar{S}_n/Z \approx S_n$, see [1]. Then, the irreducible representations or characters of $\bar{S}_n$ fall into two classes [1, 2]. Those, which have Z in their kernel, are referred to as ordinary representations or characters. The irreducible representations and characters are indexed by partitions λ of n and the character is denoted by $[\lambda]$. And the representations which do not have Z in their kernel are called the spin representation of S_n. The irreducible spin representations are indexed by partitions of n with distinct parts which are called bar partitions of n and denoted by $\langle\lambda\rangle$, see [2,3].

In fact, if $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$, $\lambda \mapsto n$ and if $n-m$ is even, then there is one irreducible spin character denoted by $\langle\lambda\rangle$ which is self-associate, and if $n-m$ is odd, then there are two associate spin characters denoted by $\langle\lambda\rangle$ and $\langle\lambda\rangle'$. The degree of these characters $\langle\lambda\rangle$ and $\langle\lambda\rangle'$ is [1,4]:

$$2^{\left[\frac{(n-m)}{2}\right]}\frac{n!}{\prod_{i=1}^{m}\lambda!}\prod_{1\le i<j\le m}\frac{\lambda_i-\lambda_j}{\lambda_i+\lambda_j}. \qquad (1)$$

The decomposition matrix gives the relationship between the irreducible spin characters and projective indecomposable spin characters of S_n.

In this paper, we determined the irreducible modular spin characters of the symmetric group S_{22} *modulo* 13 by using the method (r,r')-inducing (restricting) in [3], to distribute the spin characters into p-blocks and use Morris-Humphreys theorem [5]. The Brauer trees for spin characters of S_n, $13 \le n \le 20$ *modulo* $p = 13$ are found by Taban and Jawad [6], and for $n = 21$ was found by Yaseen [7].

2 Preliminaries

The fundamental theorem of the modular spin characters of symmetric groups S_n, which distributes the spin irreducible characters into p-block is called Morris-Humphreys Theorem [5]. Morris formulated this conjecture on how the irreducible spin characters of $\hat{S}_n$ are assigned into p-blocks [4], and proved by Humphreys [8].

Theorem 2.1. (Morris-Humphreys Theorem): Let λ and μ be bar partitions such that $\lambda \neq \mu$; then $\langle\lambda\rangle$ and $\langle\mu\rangle$ are in the same p-block if and only if $\lambda(\bar{p}) = \mu(\bar{p})$, (where p is an odd prime). The associative irreducible spin characters $\langle\lambda\rangle$ and $\langle\lambda\rangle'$ are in the same p-block if $\lambda(\bar{p}) \neq \lambda$, see [5].

Now, if $\varphi = \sum d\lambda\langle\lambda\rangle + d\lambda'\langle\lambda'\rangle$ is projective indecomposable spin character of S_n, (where $d\lambda' = 0$ if $\langle\lambda\rangle = \langle\lambda\rangle'$), then $\varphi \uparrow S_{n+1}$ is a projective spin character of S_{n+1} which is in general not indecomposable [3].

The following results are very useful to find the modular characters:

1. Every spin (modular, projective) character of S_n can be written as a linear combination with non-negative integer coefficients of the irreducible spin (irreducible modular, projective indecomposable) characters

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respectively [9].

2. Let H be a subgroup of the group G, then [10]:
   a. If φ is a modular (principle) character of a subgroup H of G, then $\varphi \uparrow G$ is a modular (principal) character of G, (where $\uparrow$ denotes inducing).
   b. If ψ is a modular (principal) character of group G, then $\psi \downarrow H$ is a modular (principal) character of a subgroup H, (where $\downarrow$ denotes the restricting).

3. Let B be a p-block G of defect one and let b be the number of p-conjugate characters to the irreducible ordinary character χ of G, then [11]:
   a. There exists a positive integer number N such that the irreducible ordinary characters lying in the block B can be partitioned into two disjoint classes: $B_1 = \{x \in B \mid b\ deg\ x \equiv N\ mod\ p^a\}$, $B_2 = \{x \in B \mid b\ deg\ x \equiv -N\ mod\ p^a\}$.
   b. Each coefficient of the decomposition matrix of the block B is 1 or 0.
   c. If α_1 and α_2 are not p-conjugate characters and belong to the same partition class B_1 or B_2 above, then they have no irreducible modular character in common.
   d. For every irreducible ordinary character χ in B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with multiplicity one.

4. Let G be a group of order $m = m_0 p^a$, where $(p, m_0) = 0$. If c is a principal character of H, then $deg\ c \equiv 0\ mod\ p^a$, see [12].

5. If c is a principal character of G for an odd prime p and all entries in c are divisible by positive integer q, then c/q is a principal character of G, see [10].

6. Let p be odd and n be even, then from [3]:
   a. If $p \nmid n$, then $\langle n\rangle = \varphi\langle n\rangle$ and $\langle n\rangle' = \varphi\langle n\rangle'$ are distinct irreducible modular spin characters of degree $2^{(n-2)/2}$.
   b. If $p \nmid n$ and $p \nmid (n-1)$, then $\langle n-1, 1\rangle = \varphi\langle n-1, 1\rangle^*$ is an irreducible modular spin characters of degree $2^{(n-2)/2}(n-2)$.
7. Let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$ be a bar partition of n, not a p-bar core, and B be the block containing $\langle\alpha\rangle$, then:
   a. If $n - m - m_0$ is even, then all irreducible modular spin characters in B are double.
   b. If $n - m - m_0$ is odd, then all irreducible modular spin characters in B are associate, (where m_0 is the number of parts of α divisible by p) [13]. For more details, see [14, 15, 16, 17].

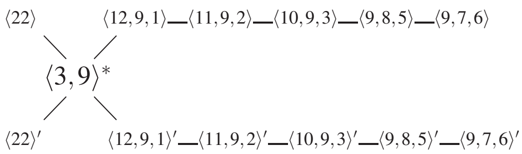
We shall use the following notations next: Irreducible modular spin characters (I.M.S), Modular spin characters (M.S), Principal indecomposable spin character (P.I.S), and Principal spin character (P.S).

3 The Brauer trees of the symmetric group $\bar{S}_n$, p=13

The group S_{22} has 133 irreducible spin characters and 121 of $(13, \alpha)$–regular classes, then the decomposition matrix of the spin character of S_{22}, $p = 13$ has 133 rows and 121 columns [3]. There are fifty seven 13-block, (Morris and Humphreys Theorem), eight of them B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8 of defect 1. All the 49 remaining characters form their own blocks $B_9, B_{10}, B_{11}, ..., B_{57}$ of defect 0, see [10], which are irreducible modular spin characters.

The principal block B_1, (The block which contains the spin character $\langle n\rangle$ or $\langle n\rangle'$), where B_1 contains the irreducible spin characters:
$\{\langle 22\rangle, \langle 22\rangle', \langle 13,9\rangle^*, \langle 12,9,1\rangle, \langle 12,9,1\rangle', \langle 11,9,2\rangle, \langle 11,9,2\rangle', \langle 10,9,3\rangle, \langle 10,9,3\rangle', \langle 9,8,5\rangle, \langle 9,8,5\rangle', \langle 9,7,6\rangle, \langle 9,7,6\rangle'\}$, has 13-bar core $\langle 9\rangle$.
B_2 contains the irreducible spin characters:
$\{\langle 14,8\rangle^*, \langle 13,8,1\rangle, \langle 13,8,1\rangle', \langle 11,8,2,1\rangle^*, \langle 10,8,3,1\rangle^*, \langle 9,8,4,1\rangle^*, \langle 8,7,6,1\rangle^*\}$, has 13-bar core $\langle 8,1\rangle$.
B_3 contains the irreducible spin characters:
$\{\langle 20,2\rangle, \langle 15,7\rangle^*, \langle 13,7,2\rangle, \langle 13,7,2\rangle', \langle 12,7,2,1\rangle^*, \langle 10,7,3,2\rangle^*, \langle 9,7,4,2\rangle^*, \langle 8,7,5,2\rangle^*\}$, has 13-bar core $\langle 7,2\rangle$.
B_4 contains the irreducible spin characters:
$\{\langle 19,3\rangle^*, \langle 16,6\rangle^*, \langle 13,6,3\rangle, \langle 13,6,3\rangle', \langle 12,6,3,1\rangle^*, \langle 11,6,3,2\rangle^*, \langle 9,6,4,3\rangle^*, \langle 8,6,5,3\rangle^*\}$, has 13-bar core $\langle 6,3\rangle$.
B_5 contains the irreducible spin characters:
$\{\langle 19,2,1\rangle, \langle 19,2,1\rangle', \langle 15,6,1\rangle, \langle 15,6,1\rangle', \langle 14,6,2\rangle, \langle 14,6,2\rangle', \langle 13,6,2,1\rangle^*, \langle 10,6,3,2,1\rangle, \langle 10,6,3,2,1\rangle', \langle 9,6,4,2,1\rangle, \langle 9,6,4,2,1\rangle', \langle 8,6,5,2,1\rangle, \langle 8,6,5,2,1\rangle'\}$, has 13-bar core $\langle 6,2,1\rangle$.
B_6 contains the irreducible spin characters:
$\{\langle 18,4\rangle^*, \langle 17,5\rangle^*, \langle 13,5,4\rangle, \langle 13,5,4\rangle', \langle 12,5,4,1\rangle^*, \langle 11,5,4,2\rangle^*, \langle 10,5,4,3\rangle^*, \langle 7,6,5,4\rangle^*\}$, has 13-bar core $\langle 5,4\rangle$.
B_7 contains the irreducible spin characters:
$\{\langle 18,3,1\rangle, \langle 18,3,1\rangle', \langle 16,5,1\rangle, \langle 16,5,1\rangle', \langle 14,5,3\rangle, \langle 14,5,3\rangle', \langle 13,5,3,1\rangle^*, \langle 11,5,3,2,1\rangle, \langle 11,5,3,2,1\rangle', \langle 9,5,4,3,1\rangle, \langle 9,5,4,3,1\rangle', \langle 7,6,5,3,1\rangle, \langle 7,6,5,3,1\rangle'\}$, has 13-bar core $\langle 5,3,1\rangle$.
B_8 contains the irreducible spin characters:
$\{\langle 17,3,2\rangle, \langle 17,3,2\rangle', \langle 16,4,2\rangle, \langle 16,4,2\rangle', \langle 15,4,3\rangle, \langle 15,4,3\rangle', \langle 13,4,3,2\rangle^*, \langle 12,4,3,2,1\rangle, \langle 12,4,3,2,1\rangle', \langle 8,5,4,3,2\rangle, \langle 8,5,4,3,2\rangle', \langle 7,6,4,3,2\rangle, \langle 7,6,4,3,2\rangle'\}$, has 13-bar core $\langle 4,3,2\rangle$.

Proposition 3.1. The Brauer tree for the principal block B_1 is:



Proof.

$deg\ \{\langle 22\rangle, \langle 22\rangle', \langle 12,9,1\rangle, \langle 12,9,1\rangle', \langle 10,9,3\rangle, \langle 10,9,3\rangle', \langle 9,7,6\rangle, \langle 9,7,6\rangle'\} \equiv 10\ mod 13$;

deg $\{\langle 13,9\rangle, \langle 11,9,2\rangle, \langle 11,9,2\rangle', \langle 9,8,5\rangle, \langle 9,8,5\rangle'\} \equiv -10\, mod 13$.
By using $(9,5)$-inducing of P.I.S of S_{21} to S_{22}, see Table (1), we have:

$$d_1 \uparrow^{(9,5)} S_{22} = \langle 22\rangle + \langle 22\rangle' + 2\langle 13,9\rangle^* = K_1 = D_1 + D_2 \quad (2)$$

$$d_2 \uparrow^{(9,5)} S_{22} = 2\langle 13,9\rangle^* + \langle 12,9,1\rangle + 2\langle 12,9,1\rangle' = K_2 = D_3 + D_4 \quad (3)$$

$$d_3 \uparrow^{(9,5)} S_{22} = \langle 12,9,1\rangle + \langle 12,9,1\rangle' + \langle 11,9,2\rangle + \langle 11,9,2\rangle' = K_3 = D_5 + D_6 \quad (4)$$

$$d_4 \uparrow^{(9,5)} S_{22} = \langle 11,9,2\rangle + \langle 11,9,2\rangle' + \langle 10,9,3\rangle + \langle 10,9,3\rangle' = K_4 = D_7 + D_8 \quad (5)$$

$$d_5 \uparrow^{(9,5)} S_{22} = \langle 10,9,3\rangle + \langle 10,9,3\rangle' + \langle 9,8,5\rangle + \langle 9,8,5\rangle' = K_5 = D_9 + D_{10} \quad (6)$$

$$d_6 \uparrow^{(9,5)} S_{22} = \langle 9,8,5\rangle + \langle 9,8,5\rangle' + \langle 9,7,6\rangle + \langle 9,7,6\rangle' = K_6 = D_{11} + D_{12} \quad (7)$$

Table 1: $D^{(1)}_{21,13}$

The spin characters	The decomposition matrix for the block B_1					
$\langle 21\rangle^*$	1					
$\langle 13,8\rangle$	1	1				
$\langle 13,8\rangle'$	1	1				
$\langle 12,8,1\rangle^*$		1	1			
$\langle 11,8,2\rangle^*$			1	1		
$\langle 10,8,3\rangle^*$				1	1	
$\langle 9,8,4\rangle^*$					1	1
$\langle 8,7,6\rangle^*$						1
	d_1	d_2	d_3	d_4	d_5	d_6

Since $\langle 22\rangle \neq \langle 22\rangle'$ on $\langle 13,\alpha\rangle$ regular classes, then $K_1 = D_1 + D_2$ preliminaries $6(a)$. Since $\langle 12,9,1\rangle \neq \langle 12,9,1\rangle'$, $\langle 11,9,2\rangle \neq \langle 11,9,2\rangle'$, $\langle 10,9,3\rangle \neq \langle 10,9,3\rangle'$, $\langle 9,8,5\rangle \neq \langle 9,8,5\rangle'$, and $\langle 9,7,6\rangle \neq \langle 9,7,6\rangle'$ on $\langle 13,\alpha\rangle$ regular classes, then K_1, K_2, K_3, K_4, K_5 and K_6 are splits, respectively. So we have the Brauer tree for B_1, and the decomposition matrix for this block $D^{(1)}_{22,13}$ in Table (2).

Proposition 3.2. The Brauer tree for the block B_2 is:
$\langle 21,1\rangle^*$—$\langle 14,8\rangle^*$—$\langle 13,8,1\rangle = \langle 13,8,1\rangle'$—$\langle 11,8,2,1\rangle^*$—$\langle 10,8,3,1\rangle^*$—$\langle 9,8,4,1\rangle^*$—$\langle 8,7,6,1\rangle^*$

Table 2: $D^{(1)}_{22,13}$

The spin characters	The decomposition matrix for the block B_1											
$\langle 22\rangle$	1											
$\langle 22\rangle'$		1										
$\langle 13,9\rangle^*$	1	1	1	1								
$\langle 12,9,1\rangle$			1		1							
$\langle 12,9,1\rangle'$				1		1						
$\langle 11,9,2\rangle$					1		1					
$\langle 11,9,2\rangle'$						1		1				
$\langle 10,9,3\rangle$							1		1			
$\langle 10,9,3\rangle'$								1		1		
$\langle 9,8,5\rangle$									1		1	
$\langle 9,8,5\rangle'$										1		1
$\langle 9,7,6\rangle$											1	
$\langle 9,7,6\rangle'$												1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}

Proof.
deg $\{\langle 21,1\rangle^*, \langle 13,8,1\rangle + \langle 13,8,1\rangle', \langle 10,8,3,1\rangle^*, \langle 8,7,6,1\rangle^*\} \equiv 5\, mod 13$.
deg $\{\langle 14,8\rangle^*, \langle 11,8,2,1\rangle^*, \langle 9,8,4,1\rangle^*\} \equiv -5\, mod 13$.
By using $(8,6)$-inducing of P.I.S of S_{21} to S_{22}, see Table (3), we have:

Table 3: $D^{(2)}_{21,13}$

The spin characters	The decomposition matrix for the block B_2											
$\langle 20,1\rangle$	1											
$\langle 20,1\rangle'$		1										
$\langle 14,7\rangle$	1		1									
$\langle 14,7\rangle'$		1		1								
$\langle 13,7,1\rangle^*$			1	1	1	1						
$\langle 11,7,2,1\rangle$					1		1					
$\langle 11,7,2,1\rangle'$						1		1				
$\langle 10,7,3,1\rangle$							1		1			
$\langle 10,7,3,1\rangle'$								1		1		
$\langle 9,7,4,1\rangle$									1		1	
$\langle 9,7,4,1\rangle'$										1		1
$\langle 8,7,5,1\rangle$											1	
$\langle 8,7,5,1\rangle'$												1
	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}

$$d_7 \uparrow^{(8,6)} S_{22} = \langle 21,1\rangle^* + \langle 14,8\rangle^* = D_{13} \quad (8)$$

$$d_9 \uparrow^{(8,6)} S_{22} = \langle 14,8\rangle + \langle 13,8,1\rangle + \langle 13,8,1\rangle' = D_{14} \quad (9)$$

$$d_{11} \uparrow^{(8,6)} S_{22} = \langle 13,8,1\rangle + \langle 13,8,1\rangle' + \langle 11,8,2,1\rangle^* = D_{15} \quad (10)$$

$$d_{13}\uparrow^{(8,6)} S_{22} = \langle 11,8,2,1\rangle^* + \langle 10,8,3,1\rangle^* = D_{16} \quad (11)$$

$$d_{15}\uparrow^{(8,6)} S_{22} = \langle 10,8,3,1\rangle^* + \langle 9,8,4,1\rangle^* = D_{17} \quad (12)$$

$$d_{17}\uparrow^{(8,6)} S_{22} = \langle 9,8,4,1\rangle^* + \langle 8,7,6,1\rangle^* = D_{18} \quad (13)$$

Hence, we have the Brauer tree for B_2, and the decomposition matrix for this block $D^{(2)}_{22,13}$ in Table (4).

Table 4: $D^{(2)}_{22,13}$

The spin characters	The decomposition matrix for the block B_2					
$\langle 21,1\rangle^*$	1					
$\langle 14,8\rangle$	1	1				
$\langle 13,8,1\rangle$		1	1			
$\langle 13,8,1\rangle'$		1	1			
$\langle 11,8,2,1\rangle^*$			1	1		
$\langle 10,8,3,1\rangle^*$				1	1	
$\langle 9,8,4,1\rangle^*$					1	1
$\langle 8,7,6,1\rangle^*$						1
	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}

Proposition 3.3. The Brauer tree for the block B_3 is:
$\langle 20,2\rangle^*$—$\langle 15,7\rangle^*$—$\langle 13,7,2\rangle = \langle 13,7,2\rangle'$—$\langle 12,7,2,1\rangle^*$—$\langle 10,7,3,2\rangle^*$—$\langle 9,7,4,2\rangle^*$—$\langle 8,7,5,2\rangle^*$

Proof.

$deg\ \{\langle 20,2\rangle^*, \langle 13,7,2\rangle + \langle 13,7,2\rangle', \langle 10,7,3,2\rangle^*, \langle 8,7,5,2\rangle^*\} \equiv 5\ mod 13$.
$deg\ \{\langle 15,7\rangle^*, \langle 12,7,2,1\rangle^*, \langle 9,7,4,2\rangle^*\} \equiv -5\ mod 13$.
Now, by using $(2,12)$-inducing of P.I.S of S_{21} to S_{22}, see Table (3) $D_{21,13}$, we have:

$$d_7\uparrow^{(2,12)} S_{22} = \langle 20,2\rangle^* + \langle 15,7\rangle^* = D_{19} \quad (14)$$

$$d_9\uparrow^{(2,12)} S_{22} = \langle 15,7\rangle^* + \langle 13,7,2\rangle + \langle 13,7,2\rangle' = D_{20} \quad (15)$$

$$d_{11}\uparrow^{(2,12)} S_{22} = \langle 13,7,2\rangle + \langle 13,7,2\rangle' + \langle 12,7,2,1\rangle^* = D_{21} \quad (16)$$

$$d_{13}\uparrow^{(2,12)} S_{22} = \langle 12,7,2,1\rangle^* + \langle 10,7,3,2\rangle^* = D_{22} \quad (17)$$

$$d_{15}\uparrow^{(2,12)} S_{22} = \langle 10,7,3,2\rangle^* + \langle 9,7,4,2\rangle^* = D_{23} \quad (18)$$

$$d_{17}\uparrow^{(2,12)} S_{22} = \langle 9,7,4,2\rangle^* + \langle 8,7,5,2\rangle^* = D_{24} \quad (19)$$

Then, we have the Brauer tree for B_3, and the decomposition matrix for this block $D^{(3)}_{22,13}$ in Table (5).

Table 5: $D^{(3)}_{22,13}$

The spin characters	The decomposition matrix for the block B_3					
$\langle 20,2\rangle^*$	1					
$\langle 15,7\rangle$	1	1				
$\langle 13,7,2\rangle$		1	1			
$\langle 13,7,2\rangle'$		1	1			
$\langle 12,7,2,1\rangle^*$			1	1		
$\langle 10,7,3,2\rangle^*$				1	1	
$\langle 9,7,4,2\rangle^*$					1	1
$\langle 8,7,5,2\rangle^*$						1
	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}

Proposition 3.4. The Brauer tree for the block B_4 is:
$\langle 19,3\rangle^*$—$\langle 16,6\rangle^*$—$\langle 13,6,3\rangle = \langle 13,6,3\rangle'$—$\langle 12,6,3,1\rangle^*$—$\langle 11,6,3,2\rangle^*$—$\langle 8,6,5,3\rangle^*$

Proof.

$deg\ \{\langle 19,3\rangle^*, \langle 13,6,2\rangle + \langle 13,6,2\rangle', \langle 11,6,3,2\rangle^*, \langle 8,6,5,3\rangle^*\} \equiv 7\ mod 13$.
$deg\ \{\langle 16,6\rangle^*, \langle 12,6,3,1\rangle^*, \langle 9,6,4,1\rangle^*\} \equiv -7\ mod 13$.
We apply $(3,11)$-inducing of P.I.S of S_{21} to S_{22}, see Table (6) $D_{21,3}$, we have:

Table 6: $D^{(3)}_{21,13}$

The spin characters	The decomposition matrix for the block B_3											
$\langle 19,2\rangle$	1											
$\langle 19,2\rangle'$		1										
$\langle 15,6\rangle$	1		1									
$\langle 15,6\rangle'$		1		1								
$\langle 13,6,2\rangle^*$			1	1	1	1						
$\langle 12,6,2,1\rangle$					1		1					
$\langle 12,6,2,1\rangle'$						1		1				
$\langle 10,6,3,2\rangle$							1		1			
$\langle 10,6,3,2\rangle'$								1		1		
$\langle 9,6,4,2\rangle$									1		1	
$\langle 9,6,4,2\rangle'$										1		1
$\langle 8,6,5,2\rangle$											1	
$\langle 8,6,5,2\rangle'$												1
	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

$$d_{19}\uparrow^{(3,11)} S_{22} = \langle 19,3\rangle^* + \langle 16,6\rangle^* = D_{25} \quad (20)$$

$$d_{21}\uparrow^{(3,11)} S_{22} = \langle 16,6\rangle^* + \langle 13,6,1\rangle + \langle 13,6,1\rangle' = D_{26} \quad (21)$$

$$d_{23}\uparrow^{(3,11)} S_{22} = \langle 13,6,1\rangle + \langle 13,6,1\rangle' + \langle 12,6,3,1\rangle^* = D_{27} \quad (22)$$

$$d_{25}\uparrow^{(3,11)} S_{22} = \langle 12,6,3,1\rangle^* + \langle 11,6,3,2\rangle^* = D_{28} \quad (23)$$

$$d_{27}\uparrow^{(3,11)} S_{22} = \langle 11,6,3,2\rangle^* + \langle 9,6,4,3\rangle^* = D_{29} \quad (24)$$

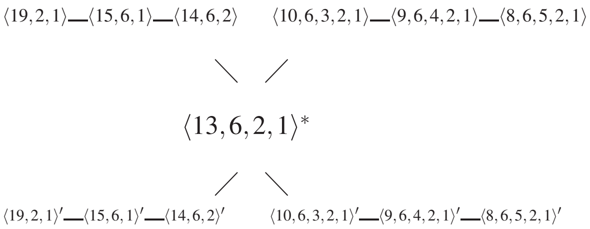
$$d_{29}\uparrow^{(3,11)} S_{22} = \langle 9,6,4,3\rangle^* + \langle 8,6,5,3\rangle^* = D_{30} \quad (25)$$

So we have the Brauer tree for B_4, and the decomposition matrix for this block $D^{(4)}_{22,13}$ in Table (7).

Table 7: $D^{(4)}_{22,13}$

The spin characters	The decomposition matrix for the block B_4					
$\langle 19,3\rangle^*$	1					
$\langle 16,6\rangle^*$	1	1				
$\langle 13,6,3\rangle$		1	1			
$\langle 13,6,3\rangle'$		1	1			
$\langle 12,6,3,1\rangle^*$			1	1		
$\langle 11,6,3,2\rangle^*$				1	1	
$\langle 9,6,4,3\rangle^*$					1	1
$\langle 8,6,5,3\rangle^*$						1
	D_{25}	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}

Proposition 3.5. The Brauer tree for the block B_5 is:



Proof.

deg $\{\langle 19,2,1\rangle, \langle 19,2,1\rangle', \langle 14,6,2\rangle, \langle 14,6,2\rangle', \langle 10,6,3,2,1\rangle, \langle 10,6,3,2,1\rangle', \langle 8,6,5,2,1\rangle, \langle 8,6,5,2,1\rangle'\} \equiv 7 \; mod 13$;
deg $\{\langle 15,6,1\rangle, \langle 15,6,1\rangle', \langle 13,6,2,1\rangle^*, \langle 9,6,4,2,1\rangle, \langle 9,6,4,2,1\rangle'\} \equiv -7 \; mod 13$.
Now, by using $(1,0)$-inducing of P.I.S of S_{21} to S_{22}, see the Table (6) of $D_{21,13}$, we have:

$$d_{19}\uparrow^{(1,0)} S_{22} = \langle 19,2,1\rangle + \langle 15,6,1\rangle = D_{31} \quad (26)$$

$$d_{20}\uparrow^{(1,0)} S_{22} = \langle 19,2,1\rangle' + \langle 15,6,1\rangle' = D_{32} \quad (27)$$

$$d_{21}\uparrow^{(1,0)} S_{22} = \langle 15,6,1\rangle + \langle 14,6,2\rangle + \langle 14,6,2\rangle' + \langle 13,6,2,1\rangle^* = K_1 \quad (28)$$

$$d_{22}\uparrow^{(1,0)} S_{22} = \langle 15,6,1\rangle' + \langle 14,6,2\rangle + \langle 14,6,2\rangle' + \langle 13,6,2,1\rangle^* = K_2 \quad (29)$$

$$d_{23}\uparrow^{(1,0)} S_{22} = \langle 14,6,2\rangle' + \langle 14,6,2\rangle' + 2\langle 13,6,2,1\rangle^* = K_3 \quad (30)$$

$$d_{25}\uparrow^{(1,0)} S_{22} = \langle 13,6,2,1\rangle^* + \langle 10,6,3,2,1\rangle = D_{37} \quad (31)$$

$$d_{26}\uparrow^{(1,0)} S_{22} = \langle 13,6,2,1\rangle^* + \langle 10,6,3,2,1\rangle' = D_{38} \quad (32)$$

$$d_{27}\uparrow^{(1,0)} S_{22} = \langle 10,6,3,2,1\rangle + \langle 9,6,4,2,1\rangle = D_{39} \quad (33)$$

$$d_{28}\uparrow^{(1,0)} S_{22} = \langle 10,6,3,2,1\rangle' + \langle 9,6,4,2,1\rangle' = D_{40} \quad (34)$$

$$d_{29}\uparrow^{(1,0)} S_{22} = \langle 9,6,4,2,1\rangle + \langle 8,6,5,2,1\rangle = D_{41} \quad (35)$$

$$d_{30}\uparrow^{(1,0)} S_{22} = \langle 9,6,4,2,1\rangle' + \langle 8,6,5,2,1\rangle' = D_{42} \quad (36)$$

$$\langle 14,6,2,1\rangle\downarrow_{(1,0)} S_{22} = \langle 13,6,2,1\rangle^* + \langle 14,6,2\rangle = D_{35}, \quad (37)$$

since $\langle 14,6,2,1\rangle$ I.M.S in S_{23}, and

$$\langle 14,6,2,1\rangle'\downarrow_{(1,0)} S_{22} = \langle 13,6,2,1\rangle^* + \langle 14,6,2\rangle' = D_{36}, \quad (38)$$

since $\langle 14,6,2,1\rangle'$ I.M.S in S_{23}. Then K_3 splits to D_{35}, D_{36}, and

$$\langle 15,6,2\rangle\downarrow_{(2,12)} S_{22} = \langle 14,6,2\rangle + \langle 15,6,1\rangle = D_{33} = K_1 - D_{36}, \quad (39)$$

since $\langle 15,6,2\rangle$ I.M.S in S_{23}, and

$$\langle 15,6,2\rangle'\downarrow_{(2,12)} S_{22} = \langle 14,6,2\rangle' + \langle 15,6,1\rangle' = D_{34} = K_2 - D_{35}, \quad (40)$$

since $\langle 15,6,2\rangle'$ I.M.S in S_{23}. Therefore, we have the Brauer tree for B_5, and the decomposition matrix for this block $D^{(5)}_{22,13}$ in Table (8).

Table 8: $D^{(5)}_{22,13}$

The spin characters	The decomposition matrix for the block B_5											
$\langle 19,2,1\rangle$	1											
$\langle 19,2,1\rangle'$		1										
$\langle 15,6,1\rangle$	1		1									
$\langle 15,6,1\rangle'$		1		1								
$\langle 14,6,2\rangle$			1		1							
$\langle 14,6,2\rangle'$				1		1						
$\langle 13,6,2,1\rangle^*$					1	1	1	1				
$\langle 10,6,3,2,1\rangle$							1		1			
$\langle 10,6,3,2,1\rangle'$								1		1		
$\langle 9,6,4,2,1\rangle$									1		1	
$\langle 9,6,4,2,1\rangle'$										1		1
$\langle 8,6,5,2,1\rangle$											1	
$\langle 8,6,5,2,1\rangle'$												1
	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}	D_{41}	D_{42}

Proposition 3.6. The Brauer tree for the block B_6 is:

$\langle 18,4\rangle^*$—$\langle 17,5\rangle^*$—$\langle 13,5,4\rangle = \langle 13,5,4\rangle'$—$\langle 11,5,4,2\rangle^*$—$\langle 10,5,4,3,1\rangle^*$—$\langle 7,6,5,4\rangle^*$

Proof.

$deg\ \{\langle 18,4\rangle^*, \langle 13,5,4\rangle + \langle 13,5,4\rangle', \langle 11,5,4,2\rangle^*, \langle 7,6,5,4\rangle^*\} \equiv 10\ mod 13.$
$deg\ \{\langle 17,5\rangle^*, \langle 12,5,4,1\rangle^*, \langle 10,5,4,3\rangle^*\} \equiv -10\ mod 13.$
Now, by using $(4,10)$-inducing of P.I.S of S_{21} to S_{22}, see Table (9) $D_{21,13}$, we have:

Table 9: $D^{(4)}_{21,13}$

The spin characters	The decomposition matrix for the block B_4											
$\langle 18,3\rangle$	1											
$\langle 18,3\rangle'$		1										
$\langle 16,5\rangle$	1		1									
$\langle 16,5\rangle'$		1		1								
$\langle 13,5,3\rangle^*$			1	1	1	1						
$\langle 12,5,3,1\rangle$					1		1					
$\langle 12,5,3,1\rangle'$						1		1				
$\langle 11,5,3,2\rangle$							1		1			
$\langle 11,5,3,2\rangle'$								1		1		
$\langle 9,5,4,3\rangle$									1		1	
$\langle 9,5,4,3\rangle'$										1		1
$\langle 7,6,5,3\rangle$											1	
$\langle 7,6,5,3\rangle'$												1
	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}

$$d_{31} \uparrow^{(4,10)} S_{22} = \langle 18,4\rangle^* + \langle 17,5\rangle^* = D_{43} \quad (41)$$

$$d_{33} \uparrow^{(4,10)} S_{22} = \langle 17,5\rangle^* + \langle 13,5,4\rangle + \langle 13,5,4\rangle' = D_{44} \quad (42)$$

$$d_{35} \uparrow^{(4,10)} S_{22} = \langle 13,5,4\rangle + \langle 13,5,4\rangle' + \langle 12,5,4,1\rangle^* = D_{45} \quad (43)$$

$$d_{37} \uparrow^{(4,10)} S_{22} = \langle 12,5,4,1\rangle^* + \langle 11,5,4,2\rangle^* = D_{46} \quad (44)$$

$$d_{39} \uparrow^{(4,10)} S_{22} = \langle 11,5,4,2\rangle^* + \langle 10,5,4,3\rangle^* = D_{47} \quad (45)$$

$$d_{41} \uparrow^{(4,10)} S_{22} = \langle 10,5,4,3\rangle^* + \langle 7,6,5,4\rangle^* = D_{48} \quad (46)$$

Then, we have the Brauer tree for B_6, and the decomposition matrix for this block $D^{(6)}_{22,13}$ in Table (10).

Proposition 3.7. The Brauer tree for the block B_7 is:

$\langle 18,3,1\rangle$—$\langle 16,5,1\rangle$—$\langle 14,5,3\rangle$ $\langle 11,5,3,2,1\rangle$—$\langle 9,5,4,3,1\rangle$—$\langle 7,6,5,3,1\rangle$

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$\langle 13,5,3,1\rangle^*$

Table 10: $D^{(6)}_{22,13}$

The spin characters	The decomposition matrix for the block B_6					
$\langle 18,4\rangle^*$	1					
$\langle 17,5\rangle^*$	1	1				
$\langle 13,5,4\rangle$		1	1			
$\langle 13,5,4\rangle'$		1	1			
$\langle 12,5,4,1\rangle^*$			1	1		
$\langle 11,5,4,2\rangle^*$				1	1	
$\langle 10,5,4,3\rangle^*$					1	1
$\langle 7,6,5,4\rangle^*$						1
	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}

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$\langle 18,3,1\rangle'$—$\langle 16,5,1\rangle'$—$\langle 14,5,3\rangle'$ $\langle 11,5,3,2,1\rangle'$—$\langle 9,5,4,3,1\rangle'$—$\langle 7,6,5,3,1\rangle'$

Proof.

$deg\ \{\langle 18,3,1\rangle, \langle 18,3,1\rangle', \langle 14,5,3\rangle, \langle 14,5,3\rangle', \langle 11,5,3,2,1\rangle, \langle 11,5,3,2,1\rangle', \langle 7,6,5,3,1\rangle, \langle 7,6,5,3,1\rangle'\} \equiv 2\ mod 13;$
$deg\ \{\langle 16,5,1\rangle, \langle 16,5,1\rangle', \langle 13,5,3,1\rangle^*, \langle 9,5,4,3,1\rangle, \langle 9,5,4,3,1\rangle'\} \equiv -2\ mod 13.$
Now, by using $(1,0)$-inducing of P.I.S of S_{21} to S_{22}, see Table (9) of $D_{21,13}$, we have:

$$d_{31} \uparrow^{(1,0)} S_{22} = \langle 18,3,1\rangle + \langle 16,5,1\rangle = D_{49} \quad (47)$$

$$d_{32} \uparrow^{(1,0)} S_{22} = \langle 18,3,1\rangle' + \langle 16,5,1\rangle' = D_{50} \quad (48)$$

$$d_{33} \uparrow^{(1,0)} S_{22} = \langle 16,5,1\rangle + \langle 14,5,3\rangle + \langle 14,5,3\rangle' + \langle 13,5,3,1\rangle^* = K_1 \quad (49)$$

$$d_{34} \uparrow^{(1,0)} S_{22} = \langle 16,5,1\rangle' + \langle 14,5,3\rangle + \langle 14,5,3\rangle' + \langle 13,5,3,1\rangle^* = K_2 \quad (50)$$

$$d_{35} \uparrow^{(1,0)} S_{22} = \langle 14,5,3\rangle + \langle 14,5,3\rangle' + 2\langle 13,5,3,1\rangle^* = K_3 \quad (51)$$

$$d_{37} \uparrow^{(1,0)} S_{22} = \langle 13,5,3,1\rangle^* + \langle 11,5,3,2,1\rangle = D_{55} \quad (52)$$

$$d_{38} \uparrow^{(1,0)} S_{22} = \langle 13,5,3,1\rangle^* + \langle 11,5,3,2,1\rangle' = D_{56} \quad (53)$$

$$d_{39} \uparrow^{(1,0)} S_{22} = \langle 11,5,3,2,1\rangle + \langle 9,5,4,3,1\rangle = D_{57} \quad (54)$$

$$d_{40} \uparrow^{(1,0)} S_{22} = \langle 11,5,3,2,1\rangle' + \langle 9,5,4,3,1\rangle' = D_{58} \quad (55)$$

$$d_{41} \uparrow^{(1,0)} S_{22} = \langle 9,5,4,3,1\rangle + \langle 7,6,5,3,1\rangle = D_{59} \quad (56)$$

$$d_{42} \uparrow^{(1,0)} S_{22} = \langle 9,5,4,3,1\rangle' + \langle 7,6,5,3,1\rangle' = D_{60} \quad (57)$$

$$\langle 14,5,3,1\rangle \downarrow_{(1,0)} S_{22} = \langle 14,5,3\rangle + \langle 13,5,3,1\rangle^* = D_{53}, \quad (58)$$

since $\langle 14,5,3,1\rangle$ I.M.S in S_{23}, and

$$\langle 14,5,3,1\rangle \downarrow_{(1,0)} S_{22} = \langle 14,5,3\rangle' + \langle 13,5,3,1\rangle^* = D_{54}, \quad (59)$$

since $\langle 16,5,1\rangle \neq \langle 16,5,1\rangle'$, $\langle 14,5,3\rangle \neq \langle 14,5,3\rangle'$ on $(13,\alpha)$ regular classes, then $K_1 - D_{54} = D_{51}$ and $K_2 - D_{53} = D_{52}$. Then, we have the Brauer tree for B_7, and the decomposition matrix for this block $D^{(7)}_{22,13}$ in Table (11).

Table 11: $D^{(7)}_{22,13}$

The spin characters	The decomposition matrix for the block B_7											
$\langle 18,3,1\rangle$	1											
$\langle 18,3,1\rangle'$		1										
$\langle 16,5,1\rangle$	1		1									
$\langle 16,5,1\rangle'$		1		1								
$\langle 14,5,3\rangle$			1		1							
$\langle 14,5,3\rangle'$				1		1						
$\langle 13,5,3,1\rangle^*$					1	1	1	1				
$\langle 11,5,3,2,1\rangle$							1		1			
$\langle 11,5,3,2,1\rangle'$								1		1		
$\langle 9,5,4,3,1\rangle$									1		1	
$\langle 9,5,4,3,1\rangle'$										1		1
$\langle 7,6,5,3,1\rangle$											1	
$\langle 7,6,5,3,1\rangle'$												1
	D_{49}	D_{50}	D_{51}	D_{52}	D_{53}	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}

Proposition 3.8. The Brauer tree for the block B_8 is:

$\langle 17,3,2\rangle$—$\langle 16,4,2\rangle$—$\langle 15,4,3\rangle$ $\langle 12,4,3,2,1\rangle$—$\langle 8,5,4,3,2\rangle$—$\langle 7,6,4,3,2\rangle$

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$\langle 13,4,3,2\rangle^*$

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$\langle 17,3,2\rangle'$—$\langle 16,4,2\rangle'$—$\langle 15,4,3\rangle'$ $\langle 12,4,3,2,1\rangle'$—$\langle 8,5,4,3,2\rangle'$—$\langle 7,6,4,3,2\rangle'$

Proof.
$deg\ \{\langle 17,3,2\rangle, \langle 17,3,2\rangle', \langle 15,4,3\rangle, \langle 15,4,3\rangle', \langle 12,4,3,2,1\rangle, \langle 12,4,3,2,1\rangle', \langle 7,6,4,3,2\rangle, \langle 7,6,4,3,2\rangle'\} \equiv 8\ mod 13$;
$deg\ \{\langle 16,4,2\rangle, \langle 16,4,2\rangle', \langle 13,4,3,2\rangle^*, \langle 8,5,4,3,2\rangle, \langle 8,5,4,3,2\rangle'\} \equiv -8\ mod 13$.
Now, by using $(r,\bar{r})$-inducing of P.I.S of S_{21} to S_{22}, see Table (12) of $D_{21,13}$, we have:

Table 12: $D^{(6)}_{21,13}$

The spin characters	The decomposition matrix for the block B_6					
$\langle 17,3,1\rangle^*$	1					
$\langle 16,4,1\rangle^*$	1	1				
$\langle 14,4,3\rangle^*$		1	1			
$\langle 13,4,3,1\rangle$			1	1		
$\langle 13,4,3,1\rangle'$			1	1		
$\langle 11,4,3,2,1\rangle^*$				1	1	
$\langle 8,5,4,3,1\rangle^*$					1	1
$\langle 7,6,4,3,1\rangle^*$						1
	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}

$$d_{49} \uparrow^{(2,12)} S_{22} = \langle 17,3,2\rangle + \langle 17,3,2\rangle' + \langle 16,4,2\rangle + \langle 16,4,2\rangle' = K_1 = D_{61} + D_{62} \quad (60)$$

$$d_{50} \uparrow^{(2,12)} S_{22} = \langle 16,4,2\rangle + \langle 16,4,2\rangle' + \langle 15,4,3\rangle + \langle 15,4,3\rangle' = K_2 = D_{63} + D_{64} \quad (61)$$

$$d_{51} \uparrow^{(2,12)} S_{22} = \langle 15,4,3\rangle + \langle 15,4,3\rangle' + 2\langle 13,4,3,2\rangle^* = K_3 = D_{65} + D_{66} \quad (62)$$

$$d_{52} \uparrow^{(2,12)} S_{22} = 2\langle 13,4,3,2\rangle^* + \langle 12,4,3,2,1\rangle + \langle 12,4,3,2,1\rangle' = K_4 = D_{67} + D_{68} \quad (63)$$

$$d_{53} \uparrow^{(2,12)} S_{22} = \langle 12,4,3,2,1\rangle + \langle 12,4,3,2,1\rangle' + \langle 8,5,4,3,2\rangle + \langle 8,5,4,3,2\rangle' = K_5 = D_{69} + D_{70} \quad (64)$$

$$d_{54} \uparrow^{(2,12)} S_{22} = \langle 8,5,4,3,2\rangle + \langle 8,5,4,3,2\rangle' + \langle 7,6,4,3,2\rangle + \langle 7,6,4,3,2\rangle' = K_6 = D_{71} + D_{72} \quad (65)$$

Since $\langle 17,3,2\rangle \neq \langle 17,3,2\rangle'$, $\langle 16,4,2\rangle \neq \langle 16,4,2\rangle'$, $\langle 15,4,3\rangle \neq \langle 15,4,3\rangle'$, $\langle 12,4,3,2,1\rangle \neq \langle 12,4,3,2,1\rangle'$, $\langle 8,5,4,3,2\rangle \neq \langle 8,5,4,3,2\rangle'$, $\langle 7,6,4,3,2\rangle \neq \langle 7,6,4,3,2\rangle'$ on $(13,\alpha)$ regular classes, then K_1,K_2,K_3,K_4,K_5 and K_6 are splits, respectively. Thus, we have the Brauer tree for B_8, and the decomposition matrix for this block $D^{(8)}_{22,13}$ in Table (13).

Table 13: $D^{(8)}_{22,13}$

The spin characters	The decomposition matrix for the block B_8											
$\langle 17,3,2\rangle$	1											
$\langle 17,3,2\rangle'$		1										
$\langle 16,4,2\rangle$	1		1									
$\langle 16,4,2\rangle'$		1		1								
$\langle 15,4,3\rangle$			1		1							
$\langle 15,4,3\rangle'$				1		1						
$\langle 13,4,3,2\rangle^*$					1	1	1	1				
$\langle 12,4,3,2,1\rangle$							1		1			
$\langle 12,4,3,2,1\rangle'$								1		1		
$\langle 8,5,4,3,2\rangle$									1		1	
$\langle 8,5,4,3,2\rangle'$										1		1
$\langle 7,6,4,3,2\rangle$											1	
$\langle 7,6,4,3,2\rangle'$												1
	D_{61}	D_{62}	D_{63}	D_{64}	D_{65}	D_{66}	D_{67}	D_{68}	D_{69}	D_{70}	D_{71}	D_{72}

4 Conclusion

In this work, motivated by previous results given in the papers [3,7,10,11], we conclude that all blocks of defect one and the decomposition numbers are zero or one. Also we compute the Brauer trees of the symmetric group S_{22} modulo $P = 13$. Finally, all the 13-decomposition matrices of spin characters of S_{22} are found.

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References

[1] I. Schur, Uber die Darstellung der symmetrischen und der alternierenden Gruppe durch gebrochene lineare subtituttionen, j.Reine Angew.Math., **139**, 155-250 (1911).

[2] A.O. Morris, The spin representation of the symmetric group, proc. London Math. Soc., **12**, 55-76 (1962).

[3] A.O. Morris and A.K. Yaseen, Decomposition matrices for spin characters of symmetric group, Proc. Of Royal Society of Edinburgh, **108**A, 145–164 (1988).

[4] A.O. Morris, The spin representation of the symmetric group, Canada J. Math., **17**, 543-549 (1965).

[5] A.K. Yaseen. Modular spin representations of the symmetric groups. Ph.D thesis, Aberystwyth University, United Kigdom, (1987).

[6] S.A. Taban and M.M. Jawad, Brauer trees for spin characters of S_n, $13 \leq n \leq 20$ $modulo$ $p = 13$, Basrah Journal of Science (A), **35**, 106-112 (2017).

[7] A.K. Yaseen, The Brauer trees of the symmetric groups S_{21}, $modulo$ $p = 13$, Basrah Journal of Science, **37**, 126-140 (2019).

[8] J.F. Humphreys, Blocks of the Projective representations of symmetric groups, J. London math. Society, **33**, 441-452 (1986).

[9] L. Dornhoff, Group Representation Theory, Marcel Dekker Inc., New York USA, (1972).

[10] G. James and A. Kerber, The representation theory of the symmetric groups, Addison-Wesley Pub. Co., Reading, Mass., Boston USA, (1981).

[11] B.M. Puttaswamaiah and J.D. Dixon, Modular representation of finite groups, Academic press, Cambridge USA, (1977).

[12] G. James, The modular characters of Mathew groups, J. Algebra, **27**, 57-111 (1973).

[13] C. Bessenrodt, A.O. Morris and J.B. Olsson, Decomposition matrices for spin Characters of symmetric groups at characteristic 3, J. Algebra, **164**, 146-172 (1994).

[14] C.W. Curtis and I. Reiner, Representation theory of finite groups and associative algebra, Canadian Mathematical Society, Canada, (1962).

[15] I. M. Isaacs, Character Theory of Finite Group, Academic press, Cambridge USA, (1976).

[16] L. Mass. Modular spin characters of symmetric group. Ph.D Dissertation, Universität Duisburg–Essen, Jermany, (2011).

[17] D.B. Wales, Some projective representations of S_n, J. Algebra, **61**, 37-57 (1979).

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