

# Fixed point theorem in intuitionistic fuzzy 3–metric space under strict contractive conditions

H. M. Abu-Donia<sup>1</sup>, H. A. Atia<sup>1</sup>, and Omnia M. A. Khater<sup>1,2\*</sup>

<sup>1</sup>Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

<sup>2</sup>Department of Basic Science, Zagazig Higher Institute of Engineering and Technology, 10<sup>th</sup> of Ramadan City, Sharkia, Egypt

Received: 9 Feb. 2020, Revised: 12 July 2020, Accepted: 19 July 2020

Published online: 1 Nov. 2020

**Abstract:** The present paper aims to introduce the notion of intuitionistic fuzzy 3–metric space and prove some common fixed point theorems under strict contractive conditions.

**Keywords:** Fixed point, Intuitionistic fuzzy 3–metric space, Weakly compatible.

## 1 Introduction

In 1965, Zadeh [14] introduced the theory of fuzzy set which is an important and useful branch of mathematics, science and engineering. Kramosil and Michalek [9] introduced the definition of fuzzy metric space. Authors [13,6] investigated fixed point theorems in fuzzy metric space. Park described the concept of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm. In this paper [10] the authors studied the common fixed point theorems in intuitionistic fuzzy metric space under strict contractive. Jungck [8] defined the concept of weak commutativity in metric space and compatibility and proved the uniqueness of fixed point theorems. Abu-Donia et al. [2,3] investigated common fixed point theorems in intuitionistic fuzzy metric spaces and intuitionistic  $(\phi, \psi)$ -contractive mappings and fixed point theorem using  $\psi$ -contraction and  $(\phi, \phi)$ -contraction in probabilistic 2–metric spaces.

Gähler [5] introduced the concept of 2–metric space which was proposed in Euclidean space by the area function. Sharma [12] described the definition of fuzzy 2-metric space which is the generalization of the intuitionistic fuzzy metric space and proved some common fixed point theorems. Mursaleen and Lohani [7] using the idea of intuitionistic fuzzy metric space and defined the concept of intuitionistic fuzzy 2–metric space and proved the common fixed point theorems in intuitionistic fuzzy 2–metric space. Aamri [1] described

the notion property (E.A.). Shrivastava et al. [11] presented the definition of the weak compatible mappings in intuitionistic fuzzy 2-metric spaces. Chauhan and Singh [4] proved fixed point theorem in intuitionistic fuzzy-3 metric space.

In this paper we obtain some fixed point theorems in intuitionistic fuzzy 3-metric spaces on two mappings and four using the concept of weakly compatible and the property (E.A.).

## 2 preliminaries

**Definition 21** A binary operation  $*$  :  $[0, 1] \times [0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-norm if  $*$  satisfies the following conditions:

(i)  $*$  is commutative and associative;

(ii)  $*$  is continuous;

(iii)  $a * 1 = a$  for all  $a \in [0, 1]$ ;

(iv)  $a_1 * b_1 * c_1 * d_1 \leq a_2 * b_2 * c_2 * d_2$  whenever  $a_1 \leq a_2, b_1 \leq b_2, c_1 \leq c_2, d_1 \leq d_2$  and  $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 \in [0, 1]$ .

For example:  $a * b * c * d = \min\{a, b, c, d\}$  or  $a * b * c * d = a.b.c.d.$

**Definition 22** A binary operation  $\diamond$  :  $[0, 1] \times [0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-conorm if  $\diamond$  satisfies the following conditions:

(i)  $\diamond$  is commutative and associative;

\* Corresponding author e-mail: [omnia.khater1693@yahoo.com](mailto:omnia.khater1693@yahoo.com)

(ii)  $\diamond$  is continuous;  
 (iii)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ;  
 (iv)  $a_1 \diamond b_1 \diamond c_1 \diamond d_1 \leq a_2 \diamond b_2 \diamond c_2 \diamond d_2$  whenever  $a_1 \leq a_2, b_1 \leq b_2, c_1 \leq c_2, d_1 \leq d_2$  and  $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 \in [0, 1]$ .  
 For example  $a \diamond b \diamond c \diamond d = \max\{a, b, c, d\}$  or  $a \diamond b \diamond c \diamond d = \min\{a + b + c + d, 1\}$ .

**Definition 23[7]** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy 3-metric space if  $X$  is an arbitrary set,  $*$  is a continuous  $t$ -norm,  $\diamond$  is a continuous  $t$ -conorm and  $M, N$  are intuitionistic fuzzy sets on  $X^4 \times [0, \infty) \rightarrow [0, 1]$  satisfying the following conditions:

- (i)  $M(x, y, z, w, t) + N(x, y, z, w, t) \leq 1$ ,  
 (ii)  $M(x, y, z, w, 0) = 0$ ,  
 (iii)  $M(x, y, z, w, t) = 1$  for all  $t > 0$ . Only when at least two of the three simplex  $(x, y, z, w)$  degenerate,  
 (iv)  $M(x, y, z, w, t) = M(x, y, z, y, t) = M(y, z, w, x, t)$ ,  
 (v)  $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \geq M(x, y, z, u, t_1) * M(x, y, u, w, t_2) * M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$ ,  
 (vi)  $M(x, y, z, w, \cdot) : [0, \infty) \rightarrow [0, 1]$  is left continuous,  
 (vii)  $\lim_{t \rightarrow \infty} M(x, y, z, w, t) = 1$ ,  
 (viii)  $N(x, y, z, w, 0) = 1$ ,  
 (ix)  $N(x, y, z, w, t) = 0$  for all  $t > 0$ . Only when at least three simplex  $(x, y, z, w)$  degenerate,  
 (x)  $N(x, y, z, w, t) = N(x, w, z, y, t) = N(y, z, w, x, t)$ ,  
 (xi)  $N(x, y, z, t_1 + t_2 + t_3 + t_4) \leq N(x, y, z, u, t_1) \diamond N(x, y, u, w, t_2) \diamond N(x, u, z, w, t_3) \diamond N(u, y, z, w, t_4)$ ,  
 (xii)  $N(x, y, z, w, \cdot) : [0, \infty) \rightarrow [0, 1]$  is right continuous,  
 (xiii)  $\lim_{t \rightarrow \infty} N(x, y, z, w, t) = 0$ ,

for all  $x, y, z, w, u \in X$  and  $t, t_1, t_2, t_3, t_4 > 0$ . The values  $M(x, y, z, w, t)$  and  $N(x, y, z, w, t)$  may interpret the degrees of nearness and non-nearness that the volume of the quadrilateral enlarged  $(x, y, z, w)$  with respect to  $t$  respectively.

**Definition 24** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy 3-metric space. Then a sequence  $\{x_n\}$  in  $X$  is said to be convergent to a point  $x \in X$  for all  $t > 0$ ,  
 $\lim_{n \rightarrow \infty} M(x_n, x, z, w, t) = 1$  and  $\lim_{n \rightarrow \infty} N(x_n, x, z, w, t) = 0$ .

**Definition 25** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy 3-metric space. Then a sequence  $\{x_n\}$  in  $X$  is said to be Cauchy sequence if, for all  $t > 0$  and  $p > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, z, w, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, z, w, t) = 0.$$

**Definition 26** An intuitionistic fuzzy 3-metric space  $(X, M, N, *, \diamond)$  is said to be complete if and only if every Cauchy sequence in  $X$  is convergent.

**Lemma 2.1** Let  $(X, M, N, *, \diamond)$  is an intuitionistic fuzzy 3-metric space, then  $M$  and  $N$  are continuous function on  $X^4 \times (0, \infty)$ .

**Lemma 2.2** Let  $(X, M, N, *, \diamond)$  is an intuitionistic fuzzy

3-metric space, if for all  $x, y, z, w \in X, t > 0$  and for a number  $k \in (0, 1)$

$$M(x, y, z, w, kt) \geq M(x, y, z, w, t) \text{ and } N(x, y, z, w, kt) \leq N(x, y, z, w, t).$$

**Definition 27** Two self-mappings  $A$  and  $B$  of an intuitionistic fuzzy 3-metric  $(X, M, N, *, \diamond)$  are said to be weakly compatible if  $ABx = BAx$  when  $Ax = Bx$  for some  $x \in X$ .

**Definition 28** Two self-mappings  $A$  and  $B$  of an intuitionistic fuzzy 3-metric  $(X, M, N, *, \diamond)$  are said to be compatible if  $\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, z, w, t) = 1, \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, z, w, t) = 0$  for all  $z, w \in X$  and  $t > 0$  whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = x$  for some  $x \in X$ .

**Definition 29** Let  $A$  and  $B$  be two self-mappings of an intuitionistic fuzzy 3-metric  $(X, M, N, *, \diamond)$ . We say that  $A$  and  $B$  satisfy the property (E.A) if there exists a sequence  $\{x_n\}$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = x$  for some  $x \in X$ .

### 3 Main results

**Theorem 1.** Let  $(X, M, N, *, \diamond)$  be intuitionistic fuzzy 3-metric such that  $a * b = \min\{a, b\}$  and  $a \diamond b = \max\{a, b\}$  for  $a, b \in X$  and let  $A$  and  $B$  be two weakly compatible of  $X$  into itself such that

- (i)  $AX \subset BX$ ,  
 (ii)  $A$  and  $B$  satisfy the property (E.A),  
 (iii)  $M(Ax, Ay, z, w, kt) \geq \min\{M(Bx, By, z, w, t), M(Bx, Ax, z, w, t), M(By, Ay, z, w, t), M(By, Ax, z, w, t), M(Bx, Ay, z, w, t)\}$

$$N(Ax, Ay, z, w, kt) \leq \max\{N(Bx, By, z, w, t), N(Bx, Ax, z, w, t), N(By, Ay, z, w, t), N(By, Ax, z, w, t), N(Bx, Ay, z, w, t)\}$$

If  $AX$  or  $BX$  is a complete subspace of  $X$ , then  $A$  and  $B$  have a unique common fixed point.

**Proof.** Since  $A$  and  $B$  satisfy the property (E.A), there exists in  $X$  a sequence  $\{x_n\}$  satisfying

$$\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Ax_n = v, \text{ for some } v \in X.$$

Suppose that  $BX$  is complete. Then  $\lim_{n \rightarrow \infty} Bx_n = Bu$  for some  $u \in X$ . Also  $\lim_{n \rightarrow \infty} Ax_n = Bu$ . We show that  $Au = Bu$ .

Suppose that  $Au \neq Bu$ . From (iii) we take  $x = x_n, y = u$

$$M(Ax_n, Au, z, w, kt) \geq \min\{M(Bx_n, Bu, z, w, t), M(Bx_n, Ax_n, z, w, t), M(Bu, Au, z, w, t), M(Bu, Ax_n, z, w, t), M(Bx_n, Au, z, w, t)\}$$

$$N(Ax_n, Au, z, w, kt) \leq \max\{N(Bx_n, Bu, z, w, t), N(Bx_n, Ax_n, z, w, t), N(Bu, Au, z, w, t), N(Bu, Ax_n, z, w, t), N(Bx_n, Au, z, w, t)\}$$

Letting  $n \rightarrow \infty$  we get

$$M(Bu, Au, z, w, kt) \geq \min\{M(Bu, Bu, z, w, t), M(Bu, Bu, z, w, t), M(Bu, Au, z, w, t), M(Bu, Bu, z, w, t), M(Bu, Au, z, w, t)\}$$

$$N(Bu, Au, z, w, kt) \leq \max\{N(Bu, Bu, z, w, t), N(Bu, Bu, z, w, t), N(Bu, Au, z, w, t), N(Bu, Bu, z, w, t), N(Bu, Au, z, w, t)\}$$

$$M(Bu, Au, z, w, kt) \geq M(Bu, Au, z, w, t), N(Bu, Au, z, w, kt) \leq N(Bu, Au, z, w, t),$$

by using lemma 2.1 we have  $Au = Bu$ .

Since  $A$  and  $B$  are weakly compatible,  $ABu = BAu$  thus,  $AAu = ABu = BAu = BBu$ .

We show that  $Au$  is common fixed point of  $A$  and  $B$ . Suppose that  $Au \neq AAu$ . Then, we take  $x = u, y = Au$  and we have

$$M(Au, AAu, z, w, kt) \geq \min\{M(Bu, BAu, z, w, t), M(Bu, Au, z, w, t), M(BAu, AAu, z, w, t), M(BAu, Au, z, w, t), M(Bu, AAu, z, w, t)\}$$

$$N(Au, AAu, z, w, kt) \leq \max\{N(Bu, BAu, z, w, t), N(Bu, Au, z, w, t), N(BAu, AAu, z, w, t), N(BAu, Au, z, w, t), N(Bu, AAu, z, w, t)\}$$

$$M(Au, AAu, z, w, kt) \geq M(Au, AAu, z, w, t), N(Au, AAu, z, w, kt) \leq N(Au, AAu, z, w, t)$$

Hence by lemma 2.1, we have  $Au = AAu$  and  $BAu = AAu = Au$ . The proof is similar when  $AX$  is assumed to be a complete subspace of  $X$ , since  $AX \subset BX$ . Then the common fixed point is unique.

**Theorem 2.** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy 3-metric space such that  $a * b = \min\{a, b\}$ ,  $a \diamond b = \max\{a, b\}$  and  $t * t \geq t$ . Let  $A, B, P$  and  $Q$  be mappings of  $X$  into itself such that  
 (2.1)  $AX \subset QX$  and  $BX \subset PX$ ;  
 (2.2)  $(A, P)$  or  $(B, Q)$  satisfies the property (E.A);  
 (2.3)  $(A, P)$  and  $(B, Q)$  are weakly compatible;  
 (2.4) there exists a number  $k \in (0, 1)$  such that

$$M(Ax, By, z, w, kt) \geq \min\{M(Px, Qy, z, w, t), M(Px, By, z, w, t), M(Qy, By, z, w, t)\}$$

$$N(Ax, By, z, w, kt) \leq \max\{N(Px, Qy, z, w, t), N(Px, By, z, w, t), N(Qy, By, z, w, t)\}$$

For all  $x, y, z, w \in X$

(2.5) One of  $AX, BX, PX$  or  $QX$  is a complete subspace of  $X$ .

Then  $A, B, P$  and  $Q$  have a unique common fixed point in  $X$ .

**Proof.** Suppose that  $(B, Q)$  satisfies the property (E.A). Then there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Qx_n = u$  for some  $u \in X$ .

Since  $BX \subset PX$ , there exists in  $X$  a sequence  $\{y_n\}$  such that  $Bx_n = Py_n$ . Hence  $\lim_{n \rightarrow \infty} Py_n = u$ . Let us show that

$$\lim_{n \rightarrow \infty} Ay_n = u.$$

From (2.4) we have

$$M(Ay_n, Bx_n, z, w, kt) \geq \min\{M(Py_n, Qx_n, z, w, t), M(Py_n, Bx_n, z, w, t), M(Qx_n, Bx_n, z, w, t)\}$$

$$\geq \min\{M(Py_n, Qx_n, z, w, t), 1, M(Qx_n, Bx_n, z, w, t)\}$$

$$\geq M(Py_n, Qx_n, z, w, t)$$

$$N(Ay_n, Bx_n, z, w, kt) \leq \max\{N(Py_n, Qx_n, z, w, t), N(Py_n, Bx_n, z, w, t), N(Qx_n, Bx_n, z, w, t)\}$$

$$\leq \max\{N(Py_n, Qx_n, z, w, t), 0, N(Qx_n, Bx_n, z, w, t)\}$$

$$\leq N(Py_n, Qx_n, z, w, t)$$

Letting  $n \rightarrow \infty$ , we get

$$\lim_{n \rightarrow \infty} M(Ay_n, Bx_n, z, w, kt) = 1, \lim_{n \rightarrow \infty} N(Ay_n, Bx_n, z, w, kt) = 0.$$

Hence we deduce that  $\lim_{n \rightarrow \infty} Ay_n = u$ . Suppose  $PX$  is a complete subspace of  $X$ .

Then,  $Pv = u$  for some  $v \in X$ . Subsequently, we have

$$\lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Qx_n = \lim_{n \rightarrow \infty} Py_n = Pv.$$

From (2.4) we have

$$M(Av, Bx_n, z, w, kt) \geq \min\{M(Pv, Qx_n, z, w, t), M(Pv, Bx_n, z, w, t), M(Qx_n, Bx_n, z, w, t)\}$$

$$N(Av, Bx_n, z, w, kt) \leq \max\{N(Pv, Qx_n, z, w, t), N(Pv, Bx_n, z, w, t), N(Qx_n, Bx_n, z, w, t)\}$$

Letting  $n \rightarrow \infty$  we have  $\lim_{n \rightarrow \infty} M(Av, pv, z, w, kt) = 1, \lim_{n \rightarrow \infty} N(Av, pv, z, w, kt) = 0$ .

Hence, we deduce that  $Av = Pv$ . Since  $A$  and  $P$  are weakly compatible,  $APv = PAv$  and then  $AAv = APv = PAv = PPv$ .

On the other hand, since  $AX \subset QX$ , there exists a point  $s \in X$  such that  $Av = Qs$ . We show that  $Qs = Bs$ . Using (2.4) we have

$$M(Av, Bs, z, w, kt) \geq \min\{M(Pv, Qs, z, w, t), M(Pv, Bs, z, w, t), M(Qs, Bs, z, w, t)\}$$

$$\geq M(Av, Bs, z, w, t)$$

$$N(Av, Bs, z, w, kt) \leq \max\{N(Pv, Qs, z, w, t), N(Pv, Bs, z, w, t), N(Qs, Bs, z, w, t)\}$$

$$\leq N(Av, Bs, z, w, t)$$

By Lemma 2.1 we have  $Av = Bs$ , therefore  $Av = Pv = Qs = Bs$ . Since  $B$  and  $Q$  are weakly compatible implies that  $BQs = QBs$  and  $QQs = QBs = BQs = BBs$ . we show that  $Av$  common fixed point of  $A, B, P$  and  $Q$ . Using (2.4) we have

$$M(Av, AAv, z, w, kt) = M(AAv, Bs, z, w, kt) \geq \min\{M(PAv, Qs, z, w, t), M(PAv, Bs, z, w, t)$$

$$, M(Qs, BAv, z, w, t)\}$$

$$\begin{aligned}
 N(Av, AAv, z, w, kt) &= N(AAv, Bs, z, w, kt) \leq \\
 \max\{N(PAv, Qs, z, w, t), N(PAv, Bs, z, w, t)\} \\
 &\geq M(AAv, Av, z, w, t) \\
 &N(Qs, BAv, z, w, t) \\
 &\leq N(AAv, Av, z, w, t)
 \end{aligned}$$

Therefore by Lemma 2.1, we have  $Av = AAv = PAv$  and  $Av$  is common fixed point of  $A$  and  $P$ .

Similarly, we show that  $Bs$  is common fixed point of  $B$  and  $Q$ . Since  $Av = Bs$ , we conclude that  $Au$  is common fixed point of  $A, B, P$  and  $Q$ .

The proof is similar when  $QX$  is assumed to be complete subspace of  $X$ . The cases in which  $AX$  OR  $BX$  is complete subspace of  $X$  are similar to the cases in which  $PX$  or  $QX$ , respectively, is complete since  $AX \subset QX$  and  $BX \subset PX$ .

If  $Av = Bv = Pv = Qv = v$  and  $As = Bs = Ps = Qs = s$ , using (2.4), we have

$$\begin{aligned}
 M(v, s, z, w, kt) &= M(Av, Bs, z, w, kt) \geq \\
 \min\{M(Pv, Qs, z, w, t), M(Pv, Bs, z, w, t)\} \\
 &M(Qs, Bs, z, w, t) \\
 &\geq M(v, s, z, w, t) \\
 N(v, s, z, w, kt) &= N(Av, Bs, z, w, kt) \leq \\
 \max\{N(Pv, Qs, z, w, t), N(Pv, Bs, z, w, t)\} \\
 &N(Qs, Bs, z, w, t) \\
 &\leq N(v, s, z, w, t)
 \end{aligned}$$

By Lemma 2.1, we obtain  $v = s$ . Then, the common fixed point is unique.

## Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

## References

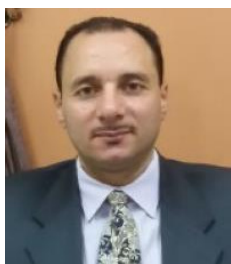
- [1] Aamri, M., and El Moutawakil, D. (2002). Some new common fixed point theorems under strict contractive conditions. *Journal of Mathematical Analysis and Applications*, 270(1), 181-188.
- [2] Abu-Donia, H.A., H.A.Atia and Omnia.M.A.Khater"Common fixed point theorems in intuitionistic fuzzy metric spaces and intuitionistic  $(\phi, \psi)$ -contractive mappings"Accepted in *Journal of Nonlinear Sciences and Applications* (2020).
- [3] Abu-Donia, H.A., H.A.Atia and Omnia.M.A.Khater"Fixed point theorem by using  $\psi$ -contraction and  $(\phi, \phi)$ -contraction in probabilistic 2–metric spaces"Alexandria Engineering Journal (2020).
- [4] Chauhan, M. S., and Singh, B. (2011). Fixed point in intuitionistic fuzzy-3 metric space. *Int. J. Engg and Techo*, 3(2), 144-150.
- [5] Gähler, S., Über die uniformisierbarkeit 2–metrische Räume, *Math.Nachr.* 28 (1965), 235-244.
- [6] Gregori, V., Miñana, J. J., and Miravet, D. (2020). Contractive sequences in fuzzy metric spaces. *Fuzzy Sets and Systems*, 379, 125-133.
- [7] Güner, E., Çetkin, V., and Aygün, H. (2018). On Intuitionistic Fuzzy 2-Metric Spaces. In *ITM Web of Conferences* (Vol. 22, p. 01024). EDP Sciences.
- [8] Jungck, G., Compatible mappings and fixed points, *Internat. J. Math. Math. Sci.*, Vol 9, No. 4(1986), pp. 771-779.
- [9] Kramosil, O., Michalek, J., Fuzzy metric and statistical metric spaces, *Kybernetika*, Vol 11, (1975), pp. 326-334.
- [10] Manro, S. (2015). A common fixed point theorem for weakly compatible maps satisfying common property (E: A:) and implicit relation in intuitionistic fuzzy metric spaces. *International Journal of Nonlinear Analysis and Applications*, 6(1), 1-8.
- [11] Qureshi, F., Modi, G., and Dubey, A. (2013). Common Fixed Point Theorem In Intuitionistic Fuzzy 2-Metric Spaces.
- [12] Sharma, S. (2003). On fuzzy metric space. *Southeast Asian Bulletin of Mathematics*, 26(1), 133-145.
- [13] Vijayaraju, P., Marudai, M., Fixed point theorems in fuzzy metric spaces?, *J. Fuzzy Math.*, Vol 8, No. 2, (2000), pp. 867- 871.
- [14] Zheng, D., and Wang, P. (2019). Meir–Keeler theorems in fuzzy metric spaces. *Fuzzy Sets and Systems*, 370, 120-128.



**H.M.Abu-Donia**

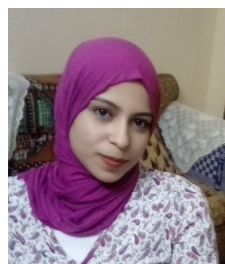
He is a professor of pure mathematics and Head of Mathematics Department at Zagazig University. He received the PH. D degree in Pure Mathematics. His research interests are in the areas of pure mathematics such as Fixed point Theory,

General Topology, Rough Sets Theory, Bitopological Spaces and Fuzzy Topology. He has published some research articles in the area of Pure Mathematics in different international journals. Also, he is a referee in some of international journals in the frame of pure mathematics.



**H. A. Atia** He is an associate professor of pure mathematics in Mathematics Department at Zagazig University. He received the Ph D degree in Pure Mathematics. His research interests are in the areas of pure mathematics such as Functional Analysis, Operator

Theory, Fixed Point Theorem. he has published some research articles in the area of Pure Mathematics in different international journals. Also, he is a referee in some of international journals in the frame of pure mathematics.



**Omnia M. A. Khater** She is a teaching assistant in Zagazig Higher Institute of Engineering and Technology. In 2015, I received the B.SC. Now, I am a master student in department of mathematics, faculty of science, Zagazig university, Egypt.