Applied Mathematics & Information Sciences
*An International Journal*

http://dx.doi.org/10.18576/amis/150103

# Distinguishing Graphs Associated to KU-Algebras using Graph Invariants

*Deena Al-Kadi*

Department of Mathematics and Statistic, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia

Received: 11 Sep. 2020, Revised: 27 Nov. 2020, Accepted: 8 Dec. 2020
Published online: 1 Jan. 2021

**Abstract:** The present paper aims to distinguish two graphs associated to $KU$-algebras. We construct $KU$-algebras graph using right zero divisors notion. Properties of right zero divisors in $KU$-algebras were studied. Moreover, it is proved that the set of right zero divisors of the identity element is a quasi R-prime ideal, whereas the sets of right zero divisors of all other elements are not quasi ideals. A condition is given for a graph to be a star and a complete graph. Then, we show that the diameter of a right zero graph of $KU$-algebras is two at most. Finally, graph isomorphism has been considered. It has been shown that $KU$-algebras graph constructed in this paper is not isomorphic to the $KU$-algebras graph constructed using $KU$-annihilator.

**Keywords:** Graph, Graph invariants, KU-algebras, Zero divisors and annihilators.

## 1 Introduction

The study of graphs was established to represent relations between objects. Many researches have been conducted to connect graph theory and algebraic structure as commutative rings (see for example [1–4] and algebras (see [5–8]) where the associated graphs for $BCK/BCI$-algebras, $IS$-algebra, $UP$-algebras and hoop algebras were studied. In [9], Beck was the first to introduce the idea of coloring a commutative ring. Then, many researches were devoted to the subject with modification to the definition of the zero divisors, see ( [10–18]).

In [19], Mostafa et al. addressed the graph of commutative $KU$-algebra, $X$ based on the $KU$-annihilator of a subset $S$ of $X$ defined as the set $\{x \in X : (x*s)*s = 0, \text{ for all } s \in S\}$ and the graph of equivalence classes.

Most studies in graph theory address simple graphs. A simple graph is a graph with no loops and the edge that connects two pairs of vertices is unique. A simple graph is considered complete if each distinct pair of vertices is connected by an edge. If no edges exist in a graph, then the graph is empty ( [20]).

In this paper, we investigate zero divisors simple graphs of $KU$-algebra. Typically, right zero divisors as the set of left zero divisors is empty-as we will check it- and so the graph associated to it is an empty graph. Moreover, we show that right zero divisors graphs are connected graphs, i.e. any two of its vertices are joined by a path. We give a condition for the right zero graph to be a star and to be complete. Moreover, we find the diameter which is the greatest distance among all minimal paths between each pair of vertices. Several properties and illustrating examples are presented throughout the paper.

## 2 Preliminaries

We start by recalling basic definition.
An algebraic structure $(X,*,0)$ of type $(2,0)$ is said to be a $KU$-algebra if
(1) $(\forall x \in X)(x*x=0)$,
(2) $(\forall x \in X)(x*0=0)$,
(3) $(\forall x \in X)(0*x=x)$,
(4) $(\forall x,z \in X)(x*z=0 \text{ and } z*x=0 \text{ imply } x=z)$,
(5) $(\forall x,y,z \in X)((x*z)*((z*y)*(x*y))=0)$.

$X$ will denote a $KU$-algebra. Define a partially ordered relation on $X$ by $x \preceq z$ if and only if $z*x=0$. Then, $0 \preceq x$, so $X$ is bounded. Having $x \preceq z$ implies $x*y \succeq z*y$. (We refer the reader to [21–23] for more information on $KU$-algebras).

* Corresponding author e-mail: dak12le@hotmail.co.uk; d.alkadi@tu.edu.sa

**Definition 2.1.** Let $J$ be a non empty subset of $X$. We say that $J$ is an ideal of $X$ if $0 \in J$ and $(\forall x,z \in X)$(if $z*x \in J$ and $x \in J$, then $z \in J$).

**Definition 2.2.** For a non empty subset $S$ of $X$,
left zero divisors are the set
$L(S) = \{x \in X : x*s = 0,\ \text{for all } s \in S\}$
and right zero divisors are
$R(S) = \{x \in X : s*x = 0,\ \text{for all } s \in S\}$.

**Definition 2.3.** Let $J$ be a non empty subset of $X$. We say that $J$ is a quasi-ideal of $X$ if
$(\forall z \in X)(\forall x \in J)$, if $z*x = 0$, then $z \in J$.

**Definition 2.4.** Let $J$ be a proper quasi-ideal of $X$. Then, $J$ is called a quasi $L$-prime ideal if $(\forall x,z \in X)$, if $L(\{x,z\}) \subseteq J$, then $x \in J$ or $z \in J$.
Similarly, a quasi $R$-prime ideal if
$(\forall x,z \in X)$, if $R(\{x,z\}) \subseteq J$, then $x \in J$ or $z \in J$.

## 3 Left and right zero divisors in KU-algebras

It is obvious from the definition of $KU$-algebra (2) that $R(\{x,z\}) \neq \phi$. Moreover, $R(\{x,0\}) = \{0\}$. Denote the set of all left and right zero divisors of an element $x \in X$ by $\mathfrak{L}_x$ and $\mathfrak{R}_x$ respectively, i.e.

$$\mathfrak{L}_x = \{z \in X : L(\{x,z\}) = \{0\}\}$$
$$\mathfrak{R}_x = \{z \in X : R(\{x,z\}) = \{0\}\}.$$

**Lemma 3.1.** If $J$ is a proper quasi-ideal in $X$, then 0 is not in $J$.

**Proof.** Let $J$ be a proper quasi-ideal containing 0. Then, having $x*0 = 0$ $(\forall x \in X)$ implies $x \in J$ $(\forall x \in X)$ and so $J = X$ which gives a contradiction. Thus, $0 \notin J$.

**Example 3.1.** Consider the $KU$-algebra $(X;*,0)$ where $X$ is the set $\{0,1,2,3\}$ and the operation $*$ is presented in Table 1.

**Table 1:** Cayley table

| $*$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 0 | 0 | 1 | 3 |
| 2 | 0 | 0 | 0 | 3 |
| 3 | 0 | 1 | 2 | 0 |

Then, $J = X$ is a quasi-ideal containing 0, whereas $I = \{0,1,2\}$ is not quasi-ideal. All possible proper quasi-ideals are $\{1,2,3\}$, $\{1,2\}$, $\{2,3\}$, $\{2\}$ and $\{3\}$.
Moreover, they are $L$-prime quasi ideals and $R$-prime quasi ideals in $X$ as Definition 2.4 is satisfied. See Table 2 where the left and right zero divisors for each pair are found. Then, for any element $x \in X$, the set of all left zero divisors, $\mathfrak{L}_x = \phi$ and the set of all right zero divisors of $0,1,2$ and $3$ are $\mathfrak{R}_0 = \{1,2,3\}$,$\mathfrak{R}_1 = \{0,3\}$, $\mathfrak{R}_2 = \{0,3\}$ and $\mathfrak{R}_3 = \{0,1,2\}$, respectively.

**Table 2:** Left and right zero divisors for each pair $\{x,z\}$

| $\{x,z\}$ | $L(\{x,z\})$ | $R(\{x,z\})$ |
|---|---|---|
| $\{0,1\}$ | $\{1,2\}$ | $\{0\}$ |
| $\{0,2\}$ | $\{2\}$ | $\{0\}$ |
| $\{0,3\}$ | $\{3\}$ | $\{0\}$ |
| $\{1,2\}$ | $\{2\}$ | $\{0,1\}$ |
| $\{1,3\}$ | $\phi$ | $\{0\}$ |
| $\{2,3\}$ | $\phi$ | $\{0\}$ |

**Remark 3.1.** For any two distinct elements $x$ and $z$ in $X$, $L(\{x,z\}) \neq \{0\}$ and so $\mathfrak{L}_x = \phi$.

The set of all left zero divisors is empty. Thus, this paper is devoted to exploring right zero divisors starting with the following properties.

**Lemma 3.2.** Let $S$ be a non empty subset of $X$. Then $R(S) = \bigcap_{s\in S} R(\{s\})$.

**Proof.** Since $S = \bigcup_{s\in S}\{s\}$, then
$R(S) = R(\bigcup_{s\in S}\{s\}) = \bigcap_{s\in S} R(\{s\})$.

**Proposition 3.1.** [5] Let $S$ be a non empty subset of $X$. Then,
(1) $S \subseteq L(R(S))$ and $S \subseteq R(L(S))$,
(2) $L(S) = L(R(L(S)))$ and $R(S) = R(L(R(S)))$.

**Proposition 3.2.** [5] If $S_1$ and $S_2$ are non empty subsets of $X$ such that $S_1 \subseteq S_2$, then
$R(S_2) \subseteq R(S_1)$.

**Proposition 3.3.** If $S_1$ and $S_2$ are non empty subsets of $X$ such that $S_1 \subseteq S_2$, then
(1) $R(S_1 \cap S_2) = R(S_1)$,
(2) $R(S_1 \cup S_2) = R(S_2)$.

**Proof.** Straightforward

**Proposition 3.4.** If $S_1$ and $S_2$ are non empty subsets of $X$ with $S_1 \cap S_2 \neq \phi$, then the following is true:
(1) $R(S_1 \cap S_2) \supseteq R(S_1) \cup R(S_2)$,
(2) $R(S_1 \cup S_2) = R(S_1) \cap R(S_2)$.

**Proof.** (1) We know that $S_1 \cap S_2 \subseteq S_1$ and $S_1 \cap S_2 \subseteq S_2$. Using Proposition 3.2, we have $R(S_1) \subseteq R(S_1 \cap S_2)$ and $R(S_2) \subseteq R(S_1 \cap S_2)$. Thus, $R(S_1) \cup R(S_2) \subseteq R(S_1 \cap S_2)$.
(2) On one hand, we know that $S_1 \subseteq S_1 \cup S_2$ and $S_2 \subseteq S_1 \cup S_2$. Using Proposition 3.2, we have $R(S_1 \cup S_2) \subseteq R(S_1)$ and $R(S_1 \cup S_2) \subseteq R(S_2)$. Thus, $R(S_1 \cup S_2) \subseteq R(S_1) \cap R(S_2)$. On the other hand, let $x \in R(S_1) \cap R(S_2)$. Then, $\dot{s}*x = 0$ for all $\dot{s} \in S_1$ and $\ddot{s}*x = 0$ for all $\ddot{s} \in S_2$. Thus, $s*x = 0$ for all $s \in S_1 \cup S_2$. Hence, $x \in R(S_1 \cup S_2)$.

**Lemma 3.3.** Let $x,z \in X$ such that $x*z = 0$. Then, $R(\{z\}) \subseteq R(\{x\})$.

**Proof.** If $m \in R(\{z\})$, then $z*m = 0$. Having $z \leq x$, we get $x*m \leq z*m$ and so $x*m \leq 0$. That is, $0*(x*m) = 0$. It follows from the definition of $KU$-algebra (3) that $x*m = 0$. Therefore, $m \in R(\{x\})$.

The converse of Lemma 3.3 is untrue in general as proved by the next example.

**Example 3.2.** Consider the $KU$-algebra $(X;*,0)$ where $X$ is the set $\{0,1,2,3,4\}$ and the operation $*$ is given by Table 3:

**Table 3:** Cayley table

| $*$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 2 | 0 | 0 | 0 | 3 | 4 |
| 3 | 0 | 1 | 2 | 0 | 4 |
| 4 | 0 | 1 | 2 | 3 | 0 |

We have $R(\{1\}) = \{0\} \subseteq \{0,3\} = R(\{3\})$ while $3 * 1 \neq 0$.

**Lemma 3.4.** For all $x \in X$ and $z \in X \setminus \{0\}$, if $x * z = 0$, then $\mathfrak{R}_x \subseteq \mathfrak{R}_z$.

**Proof.** Let $x \in X$ and $z \in X \setminus \{0\}$. Suppose that $m \in \mathfrak{R}_x$. Consider $R(\{m,z\})$. We know that $0 \in R(\{m,z\})$.Then, using Lemma 3.3, we have $R(\{m,z\})= R(\{m\}) \cap R(\{z\}) \subseteq R(\{m\}) \cap R(\{x\}) = R(\{m,x\}) = \{0\}$. Thus, $R(\{m,z\}) = \{0\}$. That is, $m \in \mathfrak{R}_z$.

In general, the converse of Lemma 3.4 is untrue as shown in the next example.

**Example 3.3.** Consider the $KU$-algebra $(X;*,0)$ where $X$ is the set $\{0,1,2,3,4,5\}$ and the operation $*$ is presented in Table 4.

**Table 4:** Cayley table

| $*$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 0 | 2 | 3 | 2 | 3 |
| 2 | 0 | 1 | 0 | 2 | 1 | 4 |
| 3 | 0 | 1 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 | 2 | 0 | 2 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Then, $\mathfrak{R}_4 = \{0\} \subseteq \{0,1\} = \mathfrak{R}_3$, but $4 * 3 = 2 \neq 0$.

**Theorem 3.1.** For any $x \in X \setminus \{0\}$, $\mathfrak{R}_x$ is not quasi ideal of $X$. Moreover, if $\mathfrak{R}_0$ is maximal, then it is a quasi $R$-prime ideal.

**Proof.** Suppose for all $x \in X \setminus \{0\}$, $\mathfrak{R}_x$ is a quasi ideal. Then, $(\forall x \in X \setminus \{0\})(\forall z \in \mathfrak{R}_x)$ with $x * z = 0$ implies $x \in \mathfrak{R}_x$ which is a contradiction. Thus, $\mathfrak{R}_x$ is not quasi ideal.

Now, let $a \in X \setminus \{0\}$ and $b \in \mathfrak{R}_0$ with $a * b = 0$. Then, $R(\{0,a\}) = R(\{0\}) \cap R(\{a\}) = R(\{0\}) = \{0\}$. Hence, $a \in \mathfrak{R}_0$, so $\mathfrak{R}_0$ is a quasi ideal of $X$.

Suppose that $\mathfrak{R}_0$ is maximal and let $a, b \in X$ such that $R(\{a,b\}) \subseteq \mathfrak{R}_0$. As $0 \notin \mathfrak{R}_0$ and $0 \in R(\{a,b\})$, then it does not exist $R(\{a,b\})$ where $R(\{a,b\}) \subseteq \mathfrak{R}_0$. Thus, $\mathfrak{R}_0$ is $R$-prime.

## 4 Right Zero divisors graph

In graph theory, a graph is a pair $(V(X), P(X))$ where $V(X)$ the set of vertices are the elements of $X$ which are connected by edges $P(X)$.

**Definition 4.1.** A right zero divisor graph of $X$ is a graph where the vertices are the elements of $X$ and an edge exists between two distinct vertices $x$ and $z$ if and only if $R(\{x,z\}) = \{0\}$.

As the graph under study is simple and indirect, we have the following corollary.

**Corollary 4.1.** For $x, z \in V(X)$, the relation "there is an edge between two distinct vertices $x$ and $z$" is irreflexive and symmetric. Moreover, it is intransitive as in Example 3.1 Table 2, we have $R(\{2,3\}) = \{0\}$ and $R(\{3,1\}) = \{0\}$, but $R(\{2,1\}) \neq \{0\}$.

The following theorem follows directly from Definition 4.1.

**Theorem 4.1.** In right zero divisor graphs, an edge exists between two distinct vertices $x$ and $z$ if and only if $x \in \mathfrak{R}_z$.

**Corollary 4.2.** In $KU$-algebras, $\mathfrak{L}_x = \phi$, for all $x \in X$, so no edge exists between any two distinct vertices $x$ and $z$. On the contrary, in right zero divisor graph, an edge exists between every vertex $x \in X \setminus \{0\}$ and 0.

Thus, we have the following.

**Theorem 4.2.** A left zero divisor graph of $KU$-algebras is an empty graph. Moreover, it is a disconnected graph.

**Theorem 4.3.** A right zero divisor graph of $KU$-algebras is a connected graph.

**Theorem 4.4.** If $\mathfrak{R}_x = \{0\}$ for all $x \in X \setminus \{0\}$, then $G(X)$ is a star graph.

**Proof.** Using Corollary 4.2, we know that for all $x \in X \setminus \{0\}$ there is an edge $0 - x$ and $deg(0) = \mid X \mid -1$ as $\mathfrak{R}_0 = X \setminus \{0\}$. Since $\mathfrak{R}_x = \{0\}$, then there is an edge $x - 0$ where $deg(x) = 1, \forall x \in X \setminus \{0\}$.

**Example 4.1.** Consider the $KU$-algebra $(X;*,0)$ where $X$ is the set $\{0,1,2,3,4,5\}$ and the operation $*$ is given by Cayley Table 5.

**Table 5:** Cayley table

| $*$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 0 | 2 | 2 | 4 | 5 |
| 2 | 0 | 0 | 0 | 1 | 4 | 5 |
| 3 | 0 | 0 | 0 | 0 | 4 | 5 |
| 4 | 0 | 0 | 0 | 1 | 0 | 5 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |

Then, $R(\{0,1\}) = R(\{0,2\}) = R(\{0,3\}) = R(\{0,4\}) = R(\{0,5\}) = \{0\}$, $R(\{1,2\}) = R(\{1,3\}) = R(\{1,4\}) = R(\{1,5\}) = \{0,1\}$, $R(\{2,3\}) = R(\{2,4\}) = R(\{2,5\}) = R(\{3,4\}) =$

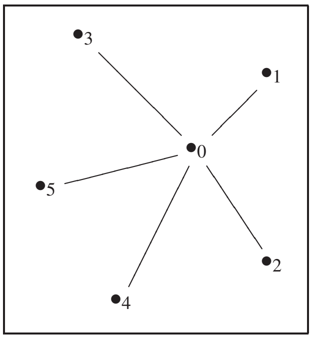


**Fig. 1:** The graph of the right zero divisors of $X$

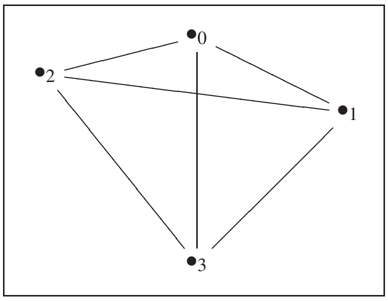


**Fig. 2:** The graph of the right zero divisors of $X$

$R(\{4,5\}) = \{0,1,2\}$ and $R(\{3,5\}) = \{0,1,2,3\}$. Thus, $\mathfrak{R}_0 = \{1,2,3,4,5\}$ and $\mathfrak{R}_1 = \mathfrak{R}_2 = \mathfrak{R}_3 = \mathfrak{R}_4 = \mathfrak{R}_5 = \{0\}$. Thus, the graph of the right zero divisors of elements of $X$ presented by Figure 1 is a star graph.

**Theorem 4.5.** If $\mathfrak{R}_x = X \setminus \{x\}$, for all $x \in X$, then $G(X)$ is a complete graph.

**Proof.** Straightforward.

**Example 4.2.** Consider the $KU$-algebra $(X;*,0)$ where $X$ is the set $\{0,1,2,3\}$ and the operation $*$ is presented in Table 6.

**Table 6:** Cayley table

| $*$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 0 | 0 | 2 | 3 |
| 2 | 0 | 1 | 0 | 3 |
| 3 | 0 | 1 | 2 | 0 |

Then, we have, $\mathfrak{R}_0 = \{1,2,3\}, \mathfrak{R}_1 = \{0,2,3\}, \mathfrak{R}_2 = \{0,1,3\}, \mathfrak{R}_3 = \{0,1,2\}$ and the graph of the right zero divisors of elements of $X$ is given by the complete graph in Figure 2.

**Theorem 4.6.** The diameter of a right zero divisor graph of $X$ is two at most.

**Proof.** Let $a,b \in X$. Then, either $a \in \mathfrak{R}_b$ or $a \notin \mathfrak{R}_b$. If $a \in \mathfrak{R}_b$, then we have an edge $a-b$. If $a \notin \mathfrak{R}_b$, then there exists a path $a-0-b$. Therefore, the maximum distance between $a$ and $b$ is two. The theorem is proved.

## 5 Graph isomorphism

For algebras $X$ and $Y$, two graphs $G(X) = (V(X), P(X))$ and $G(Y) = (V(Y), P(Y))$ are isomorphic if and only if
(1) There is a bijection $\hbar : V(X) \to V(Y)$,
(2) If $a,b \in V(X)$ such that $b \in \mathfrak{R}_a$, then $\hbar(b) \in \mathfrak{R}_{\hbar(a)}$.

**Lemma 5.1.** Let $G(X)$ and $G(Y)$ be two isomorphic graphs. Then, if $\hbar(x) = z$, then $\hbar(\mathfrak{R}_x) = \mathfrak{R}_z$ (for all $x \in V(X), z \in V(Y)$).

**Proof.** If $\hbar(x) = z$, then $\hbar(\mathfrak{R}_x) = \mathfrak{R}_{\hbar(x)} = \mathfrak{R}_z$.

**Example 5.1.** Consider two $KU$-algebras $(X_1;*_1,0)$, $(X_2;*_2,0)$ where $X_1 = X_2 = \{0,1,2,3,4\}$ and the operations $*_1$ and $*_2$ are presented in Table 7 and Table 8, respectively.

**Table 7:** Cayley table

| $*_1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 3 | 0 | 3 | 4 |
| 3 | 0 | 1 | 2 | 0 | 1 |
| 4 | 0 | 0 | 0 | 0 | 0 |

**Table 8:** Cayley table

| $*_2$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 0 | 0 | 2 | 3 | 3 |
| 2 | 0 | 1 | 0 | 1 | 4 |
| 3 | 0 | 0 | 0 | 0 | 3 |
| 4 | 0 | 0 | 0 | 0 | 0 |

Then, $R(\{0,1\}) = R(\{0,2\}) = R(\{0,3\}) = R(\{0,4\}) = R(\{2,3\}) = \{0\}$,
$R(\{1,2\}) = R(\{2,4\}) = \{0,2\}$,
$R(\{1,3\}) = R(\{3,4\}) = \{0,3\}$ and
$R(\{1,4\}) = \{0,1,2,3\}$ and so
$\mathfrak{R}_0 = \{1,2,3,4\}, \mathfrak{R}_2 = \{0,3\}, \mathfrak{R}_3 = \{0,2\}$ and $\mathfrak{R}_1 = \mathfrak{R}_4 = \{0\}$. Thus, the graph of the right zero divisors of the elements of $X_1$ is presented in Figure 3.

Similarly, we have the following in $X_2$,
$R(\{0,1\}) = R(\{0,2\}) = R(\{0,3\}) = R(\{0,4\}) = R(\{1,2\}) = \{0\}$,
$R(\{2,3\}) = R(\{2,4\}) = \{0,2\}$,
$R(\{1,3\}) = R(\{1,4\}) = \{0,1\}$ and
$R(\{3,4\}) = \{0,1,2,3\}$ and so

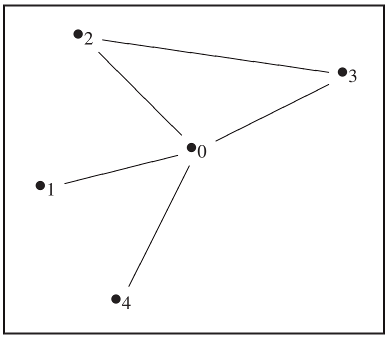


**Fig. 3:** The graph of the right zero divisors of $X_1$

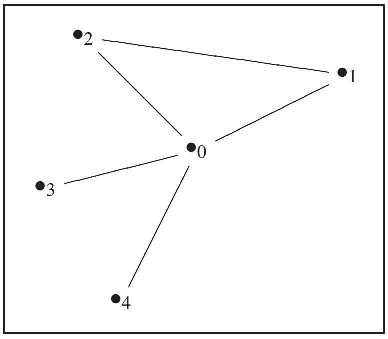


**Fig. 4:** The graph of the right zero divisors of $X_2$

$\bar{\mathfrak{R}}_0 = \{1,2,3,4\}, \bar{\mathfrak{R}}_1 = \{0,2\}, \bar{\mathfrak{R}}_2 = \{0,1\}$, and $\bar{\mathfrak{R}}_3 = \bar{\mathfrak{R}}_4 = \{0\}$. Thus, the graph of the right zero divisors of elements of $X_2$ is given by Figure 4 which is an isomorphic graph of Figure 3 as there exists a bijection $\hbar$ such that $\hbar(0) = 0, \hbar(1) = 3$, $\hbar(2) = 2, \hbar(3) = 1$ and $\hbar(4) = 4$ and $\hbar(\mathfrak{R}_0) = \bar{\mathfrak{R}}_0, \hbar(\mathfrak{R}_1) = \bar{\mathfrak{R}}_3, \hbar(\mathfrak{R}_2) = \bar{\mathfrak{R}}_2, \hbar(\mathfrak{R}_3) = \bar{\mathfrak{R}}_1$ and $\hbar(\mathfrak{R}_4) = \bar{\mathfrak{R}}_4$.

Although the $KU$-graphs constructed in [19] using $KU$-annihilator are identical to the graphs constructed in this paper using right zero divisors, they are not isomorphic. Recall the following structural property of the former graph.

**Theorem 5.1.** [19] The diameter of $KU$-graph in which two distinct elements are adjacent if and only if $(z*x)*x = 0$ is three at most.

Thus, Theorem 4.6 and Theorem 5.1 indicate that the diameter of the graphs is not preserved. Since graph diameter is invariant under graph isomorphism, it follows that the graph of $KU$-algebras constructed using $KU$-annihilator and the graph of $KU$-algebras constructed using right zero divisors are not isomorphic, which results in the following:

**Corollary 5.1.** For $KU$-algebras $X$, where $G(X)$ is the graph constructed using $KU$-annihilator and $\widetilde{G}(X)$ is the graph constructed using right zero divisors, we have $diam(G(X)) \neq diam(\widetilde{G}(X))$. Hence, these two graphs are not isomorphic.

## Acknowledgement

The author would like to thank the anonymous referee for the constructive comments that improved the paper.

## Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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**Deena Al-Kadi** is an Associate Professor of Pure Mathematics at Taif University in Saudi Arabia . She received her Phd. degree in the representation theory of algebras, in the study of Self-injective algebras and the second Hochschild cohomology group from Leicester University (UK). She has published research articles in the area of Pure Mathematics in different reputed international journals.