Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/150203

An Investigation on Int-Soft Interior Hyperideals in Ordered Ternary Semihypergroups

Aakif Fairooze Talee[1], *Mohammad Yahya Abbasi*[1] *and Kostaq Hila*[2,*]

[1]Department of Mathematics, Jamia Millia Islamia, New Delhi-110025, India
[2]Department of Mathematical Engineering, Polytechnic University of Tirana, Tirana 1001, Albania

Received: 12 Dec. 2020, Revised: 2 Jan. 2021, Accepted: 25 Jan. 2021
Published online: 1 Mar. 2021

Abstract: The present paper aims to study ordered ternary semihypergroups in terms of the int-soft interior hyperideals. We introduce the notion of int-soft interior hyperideals over ordered ternary semihypergroups. Also, some related properties are investigated. Some characterizations of interior hyperideals in terms of int-soft interior hyperideals are provided. We prove that every int-soft hyperideal is an int-soft interior hyperideal. The converse is not true in general. Examples that illustrate the results are provided.

Keywords: Int-soft interior hyperideal, ordered ternary semihypergroup, soft set.

1 Introduction and Preliminaries

Marty [1] introduced the natural generalization of classical algebraic structures, i.e. algebraic hyperstructures. In a classical algebraic structure, binary operation on a set A is a map from $A \times A$ to A. However, in an algebraic hyperstructure, binary hyperoperation on a set A is a map from $A \times A$ to the power set of A excluding empty set. Hila et al. [2–7] addressed ternary semihypergroups which are a generalization of ternary semigroup. The concept of ordering hypergroups was investigated by Chavlina [8] as a special class of hypergroups. Heidari and Davvaz [9] explored a semihypergroup $(S, \circ)$ besides a binary relation $' \leq'$, where $' \leq'$ is a partial order relation such that it satisfies the monotone condition.

A soft set is a parameterized family of sets and it was introduced by Molodsov [10] as a mathematical weapon for dealing with hesitant, fuzzy, unpredictable and unsure articles. A soft set is a collection of approximate descriptions of an object. Each approximate description has two parts: a predicate and an approximate value set. Furthermore, several operations on soft sets were introduced by Maji et al. [11] and Cagman et al. [12]. Anvariyeh et al. [13] initiated soft semihypergroups using the soft set theory. They introduced soft semihypergroups, soft subsemihypergroups, soft hyperideals, hyperidealistic soft semihypergroups and soft semihypergroup homomorphisms and studied several related properties. Sezgin et al. [14] defined soft intersection interior ideals, as a new approach to the classical semigroup theory via soft set. In other words, the parameter set of the soft set is semigroup, whereas the universe set is any set. Naz and Shabir [15] introduced the basic properties of soft sets and compared soft sets to the related concepts of semihypergroups. In [16], int-soft interior hyperideals of ordered semihypergroups were handled. Hila et al. [17, 18] studied ternary and m-ary semihypergroups in terms of soft sets. In [19], int-soft hyperideals were introduced and studied in ordered ternary semihypergroups.

The present paper aims to apply soft set theory in hyperstructure theory. We introduce the notion of int-soft interior hyperideals over ordered ternary semihypergroups. Also, some related properties are investigated. We characterize interior hyperideals by means of int-soft interior hyperideals. We prove that every int-soft hyperideal is an int-soft interior hyperideal. The converse is not true in general. Examples illustrating the results are provided.

Throughout the present paper, ordered ternary semihypergroup will be denoted by po-ternary semihypergroup. To develop our main results, we need the following notions. In this paper, the parameter set of

* Corresponding author e-mail: kostaq_hila@yahoo.com

the soft set is po-ternary semihypergroup, whereas the universe set is any set.

For any non-empty set S, let $\mathscr{P}(S)$ be the set of all subsets of S and $\mathscr{P}^*(S)$ the set of all non-empty subsets of S. A map $\circ : S \times S \to \mathscr{P}^*(S)$ is called hyperoperation on the set S and the couple $(S, \circ)$ is called a hypergroupoid.

A hypergroupoid $(S, \circ)$ is called a semihypergroup if for all x, y, z of S we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$$

If $x \in S$ and A and B are non-empty subsets of S, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, x\ o\ A = \{x\} \circ A \text{ and } A\ o\ x = A \circ \{x\}.$$

A map $T : S \times S \times S \to p^*(S)$ is called a ternary hyperoperation on the set S, where S is a non-empty set and $p^*(S)$ denotes the set of all non-empty subsets of S.

A ternary hypergroupoid is the pair (S, T) where T is a ternary hyperoperation on the set S.

If A, B, C are non-empty subsets of S, then we define

$$T(A, B, C) = \bigcup_{a \in A, b \in B, c \in C} T(a, b, c).$$

Definition 1. *A ternary hypergroupoid (S, T) is called a ternary semihypergroup [20] if for all $a_1, a_2, ..., a_5 \in S$, we have*

$$T(T(a_1, a_2, a_3), a_4, a_5) = T(a_1, T(a_2, a_3, a_4), a_5) = T(a_1, a_2, T(a_3, a_4, a_5)).$$

Definition 2. *Let (S, T) be a ternary semihypergroup and A a non-empty subset of S. Then A is called a ternary subsemihypergroup of S if and only if $T(A, A, A) \subseteq A$.*

Definition 3. *A non-empty subset I of a ternary semihypergroup S is called a left (right, lateral) hyperideal of S if $T(S, S, I) \subseteq I(T(I, S, S) \subseteq I, T(S, I, S) \subseteq I)$.*

A non-empty subset M of a ternary semihypergroup S is called a hyperideal of S if it is a left, right and lateral hyperideal of S. A non-empty subset I of a ternary semihypergroup H is called two-sided hyperideal of S if it is a left and right hyperideal of S. A lateral hyperideal I of a ternary semihypergroup S is called a proper lateral hyperideal of S if $I \neq S$.

Definition 4. *[20] Let (S, T) be a ternary semihypergroup. A binary relation ρ is called:*

1. *Compatible on the left if $a\ \rho\ b$ and $x \in T(x_1, x_2, a)$ imply that there exists $y \in T(x_1, x_2, b)$ such that $x\ \rho\ y$,*
2. *Compatible on the right if $a\ \rho\ b$ and $x \in T(a, x_1, x_2)$ imply that there exists $y \in T(b, x_1, x_2)$ such that $x\ \rho\ y$,*
3. *Compatible on the lateral if $a\ \rho\ b$ and $x \in T(x_1, a, x_2)$ imply that there exists $y \in T(x_1, b, x_2)$ such that $x\ \rho\ y$,*
4. *Compatible on the two-sided if $a_1\ \rho\ b_1$, $a_2\ \rho\ b_2$, and $x \in T(a_1, z, a_2)$ imply that there exists $y \in T(b_1, z, b_2)$ such that $x\ \rho\ y$,*
5. *Compatible if $a_1\ \rho\ b_1$, $a_2\ \rho\ b_2$, $a_3\ \rho\ b_3$ and $x \in T(a_1, a_2, a_3)$ imply that there exists $y \in T(b_1, b_2, b_3)$ such that $x\ \rho\ y$.*

Definition 5. *[7] A ternary semihypergroup (S, T) is called a partially ordered ternary semihypergroup if there exists a partially ordered relation $'\leq'$ on S such that $'\leq'$ is compatible on the left, compatible on the right, compatible on the lateral and compatible on the two-sided.*

Let $(S, T, \leq)$ be a po-ternary semihypergroup. Then for any subset R of a po-ternary semihypergroup S, we denote $(R] := \{s \in S | s \leq r$ for some $r \in R\}$. If $R = \{a\}$, we also write $(\{a\}]$ as $(a]$. If A and B are non-empty subsets of S, then we say that $A \leq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$.

Definition 6. *A non-empty subset A of a po-ternary semihypergroup $(S, T, \leq)$ is said to be a po-ternary subsemihypergroup of S if $(T(A, A, A)] \subseteq A$.*

Definition 7. *[7] A non-empty subset I of $(S, T, \leq)$ is called a right (lateral, left) hyperideal of S if*

1. *$T(I, S, S) \subseteq I$ $(T(S, I, S) \subseteq I$, $T(S, S, I) \subseteq I)$,*
2. *If $i \in I$ and $h \leq i$, then $h \in I$ for every $h \in S$.*

Definition 8. *Let $(S, T, \leq)$ be a po-ternary semihypergroup. A non-empty subset A of S is called an interior hyperideal of S, if*

1. *$T(t(S, S, A), S, S) \subseteq A$ and*
2. *If $a \in A, b \in S$ and $b \leq a$, then $b \in S$.*

2 Main Results

In what follows, we take $E = S$ as the set of parameters, which is a po-ternary semihypergroup and U is an initial universe set, unless otherwise specified.

Definition 9. *[12] A soft set f_A over U is defined as $f_A : E \to \mathscr{P}(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Hence f_A is also called an approximation function. A soft set f_A over U can be represented by the set of ordered pairs $f_A = (x, fA(x)) | x \in E, f_A(x) \in \mathscr{P}(U)$.*

It is clear to see that a soft set is a parametrized family of subsets of the set U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 10. *[12] Let $f_A, f_B \in S(U)$. Then, f_A is called a soft subset of f_B and denoted by $f_A \sqsubseteq f_B$, if $f_A(x) \subseteq f_B(x)\ \forall\ x \in E$.*

Definition 11. *[12] Let $f_A, f_B \in S(U)$. Then, union of f_A and f_B , denoted by $f_A \sqcup f_B$, is defined as $f_A \sqcup f_B = f_{A \sqcup B}$, where $f_{A \sqcup B}(x) = f_A(x) \cup f_B(x)\ \forall\ x \in E$.*

Definition 12. *[12] Let $f_A, f_B \in S(U)$. Then, intersection of f_A and f_B , denoted by $f_A \sqcap f_B$, is defined as $f_A \sqcap f_B = f_{A \sqcap B}$, where $f_{A \sqcap B}(x) = f_A(x) \cap f_B(x)\ \forall\ x \in E$.*

2.1 Int-Soft Product and Soft Characteristic Function

For any element a of S, we define

$$A_a = \{(x,y,z) \in S \times S \times S : a \preceq T(x,y,z)\}$$

Definition 13. *Let f_S, g_S and h_S be soft sets over the common universe U. Then, int-soft product $f_S \hat{\diamond} g_S \hat{\diamond} h_S$ is defined by*

$$(f_S \hat{\diamond} g_S \hat{\diamond} h_S)(a) = \begin{cases} \bigcup\limits_{(x,y,z)\in A_a} \{f_S(x) \cap g_S(y) \cap h_S(z)\}, & if\ A_a \neq \emptyset \\ \emptyset, & otherwise. \end{cases}$$

Definition 14. *Let A be any non empty subset of S. S_A the soft characteristic function of A is defined, as follows:*

$$S_A : S \to \mathscr{P}(U)\ ,\ a \mapsto \begin{cases} U, & if\ a \in A, \\ \emptyset, & if\ a \notin A. \end{cases}$$

It is obvious that the soft characteristic function is a soft set over U.

The soft set S_S, where $S_S(a) = U\ \forall\ a \in S$, is called the identity soft set over U. We denote it by $S_S = \mathbb{S}$, i.e. $\mathbb{S}(a) = U\ \forall\ a \in S$.

Definition 15. *Let S be a po-ternary semihypergroup and f_S be a soft set over U. Then, f_S is called*

1. *An int-soft ternary subsemihypergroup of S, if for all $a,b,c \in S$ the following assertions are satisfied:*
   (a) $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(a) \cap f_S(b) \cap f_S(c)$;
   (b) $a \leq b$ *implies* $f_S(a) \supseteq f_S(b)$.
2. *An int-soft left hyperideal of S if for all $a,b,c \in S$, the following assertions are satisfied:*
   (a) $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(c)$;
   (b) $a \leq b$ *implies* $f_S(a) \supseteq f_S(b)$.
3. *An int-soft right hyperideal of S if for all $a,b,c \in S$, the following assertions are satisfied:*
   (a) $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(a)$;
   (b) $a \leq b$ *implies* $f_S(a) \supseteq f_S(b)$.
4. *An int-soft lateral hyperideal of S if for all $a,b,c \in S$, the following assertions are satisfied:*
   (a) $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(b)$;
   (b) $a \leq b$ *implies* $f_S(a) \supseteq f_S(b)$.
5. *An int-soft hyperideal of S if f_S is an int-soft left hyperideal, an int-soft lateral hyperideal and an int-soft right hyperideal of S.*

Definition 16. *Let S be a po-ternary semihypergroup. An int-soft ternary subsemihypergroup f_S of S over U is called an int-soft interior hyperideal of S over U if for all $a,b,c,d,e \in S$, the following assertions are satisfied:*

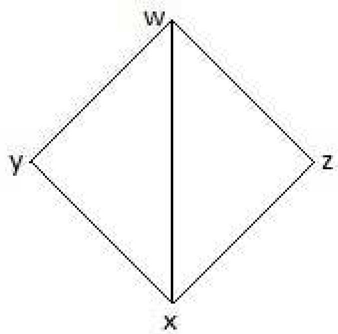
(1) $\bigcap\limits_{y\in T(T(a,b,c),d,e)} f_S(y) \supseteq f_S(c)$;
(2) $a \leq b$ *implies* $f_S(a) \supseteq f_S(b)$.

Example 1. Let $(S,T,\leq)$ be a po-ternary semihypergroup on $S = \{x,y,z,w\}$ with the ternary hyperoperation T given by $T(a,b,c) = (a \circ b) \circ c$, where $\circ$ is the binary hyperoperation given by the table

$\circ$	x	y	z	w
x	{x}	{x}	{x}	{x}
y	{x}	{x}	{x}	{x}
z	{x}	{x}	{x}	{x, y}
w	{x}	{x}	{x, y}	{x, y, z}

Order relation is defined by $\leq := \{(x,x),(y,y),(z,z),(w,w),(x,y),(x,z),(x,w),(y,w),(z,w)\}$. We give the covering relation " $\prec$" and the figure of S as follows:

$$\prec = \{(x,y),(x,z),(x,w),(y,w),(z,w)\}.$$



Let $U = D_3 = \{R_0, R_{120}, R_{240}, A_1, A_2, A_3\}$, dihedral group of order 6, be the universal set. Define the soft set f_S over $U = D_3$ such that $f_S(x) = D_3, f_S(y) = \{R_0, R_{120}, R_{240}\}, f_S(z) = \{R_0\}$ and $f_S(w) = \emptyset$. Then, f_S is an int-soft interior hyperideal of S over U.

Theorem 1. *Let S be a po-ternary semihypergroup and A be a nonempty subset of S. Then A is an interior hyperideal of S if and only if S_A is an int-soft interior hyperideal of S over U.*

Proof. Assume that A is an interior hyperideal of S, so A is a subsemihypergroup of S. Let p,q and r be any elements of S. We claim that $\bigcap\limits_{y\in T(p,q,r)} S_A(y) \supseteq S_A(p) \cap S_A(q) \cap S_A(r)$. If $T(p,q,r) \nsubseteq A$, then there exists $y \in T(p,q,r)$ such that $y \notin A$ and we have $\bigcap\limits_{y\in T(p,q,r)} S_A(y) = \emptyset$. Moreover, $T(p,q,r) \nsubseteq A$ implies that $p \notin A$ or $q \notin A$ or $r \notin A$. Then $S_A(p) = \emptyset$ or $S_A(q) = \emptyset$ or $S_A(r) = \emptyset$, so $\bigcap\limits_{y\in T(p,q,r)} S_A(y) = S_A(p) \cap S_A(q) \cap S_A(r)$. Let $T(p,q,r) \subseteq A$. Then $S_A(y) = U$, for any $y \in T(p,q,r)$.

It implies that $\bigcap_{y\in T(p,q,r)} S_A(y) = U$. Since we have $S_A(p) \subseteq U$ for any $p \in A$, then $\bigcap_{y\in T(p,q,r)} S_A(y) \supseteq S_A(p)\cap S_A(q)\cap S_A(r)$. Hence S_A is an int-soft semihypergroup of S. Let p,q,r,s and a be any elements of S. If $a\in A$, since A is an interior hyperideal of S, then $S_A(a) = U$. Since A is an interior hyperideal of S, then we have $y \in T(T(p,q,a),r,s) \subseteq T(T(S,S,A),S,S) \subseteq A$, then $S_A(y) = U$. Thus $\bigcap_{y\in T(T(p,q,a),r,s)} S_A(y) = U = S_A(a)$. If $a \notin A$, then $S_A(a) = \emptyset$. Since $S_A(y) \supseteq \emptyset = S_A(a)$, then $\bigcap_{y\in T(T(p,q,a),r,s)} S_A(y) \supseteq S_A(a)$. Let $a,b\in S$ such that $a \le b$. If $b\notin A$, then $S_A(b) = \emptyset \subseteq S_A(a)$. If $b\in A$, since A is an interior hyperideal of S, then we have $a \in A$ and so $S_A(a) = U \supseteq S_A(b)$ $\forall$ $a,b\in S$. Hence S_A is an int-soft interior hyperideal of S over U.

Conversely, let $\emptyset \neq A \subseteq S$ such that S_A is an int-soft interior hyperideal of S over U. We claim that $T(A,A,A) \subseteq A$. To prove the claim, let $p,q,r \in A$. By hypothesis, $\bigcap_{y\in T(p,q,r)} S_A(y) \supseteq S_A(p)\cap S_A(q)\cap S_A(r) = U$ which implies that $S_A(y) \supseteq U$ for any $y \in T(p,q,r)$. On the other hand $S_A(p) \subseteq U$ for all $p \in S$. Thus for any $y \in T(p,q,r)$, $S_A(y) = U$ implies that $y \in A$. Hence, it follows that $T(A,A,A) \subseteq A$. Let $y \in T(T(S,S,A),S,S)$, then there exists, $p,q,r,s \in S$ and $a \in A$ such that $y\in T(p,q,a),r,s)$. Since $\bigcap_{y\in T(T(p,q,a),r,s)} S_A(y) \supseteq S_A(a)$ and $a \in A$, then we have $S_A(y) = U$. Hence for each $y \in T(T(S,S,A),S,S)$, we have $S_A(y) = U$, and so $y \in A$. Thus $T(T(S,S,A),S,S) \subseteq A$. let $a \in A$ such that $S \ni b \le a$. Since S_A is a int-soft interior hyperideal of S, then we have $S_A(b) \supseteq S_A(a) = U$ and so $S_A(b) = U$. Thus, $b\in A$. Hence, A is an interior hyperideal of S.

Theorem 2. *Let S be a po-ternary semihypergroup and f_S be an int-soft hyperideal of S over U. Then, f_S is an int-soft interior hyperideal of S over U.*

Proof. Let $p,q,a,r,s \in S$. Since f_S is an int-soft hyperideal of S over U, then for any $y \in T(T(p,q,a),r,s)$, we have $\bigcap_{y\in T(T(p,q,a),r,s)} f_S(y) = \bigcap_{\substack{y\in T(p,q,x)\\ x\in T(a,r,s)}} f_S(y) \supseteq f_S(x) \supseteq \bigcap_{x\in T(a,r,s)} f_S(x) \supseteq f_S(a)$. Hence f_S is an int-soft interior hyperideal of S over U.

The converse of Proposition 2, is untrue in general. We can illustrate it by the following example.

Example 2. Let $(S,T,\le)$ be a po-ternary semihypergroup on $S = \{a,b,c,d\}$ with the ternary hyperoperation T given by $T(x,y,z) = (x \circ y)\circ z$, where $\circ$ is the binary hyperoperation given by the table

$\circ$	a	b	c	d
a	{a}	{a}	{a}	{a}
b	{a}	{a}	{a}	{a}
c	{a}	{a}	{a}	{a, b}
d	{a}	{a}	{a, b}	S

Order relation is defined by $\le := \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(b,d),(c,d)\}$.

Let $U = D_4 = \{R_0,R_{90},R_{180},R_{270},H,V,D,D'\}$, dihedral group of order 8, be the universal set. Define the soft set f_S over $U = D_4$ such that $f_S(a) = D_4, f_S(b) = \{R_0,R_{180},H,D\}, f_S(c) = \{R_0,H,V\}$ and $f_S(d) = \emptyset$. Then, f_S is an int-soft interior hyperideal of S over U. But f_S is not an int-soft left hyperideal of S over U as: $T(d,d,c) = \{a,b,c,d\}\circ c = a\cup a\cup a\cup \{a,b\} = \{a,b\}$. Therefore, $\bigcap_{y\in T(d,d,c)} f_S(y) = f_S(a)\cap f_S(b) = f_S(b) = \{R_0,R_{180},H,D\} \not\supseteq \{R_0,H,V\} = f_S(c)$.

Definition 17. *Let f_S be a soft set of a po-ternary semihypergroup S over U and $\delta \in U$. Then δ-inclusion of f_S, denoted by $\mathscr{U}(f_S,\delta)$, is defined as*

$$\mathscr{U}(f_S,\delta) = \{x\in S \;:\; f_S(x) \supseteq \delta\}.$$

Theorem 3. *Let f_S be a soft set of a po-ternary semihypergroup S over U and $\delta \in \mathscr{P}(U)$. Then f_S is an int-soft interior hyperideal of S over U if and only if each nonempty δ-inclusive set $\mathscr{U}(f_S,\delta)$ is an interior hyperideal of S.*

Proof. Assume that f_S is an int-soft interior hyperideal of S over U. Let $\delta \in \mathscr{P}(U)$ such that $\mathscr{U}(f_S,\delta) \neq \emptyset$. Let $a,b,c \in \mathscr{U}(f_S,\delta)$. Then $f_S(a) \supseteq \delta, f_S(b) \supseteq \delta$ *and* $f_S(c) \supseteq \delta$. By hypothesis, we have $\bigcap_{y\in T(a,b,c)} f_S(y) \supseteq f_S(a)\cap f_S(b)\cap f_S(c) \supseteq \delta\cap\delta\cap\delta = \delta$. Thus for any $y \in T(a,b,c)$, we have $f_S(y) \supseteq \delta$, implies that $y \in \mathscr{U}(f_S,\delta)$. It follows that $T(a,b,c) \subseteq \mathscr{U}(f_S,\delta)$. Hence $\mathscr{U}(f_S,\delta)$ is a ternary subsemihypergroup of S. Let $a,b,c,d \in S$ and $x \in \mathscr{U}(f_S,\delta)$. Then $f_S(x) \supseteq \delta$. Since f_S is an int-soft interior hyperideal of S over U, then $\bigcap_{y\in T(T(a,b,x),c,d)} f_S(y) \supseteq f_S(x) \supseteq \delta$. Hence $f_S(y) \supseteq \delta$ for any $y \in T(T(a,b,x),c,d)$ implies that $y \in \mathscr{U}(f_S,\delta)$. Thus $T(S,S,\mathscr{U}(f_S,\delta)),S,S) \subseteq \mathscr{U}(f_S,\delta)$. Let $a \in \mathscr{U}(f_S,\delta)$ and $b \in S$ with $b \le a$. Then $\delta \subseteq f_S(a) \subseteq f_S(b)$, we get $b \in \mathscr{U}(f_S,\delta)$. Therefore $\mathscr{U}(f_S,\delta)$ is an interior hyperideal of S.

Conversely, suppose that $\mathscr{U}(f_S,\delta) \neq \emptyset$ is an interior hyperideal of S. If $\bigcap_{y\in T(a,b,c)} f_S(y) \subset f_S(a)\cap f_S(b)\cap f_S(c)$ for some $a,b,c \in S$, then there exists $\delta \in \mathscr{P}(U)$ such that $\bigcap_{y\in T(a,b,c)} f_S(y) \subset \delta \subseteq f_S(a)\cap f_S(b)\cap f_S(c)$, which implies that $a,b,c \in \mathscr{U}(f_S,\delta)$ and $T(a,b,c) \not\subseteq \mathscr{U}(f_S,\delta)$. It contradicts the fact that $\mathscr{U}(f_S,\delta)$ is a ternary

semihypergroup of S. Consequently, $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(a)\cap f_S(b)\cap f_S(c)$ for all $a,b,c \in S$.
Next we show that $\bigcap\limits_{y\in T(T(a,b,x),c,d)} f_S(y) \supseteq f_S(x)$ for all $a,b,x,c,d \in S$. Choose $f_S(x) = \delta$, then $x \in \mathscr{U}(f_S,\delta)$. Since $\mathscr{U}(f_S,\delta)$ is an interior hyperideal of S, then we get $T(T(a,b,x),c,d) \subseteq \mathscr{U}(f_S,\delta)$. Therefore for every $y \in T(T(a,b,x),c,d)$, we have $f_S(y) \supseteq \delta$ and so $f_S(x) = \delta \subseteq \bigcap\limits_{y\in T(T(a,b,x),c,d)} f_S(y)$. Let $a,b \in S$ such that $a \leq b$. If $f_S(b) = \delta$, then $b \in \mathscr{U}(f_S,\delta)$. Since $\mathscr{U}(f_S,\delta)$ is an interior hyperideal of S, then we get $a \in \mathscr{U}(f_S,\delta)$. So $f_S(a) \supseteq \delta = f_S(b)$. Therefore f_S is a int-soft interior hyperideal of S over U.

*Example 3.*Let $S = \{x,y,z,w\}$ be the po-ternary semihypergroup in Example 1 and f_S be a soft set over $U = \{u_1,u_2,u_3\}$. If we define a soft set f_S over U such that $f_S(x) = U, f_S(y) = \{u_1,u_2\}, f_S(z) = \{u_1,u_3\}$ and $f_S(w) = \emptyset$, then f_S is an int-soft interior hyperideal of S over U. Then

$$\mathscr{U}(f_S,\delta) = \begin{cases} \{x,y,z\}, & \text{if} \quad \delta = \{u_1\} \\ \{x,y\}, & \text{if} \quad \delta = \{u_2\} \\ \{x,z\}, & \text{if} \quad \delta = \{u_3\} \\ \{x,y\}, & \text{if} \quad \delta = \{u_1,u_2\} \\ \{x,z\}, & \text{if} \quad \delta = \{u_1,u_3\} \\ \{x\}, & \text{if} \quad \delta = \{u_2,u_3\} \\ \{x\}, & \text{if} \quad \delta = \{u_1,u_2u_3\} \end{cases}$$

By Theorem 3, each δ-inclusive set $\mathscr{U}(f_S,\delta)$ is an interior hyperideal of S.

Theorem 4.*Let $\{f_{S_i} \mid i \in I\}$ be a family of int-soft interior hyperideals of a po-ternary semihypergroup of S over U. Then $f_S = \bigcap\limits_{i\in I} f_{S_i}$ is an int-soft interior hyperideals of a po-ternary semihypergroup of S over U where $\big(\bigcap\limits_{i\in I} f_{S_i}\big)(x) = \bigcap\limits_{i\in I} \big(f_{S_i}\big)(x)$.*

*Proof.*Let $a,b,c \in S$. Then, since each $\{f_{S_i} \mid i \in I\}$ is an int-soft interior hyperideals of S over U, then $\bigcap\limits_{y\in T(T(a,b,c)} f_{S_i}(y) \supseteq f_{S_i}(a)\cap f_{S_i}(b)\cap f_{S_i}(c)$. Thus for any $y \in T(a,b,c)$, $f_{S_i}(y) \supseteq f_{S_i}(a)\cap f_{S_i}(b)\cap f_{S_i}(c)$ and we have $f_S(y) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(y) = \bigcap\limits_{i\in I} \big(f_{S_i}(y)\big) \supseteq \bigcap\limits_{i\in I} \big(f_{S_i}(a) \cap f_{S_i}(b)\cap f_{S_i}(c)\big) = \big(\bigcap\limits_{i\in I} \big(f_{S_i}(a)\big)\big)\bigcap\big(\bigcap\limits_{i\in I} \big(f_{S_i}(b)\big)\big)\bigcap \big(\bigcap\limits_{i\in I} \big(f_{S_i}(c)\big)\big) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(a)\bigcap\big(\bigcap\limits_{i\in I} f_{S_i}\big)(b)\bigcap\big(\bigcap\limits_{i\in I} f_{S_i}\big)(c) = f_S(a)\cap f_S(b)\cap f_S(c)$, which implies that $\bigcap\limits_{y\in T(a,b,c)} f_S(y) \supseteq f_S(a)\cap f_S(b)\cap f_S(c)$. Let $a,b,x,c,d \in S$ and $\bigcap\limits_{y\in T(T(a,b,x),c,d)} f_{S_i}(y) \supseteq f_{S_i}(x)$. Thus for any $y \in T(T(a,b,x),c,d)$, $f_{S_i}(y) \supseteq f_{S_i}(x)$. Then $f_S(y) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(y) = \bigcap\limits_{i\in I} \big(f_{S_i}(y)\big) \supseteq \bigcap\limits_{i\in I} (f_{S_i}(x)) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(x) = f_S(x)$. Thus $\bigcap\limits_{y\in T(T(a,b,x),c,d)} f_S(y) \supseteq f_S(x)$.
Furthermore, if $a \leq b$, then we will prove $f_S(a) \supseteq f_S(b)$. Since every $f_{S_i}, (i \in I)$ is an int-soft interior hyperideals of a po-ternary semihypergroup of S over U, then it follows that $f_{S_i}(a) \supseteq f_{S_i}(b)$, for all $i \in I$. Thus $f_S(a) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(a) = \bigcap\limits_{i\in I} \big(f_{S_i}(a)\big) \supseteq \bigcap\limits_{i\in I} \big(f_{S_i}(b) = \big(\bigcap\limits_{i\in I} f_{S_i}\big)(b) = f_S(b)$. Hence f_S is an int-soft interior hyperideals of a po-ternary semihypergroup of S over U.

Theorem 5. *[19] Let f_S be a soft set over U. Then, f_S is an int-soft semihypergroup over U if and only if for all $a,b \in S$, we have*

1.$f_S \,\hat{\diamond}\, f_S \,\hat{\diamond}\, f_S \sqsubseteq f_S$.
2.If $a \leq b$ then $f_S(a) \supseteq f_S(b)$.

Theorem 6.*Let f_S be a soft set over U. Then, f_S is an int-soft interior hyperideal of S over U if and only if*

1.$f_S \,\hat{\diamond}\, f_S \,\hat{\diamond}\, f_S \sqsubseteq f_S$ and $\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, \mathbb{S} \sqsubseteq f_S$.
2.If $a \leq b$ then $f_S(a) \supseteq f_S(b)\ \forall\, a,b \in S$.

*Proof.*First assume that f_S is an int-soft interior hyperideal of S over U. (2) is straightforward by definition 16. We claim that $\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, \mathbb{S} \subseteq f_S$. To prove the claim, let $x \in S$. If $A_x \neq \emptyset$. Then there exists $a,b,c \in S$ such that $x \preceq T(a,b,c)$, and let $(p,q,r) \in A_a; a \preceq T(p,q,r)$ for any $p,q,r \in S$. $A_x \neq \emptyset$. Then there exists $p,q,r \in S$ such that $a \preceq T(p,q,r)$. Then $x \preceq T(T(p,q,r),b,c)$ and there exists $y \in T(T(p,q,r),b,c)$ such that $x \leq y$. Since f_S is an int-soft interior hyperideal of S over U, then $f_S(x) \supseteq f_S(y) \supseteq \bigcap\limits_{y\in T(T(p,q,r),b,c)} f_S(y) \supseteq f_S(r)$. Thus

$$\begin{aligned} &(\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, \mathbb{S})(x) \\ &= \bigcup_{x\preceq T(a,b,c)} \{(\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S)(a)\cap \mathbb{S}(b)\cap \mathbb{S}(c)\} \\ &= \bigcup_{x\preceq T(a,b,c)} \{\{\bigcup_{a\preceq T(p,q,r)} (\mathbb{S}(p)\cap \mathbb{S}(q)\cap f_S(r))\}\cap \mathbb{S}(b)\cap \mathbb{S}(c)\} \\ &= \bigcup_{x\preceq T(T(p,q,r),b,c)} (f_S(r)) \subseteq f_S(x). \end{aligned}$$

If $A_x = \emptyset$. Then, $(\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, \mathbb{S}) \subseteq f_S(x) = \emptyset \subseteq f_S(x)$. Hence, $\mathbb{S} \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, f_S \,\hat{\diamond}\, \mathbb{S} \,\hat{\diamond}\, \mathbb{S} \sqsubseteq f_S$.

Conversely, for any $a,b,c,d,e \in S$. Let $y \in T(T(a,b,c),d,e)$. Then there exists $x \in T(a,b,c)$ such that $y \in T(x,d,e)$. Since $x \leq x$ and $y \leq y$, then we have $x \preceq T(a,b,c)$ and $y \preceq T(x,d,e)$. Thus by hypothesis, we have

$$\begin{aligned} f_S(y) &\supseteq (\mathbb{S} \hat{\circ} \mathbb{S} \hat{\circ} f_S \hat{\circ} \mathbb{S} \hat{\circ} \mathbb{S})(y) \\ &= \bigcup_{y \preceq T(p,q,r)} \{(\mathbb{S} \hat{\circ} \mathbb{S} \hat{\circ} f_S)(p) \cap \mathbb{S}(q) \cap \mathbb{S}(r))\} \\ &\supseteq \{ \bigcap_{x \in T(a,b,c)} (\mathbb{S} \hat{\circ} \mathbb{S} \hat{\circ} f_S)(x)\} \cap \mathbb{S}(d) \cap \mathbb{S}(e) \\ &= \bigcap_{x \in T(a,b,c)} (\mathbb{S} \hat{\circ} \mathbb{S} \hat{\circ} f_S)(x) \\ &= \bigcap_{x \in T(a,b,c)} \{ \bigcup_{x \preceq T(l,m,n)} (\mathbb{S}(l) \cap \mathbb{S}(m) \cap f_S(n))\} \\ &\supseteq \mathbb{S}(a) \cap \mathbb{S}(b) \cap f_S(c) = f_S(c). \end{aligned}$$

Thus, $\bigcap_{y \in T(T(a,b,c),d,e)} f_S(y) \supseteq f_S(c)$ for all $a,b,c,d,e \in S$. The rest of the proof is the consequence of the Theorem 5.

For any $a \in S$, let S be a po-ternary semihypergroup and f_S be a soft set over U. We denote by I_a the subset of S defines as follows:

$$I_a = \{b \in S : f_S(b) \supseteq f_S(a)\}.$$

Theorem 7.*Let S be a po-ternary semihypergroup and f_S be an int-soft interior hyperideal of S over U. Then I_a is an interior hyperideal of S for every $a \in S$.*

*Proof.*Let $a \in S$. First of all $\emptyset \neq I_a \subseteq S$, since $a \in I_a$. Let $p,q,r,s \in S$ and $b \in I_a$. Since f_S is an int-soft interior hyperideal of S over U and $p,q,r,s,b \in S$, then we have $\bigcap_{y \in T(T(p,q,b),r,s)} f_S(y) \supseteq f_S(b)$. Since $b \in I_a$, it implies that $f_S(b) \supseteq f_S(a)$, then $\bigcap_{y \in T(T(p,q,b),r,s)} f_S(y) \supseteq f_S(a)$, it implies that $f_S(y) \supseteq f_S(a)$, so $y \in I_a$ and so $T(T(p,q,b),r,s) \subseteq I_a$. Let $b \in I_a$ and $c \in S$ with $c \leq b$. Since f_S is an int-soft interior hyperideal of S over U and $b,c \in S$ with $c \leq b$, then we have $f_S(c) \supseteq f_S(b) \supseteq f_S(a)$. So $c \in I_a$ and I_a is an interior hyperideal of S for every $a \in S$.

*Example 4.*Let $S = \{x,y,z,w\}$ be the po-ternary semihypergroup and f_S is an int-soft interior hyperideal of S over U in Example 1. Then

$$I_x = \{x\}, \ \ I_y = \{x,y\}, \ \ I_z = \{x,y,z\}, \ \ I_w = S.$$

By Theorem 7, each I_a is an interior hyperideal of S.

Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] F. Marty, Sur une generalization de la notion de group, 8th Congress des Mathematiciens Scandinaves, Stockholm, 45-49, (1934).
[2] K. Hila, B. Davvaz and K. Naka, On hyperideal structure of ternary semihypergroups, Iran. J. Math. Sci. Inform., **9**, no. 1, 81-98 (2014).
[3] K. Naka and K. Hila, On some special classes of hyperideals in ternary semihypergroups, Util. Math., **98**, 97-112 (2015).
[4] K. Naka and K. Hila, Some properties of hyperideals in ternary semihypergroups, Math. Slovaca, **63**, no. 3, 449-468 (2013).
[5] K. Naka and K. Hila, On the structure of quasi-hyperideals and bi-hyperideals in ternary semihypergroups, Afr. Mat., **26**, no. 7-8, 1573-1591 (2015).
[6] K. Naka, K. Hila, Regularity of ternary semihypergroups, Quasigroups Related Systems, **25**(2), 291-306 (2017).
[7] A.F. Talee, M.Y. Abbasi, S.A. Khan, K. Hila, Fuzzy set approach to hyperideal theory of po-ternary semihypergroups, J. Discrete Math. Sci. Cryptogr., **22**(3), 411-431 (2019).
[8] J. Chvalina, *Commutative hypergroups in the sense of Marty and ordered sets*. General algebra and ordered sets (Horní Lipová, 1994), 19-30, Palacký Univ. Olomouc Fac . Sci., Olomouc, [1994].
[9] D. Heidari and B. Davvaz, On ordered hyperstructures, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., **73**, no. 2, 85-96 (2011).
[10] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl., **37**, no. 4-5, 19-31 (1999).
[11] P.K. Maji, R., Bswas and A.R. Roy, Soft set theory, Comput. Math. Appl., **45**, 555-562 (2003).
[12] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European J. Oper. Res., **207**, 848-855 (2010).
[13] S.M. Anvariyeh, S. Mirvakili, O. Kazanci and B. Davvaz, Algebraic hyperstructures of soft sets associated to semihypergroups, Southeast Asian Bull. Math., **35**, no. 6, 911-925 (2011).
[14] A. Sezgin, N. Cagman and A.O. Atagun, Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; A New Approach to Semigroup Theory II, J. Mult.-Valued Logic Soft Comput., **23**, no. 1-2, 161-207 (2014).
[15] S. Naz and M. Shabir, On soft semihypergroups, J. Intell. Fuzzy Systems **26**, no. 5, 2203-2213 (2014).
[16] A. Khan, A. Farooq and B. Davvaz, Int-soft interior hyperideals of ordered semihypergroups, Int. J. Analysis Applic., **14**, no. 2, 193-202 (2017).
[17] K. Hila, K. Naka, V. Leoreanu-Fotea, S. Sadiku, Algebraic hyperstructure of soft sets associated to ternary semihypergroups, Ital. J. Pure Appl. Math., **30**, 349-372 (2013).
[18] K. Hila, T. Vougiouklis and K. Naka, On the structure of soft *m*-ary semihypergroups, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys., **76**, no. 4, 91-106 (2014).
[19] A.F. Talee, M.Y. Abbasi, K. Hila and S.A. Khan, A new approach towards int-soft hyperideals in ordered ternary semihypergroups, J. Discrete Math. Sci. Cryptogr. In Press. DOI: 10.1080/09720529.2020.1747674

[20] B. Davvaz, V. Leoreanu, Binary relations on ternary semihypergroups, Comm. Algebra, **38** (10), 3621-3636 (2010).

Aakif Fairooze Talee has completed his masters degree in Mathematics from University of Kashmir, Srinagar, India in 2012 with first division and M.Phil. from D.A.V.V. Indore in 2014 with 'A' grade. He received his Ph.D degree in 2018 from Jamia Millia Islamia, New Delhi. He is currently working as Lecturer in Department of School Education, Kashmir India. His main interest areas include Abstract Algebra and Fuzzy Mathematics.

M. Y. Abbasi has received his master degree in mathematics in 2000 with first division from Aligarh Muslim University, Aligarh, India. He received his Ph.D degree in 2005 from the same university. He is currently working as associate professor in Department of mathematics, Jamia Millia Islamia, New Delhi, India. He has guided 4 Ph. D. students. He has published more than 50 research articles. He has 15 years of teaching experience and 20 years of research experience. He is a life member of national bodies: Ramanujan mathematical society and Indian mathematical society. His area of interest includes Abstract algebra, module theory and fuzzy mathematics.

Kostaq Hila is professor of Mathematics at The Department of Mathematical Engineering, Polytechnic University of Tirana, Albania. He received his M.Sc. and PhD degree in Mathematics at University of Tirana, Albania. His main research interests include algebraic structures theory (in particular algebraic theory of semigroups and ordered semigroups, LA-semigroups etc.), algebraic hyperstructures theory, fuzzy-rough-soft sets and applications. He has published several research papers in various international reputed peer-reviewed journals. He is a referee of several well-known international peer-reviewed journals.