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Multiply Censored Partially Accelerated Life Testing for Power Function Model

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Abstract: The present paper explores estimating failure time data under step-stress partially accelerated life testing based on multiply censored data. The lifetime distribution of the test units is assumed to follow the Power Function distribution. The point and interval maximum-likelihood estimations are obtained for the distribution parameter and tampering coefficient. The performances of the estimators of the model parameters using multiply censored data are evaluated and compared in terms of biases and root mean squared errors using a Monte Carlo simulation study.

Keywords: Maximum Likelihood Estimation, Multiply Censored Data, Partially Accelerated Life Test, Simulation Study, Tempering Coefficient.

1 Introduction

Mukherjee and Islam [1] presented an important finite range failure-time distribution called Power Function failure model, which includes the exponential and rectangular distribution as particular cases. The model is commonly used as a simple lifetime distribution to assess system reliability. It exhibits a better fit for failure information and provides more appropriate information about the hazard rate and other reliability measures. Hence, it caught the attention of many reliability practitioners in the world. Lai and Mukherjee [2] discussed some ageing properties of this distribution and rectified the mistakes and brought to light some other interesting properties of this distribution. Lia et al. [3] mentioned this distribution in the list of some important bath-tub-shaped failure rate models. Showkat et al. [4] presented the procedure of estimating the Step-Stress Partially Accelerated Life Test for Power Function Distribution under Time Constraint. Recently, Showkat et al. [5] have introduced a location parameter of the distribution at a time α(i.e. a time before which failure cannot occur) which makes it a more useful failure distribution than the existing one.
It is difficult to collect failure record of highly reliable products with long lifetimes since only a few or even no failures occur within a limited testing time under normal operating conditions. To overcome this problem, an Accelerated life test (ALT) or partially accelerated life test (PALT) is used to induce failure information in a short time. The testing procedure involves the test units to higher stress conditions than normal to induce failures more quickly than would be observed under usual operating conditions. The relation between life and stress of units is usually known or assumed. For modern and advanced products, such life–stress relationships are unknown and cannot be assumed, i.e. ALT data cannot be extrapolated to use condition. Thus, in such cases, partially accelerated life test (PALT) is a more suitable test to be performed, for which tested units are subjected to both normal and accelerated conditions. PALTs have been successfully used to enable engineers to estimate the acceleration factor and so extrapolate the accelerated data to usual conditions. Thus, PALT is used for reliability analysis to save more time and money over the ordinary or traditional life tests.
The two types of PALTs are step-stress and constant stress). In step-stress PALT, a sample of test items is first run at use condition and, if it does not fail for a specified time, it is run at accelerated condition until pre-specified

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numbers of failures are obtained or censoring time is reached. Goel [6] considered the estimation problem of the accelerated factor using both maximum likelihood and Bayesian methods for items having exponential and uniform distributions. DeGroot and Goel [7] estimated the parameters of the exponential distribution and acceleration factor in SS-PALT using Bayesian approach, with different loss functions. Also, Bhattacharyya and Soejoeti [8] estimated the parameters of the Weibull distribution and acceleration factor using the maximum likelihood method. Bai and Chung [9] estimated the scale parameter and acceleration factor for the exponential distribution under type I censored sample using the maximum likelihood method. Abdel-Ghaly et al. [10] investigated the estimation problem of the acceleration factor and the parameters of the Weibull distribution in SS-PALT using the maximum likelihood method in two types of data, namely, type I and type II censoring. Wang et al. [11] discussed step-stress PALTs for Weibull distribution under multiply censored data. Showkat et al. [12] described the step stress partially accelerated life testing plan for competing risk using adaptive type-I progressive hybrid censoring. Rahman et al. [13] obtained the likelihood estimation of Exponentiated exponential distribution under step stress partially accelerated life testing plan using progressive type-II censoring. Recently, Lone and Ahmed [14] have presented the detailed analysis and design of Accelerated Life Testing with an Application under Rebate Warranty. More recently, Ahmed et al. [15] have discussed the statistical inference for burr type X distribution using geometric process in accelerated life testing design for time censored data. Alam et al. [16] presented a study on accelerated life test and age replacement under type-II censoring for Burr type-X distribution. Alam et al. [17] also tackled with step-stress partially accelerated life test under progressive censoring scheme using power function distributions.

This study consists of estimating the acceleration factor and parameters of Power Function distribution using the maximum likelihood method. This work was conducted for SS-PALT under multiply censored data. The precision of the estimators obtained is investigated in terms of mean bias, root mean square error and the coverage rate based on a simulation study. Moreover, the confidence intervals of the estimators are also obtained. The Power Function model is used first time under multiply censored data. No previous studies were available.

This study is organized, as follows: In section 2, the model under SS-PALT using multiply censored data is provided. In section 3, the maximum likelihood method is used to find the point and interval estimates of parameters and acceleration factor for the finite range model under multiply censored data. Simulation studies for illustrating the theoretical results are presented in section 4. Conclusion is presented in section 5.

2 The Material and Test Method

Power Function failure model is an important finite range distribution in the modern reliability practice and can be frequently preferred over mathematically more complex distribution, such as the Weibull and the lognormal because of its simplicity. The distribution has the probability function follows:

$$f(t,p,\theta,\alpha)=\frac{p}{\theta^{p}}t^{p-1}\quad p>0,\theta>0,0<t<\theta \tag{1}$$

where θ is a scale parameter and p is the shape parameter. The cumulative distribution function is:

$$F(t)=\left(\frac{t}{\theta}\right)^{p} \tag{2}$$

And the reliability function of the finite range model is given by

$$R(t)=1-\left(\frac{t}{\theta}\right)^{p} \tag{3}$$

In SS-PALT, all of the n units are first tested under normal use conditions and if the unit does not fail for a pre-specified time τ, then it runs at accelerated condition until failure occurs. This means that if the item has not failed by some pre-specified time τ, the test is switched to the higher level of stress and is continued until failure occurs. The effect of this switch is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β. In this case, switching to a higher stress level will shorten the life of the test item. Thus the total lifetime of a test item, denoted by Y, passes through two stages: the normal and accelerated stage. Therefore, the lifetime of the test unit in SS-PALT is given, as follows:

$$Y=\begin{cases}T & T\le\tau\\ \tau+\beta^{-1}(T-\tau) & T>\tau\end{cases} \tag{4}$$

Where T is the lifetime of the item at normal use condition, τ is the stress change time and $\beta(>1)$ is the acceleration factor which is the ratio of mean life of an item at use condition to that at the accelerated condition. Assume that the lifetime of the test item follows Power Function failure distribution with shape parameter θ and scale parameter p. The probability density function and cumulative density function of total lifetime, Y, of an item is given by equations (5) and (6), respectively.

$$f(y)=\begin{cases}0 & y<0\\ f_1(y) & 0<y\le\tau\\ f_2(y) & y>\tau\end{cases} \tag{5}$$

where $f_1=\frac{p}{\theta^p}t^{p-1}$ is equivalent form equation(1), and
$f_2(y)=\frac{\beta p}{\theta^p}[\tau+\beta(y-\tau)]^{p-1}$ $\theta>0$, $p>0$ and $\beta>0$

$$F(y)=\begin{cases}0 & y<0\\ F_1(y) & 0<y\le\tau\\ F_2(y) & y>\tau\end{cases} \tag{6}$$

where $F_1(y) = \left(\frac{y}{\theta}\right)^p$ and $F_2(y) = \frac{[\tau+\beta(y-\tau)]^p}{\theta^p}$; $\theta > 0, p > 0$ and $\beta > 0$

This is obtained using equations (1) and (4) by the variable transformation technique. At normal use condition, suppose $y_{1f}, y_{2f}, \cdots, y_{r_{1f}}$ are r_1 failure times and $y_{1_s}, y_{2_s}, \cdots, y_{m_{1_s}}$ are m_1 censoring times of test items. Also, at accelerated conditions, we assume that there are r_2 failures at times $y_{1f}, y_{2f}, \cdots, y_{r_{2_f}}$ and m_2 units with censoring times $y_{1s}, y_{2s}, \cdots, y_{m_2 s}$. Then, the likelihood function for the SS-PALT using multiply censored data is

$$L = C\prod_{i=1}^{r_1} f(y_{if}) \prod_{j=1}^{m_1}[1-F(y_{js})] \prod_{k=1}^{r_2} f(y_{kf}) \prod_{i=1}^{m_2}[1-F(y_{is})] \tag{7}$$

Substituting equations (5) and (6) in likelihood equation (7) and taking the logarithm, we get

$$\begin{aligned} lnL = {} & ln(c) + r_1 ln(p) - r_1 p ln(\theta) + (p-1)\sum_{i=1}^{r_1} ln(y_{i,f}) \\ & + \sum_{j=1}^{m_1} ln(\theta^p - y_{j,s}^p) + r_2 ln(\beta p) - r_2 p ln(\theta) \\ & + (p-1)\sum_{k=1}^{r_2} ln\phi_{k,f} + \sum_{l=1}^{m_2} ln(\theta^p - \phi_{l,s}^p) - (m_1+m_2) ln\theta^p \end{aligned} \tag{8}$$

where $\phi_{k,f} = \tau + \beta(y_{k,f} - \tau)$ and $\phi_{l,s} = \tau + \beta(y_{l,s} - \tau)$

3 Estimation of Parameters

The maximum likelihood estimation method is used to find an estimate of the parameters with good statistical properties. The point and interval estimation of the parameters of Power Function distribution and tempering coefficient is evaluated.

3.1 Point estimation

In this subsection, the process of obtaining the point ML estimates of parameters and tempering coefficient is discussed. The maximum likelihood estimates of β, θ and p are obtained by setting the first partial derivatives of equation (8) to zero with respect to β, θ and p respectively.

$$\frac{\partial lnL}{\partial \theta} = \sum_{j=1}^{m_1} \frac{p\theta^{p-1}}{(\theta^p) - y_{j,s}^p} - \frac{p(r_1+r_2)}{\theta} + \sum_{l=1}^{m_2} \frac{p\theta^{p-1}}{(\theta^p) - \phi_{l,s}^p} - \frac{p(m_1+m_2)}{\theta} = 0 \tag{9}$$

$$\frac{\partial lnL}{\partial \beta} = \beta^{-1} r_2 + (p-1)\sum_{k=1}^{r_2} \frac{y_{k,f} - \tau}{\phi_{k,f}} - \sum_{l=1}^{m_2} \frac{p\phi_{l,s}^{p-1}(y_{l,s} - \tau)}{\theta^p - \phi_{l,s}^p} = 0 \tag{10}$$

$$\begin{aligned} \frac{\partial lnL}{\partial p} = {} & \frac{r_1+r_2}{p} - (r_1+r_2) ln\theta \\ & - \sum_{i=1}^{r_1} lny_{i,f} + \sum_{j=1}^{m_1} \frac{\theta^p ln\theta - y_{j,s}^p ln(y_{j,s})}{\theta^p - y_{j,s}^p} + \sum_{k=1}^{r_2} \phi_{k,f} \\ & + \sum_{i=1}^{r_1} lny_{i,f} + \sum_{l=1}^{m_2} \frac{\theta^p ln\theta - y_{l,s}^p ln(y_{l,s})}{\theta^p - \phi_{l,s}^p} - (m_1+m_2) ln\theta = 0 \end{aligned} \tag{11}$$

Equations (9), (10) and (11) are non-linear functions of population parameters as well as functions of the solutions of these equations. Due to this difficulty, it impossible possible to find an exact solution. Hence, to obtain the MLEs of β,p and θ, the Newton Raphson method is used.

3.2 Interval estimates

The asymptotic variance-covariance matrix of β,p and θ is obtained by inverting the Fisher information matrix, $I = \left[-\frac{\partial^2 lnL}{\partial \omega_i \partial \omega_j}\right]$; $i,j = 1,2,3$, where $\omega_1 = \beta$, $\omega_2 = \theta$, $\omega_3 = p$. The elements of fisher information are given by:

$$\begin{aligned} \frac{\partial^2 lnL}{\partial \theta^2} = {} & -\sum_{j=1}^{m_1} \frac{p\theta^{2p-1} p(p-1) y_{j,s}^p \theta^{p-2}}{(\theta^p - y_{j,s}^p)^2} \\ & + \frac{m_1+m_2+r_1+r_2}{\theta^2} \\ & + \sum_{l=1}^{m_2} \frac{p(p-1)p\theta^{p-2}(\theta^p - \phi_{l,s}^p) - p^2\theta^{2p-1}}{(\theta^p - \phi_{l,s}^p)^2} \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\partial^2 lnL}{\partial \beta^2} = {} & -\frac{r^2}{\beta} - (p-1)\sum_{k=1}^{r_2} \phi_{k,f}^{-2}(y_{k,f} - \tau)^2 \\ & - \sum_{l=1}^{m_2} p(y_{l,s} - \tau)^2 \left[\frac{(p-1)\theta^p \phi_{l,s}^{p-2} + \phi_{l,s}^{2(p-1)}}{(\theta^p - \phi_{l,s}^p)^2}\right] \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\partial^2 lnL}{\partial p^2} = {} & \sum_{j=1}^{m_1} \frac{\theta^p y_{j,s}^p [2ln\theta ln(y_{j,s}) - ln^2 y_{j,s} - ln^2\theta]}{(\theta^p - y_{j,s}^p)^2} \\ & + \sum_{l=1}^{m_2} \frac{\theta^p \phi_{l,s}^p [2ln\theta ln(\phi_{l,s}) - ln^2\phi_{l,s} - ln^2\theta]}{(\theta^p - \phi_{l,s}^p)^2} \\ & - \frac{(r_1+r_2)}{p^2} \end{aligned} \tag{14}$$

$$\frac{\partial^2 lnL}{\partial\theta\partial\beta} = \sum_{l=1}^{m_2} \frac{p^2\theta^{p-1}\phi_{l,s}^{p-1}(y_{l,s}-\tau)}{(\theta^p-\phi_{l,s}^p)^2} \quad (15)$$

$$\frac{\partial^2 lnL}{\partial\theta\partial p} = \sum_{j=1}^{m_1} \frac{\theta^{2p-1}-\theta^{p-1}y_{j,s}^p+p\theta^{p-1}y_{j,s}(\theta^{-1}y_{j,s})}{(\theta^p-y_{j,s}^p)^2}$$
$$\sum_{l=1}^{m_2} \frac{\theta^{2p-1}-\theta^{p-1}\phi_{l,s}^p+p\theta^{p-1}\phi_{l,s}(\theta^{-1}\phi_{l,s})}{(\theta^p-\phi_{l,s}^p)^2}$$
$$+\frac{(r_1+r_2)}{\theta} \quad (16)$$

$$\frac{\partial^2 lnL}{\partial\beta\partial p} = \sum_{i=1}^{m_2} \frac{(y_{i,s}-\tau)\phi_{i,s}^{p-1}(\theta^p-\phi_{i,s}^p)(pln\phi_{i,s}+1)}{(\theta^p-\phi_{i,s}^p)^2}$$
$$-\sum_{l=1}^{m_2}\left[\frac{p(\theta^p ln\theta-\phi_{l,s}^p ln\phi_{l,s})}{(\theta^p-\phi_{l,s}^p)^2}\right]+\sum_{k=1}^{r_2}\frac{y_{k,f}}{\phi_{k,f}} \quad (17)$$

4 Simulation Study

Simulation studies are performed to discuss the performance of the ML estimators in terms of their biases, root mean square errors (RMSEs) and coverage rate for different combinations of true values of θ, β and p. For the samples of multiply censored data, the simulation algorithm included the following

Sample size $n = 20, 50, 100$; parameter true values $\theta = 4, 5$, $\beta = 6, 8$ and $p = 10, 15$ censoring level $(CL) = 0.2, 0.4$.

We first generate a sequence of multiply censored data from Power Function distribution in partially accelerated life test using the inverse CDF method. Based on the simulated data, a series of calculations that refer to MLE and performance indicator (bais, RMSE, coverage rate) will be performed. The procedure is as follows:

1. Generate n random samples $(t_1, t_2, \cdots, t_n)^T$ from a Power Function distribution with the specified values of shape and scale parameter. The generation of Power Function distribution is very simple, if U is uniform distribution U(0, 1), then $t = \theta U^{1/p}$ is a Power Function distribution.

2. Let $t_1 = (t_{11}, t_{21}, \cdots, t_{n_1 1})$ be the number of samples that fail at normal operating conditions before the specified stress change time . Again, choose n_2 samples that fail after the specified time, $/tau$, as the stress condition samples which is $t_2 = (t_{12}, t_{22}, \cdots, t_{n_2 2})$.

3. Let the number of failures at normal and accelerated stress levels be $n_{1f} = n_1(1-CL)$ and $n_{2f} = n_2(1-CL)$, respectively, where CL denotes the censoring level. For example, $CL = 0.2$ denotes 20% censored data and hence 80% failure data.

4. Let

$$\delta_{i,1} = \begin{cases} 1 & i = 1, 2, \cdots, n_{1f} \\ u_{i,1} & i = n_{1f}+1, \cdots, n_1 \end{cases}$$

and

$$\delta_{j,2} = \begin{cases} 1 & j = 1, 2, \cdots, n_{2f} \\ u_{j,2} & i = n_{2f}+1, \cdots, n_2 \end{cases}$$

where $u_{i,1} and u_{j,2}$ are drawn from the uniform distribution $U(0, 1)$.

5. Set $x_{i,1} = \delta_{i,1} \times t_{i,1,i=1,2,\cdots,n_1}$ and $x_{j,2} = \delta_{j,2} \times \left[\left(\frac{t_{j,2}-\tau}{\beta}\right)+\tau\right]$, $j = 1, 2, \cdots, n_2$

6. Finally, bias, root mean square error and coverage rate of parameters are calculated from the obtained data. The coverage rates of the 95% confidence interval for the parameters are based on 1000 replications. Simulation programs are performed using R software. The frequency coverage rate for the limit is a binomial random variable with $\beta = 0.05$ and $N = 1000$. Therefore, the confidence interval for the coverage proportion is $0.95 \pm 1.96\sqrt{0.95 \times 0.05 \div 1000}$. Hence, the limit of coverage proportion is $0.9365 - 0.9635$. The observed results are presented in the Tables [1-4].

Table 1: The simulation results for the Power Function distribution under multiply censored data $(p = 10, \beta = 6, \theta = 4, \tau = 3\&CL = 0.2)$

Sample size	Parameter	Estimated value	Bias	RMSE	95% limits Lower	95% limits Upper	Coverage rate
20	P	10.1274	0.1102	0.4879	9.4623	11.1029	89.9%
	θ	3.9197	0.1019	0.1733	3.7422	4.1066	
	β	6.1002	0.0672	0.4109	5.7268	6.4481	
50	P	10.2089	0.2074	0.5699	9.4155	11.0956	96.3%
	θ	4.0285	0.1285	0.1877	3.5221	4.2335	
	β	5.9879	0.0118	0.1226	5.3528	6.5592	
100	P	10.0365	0.0832	0.3365	9.1002	11.3870	98.4%
	θ	4.0711	0.0876	0.2189	3.4560	4.1478	
	β	5.8663	0.0832	0.1351	5.3001	6.5033	

Table 2: The simulation results for the Power Function distribution under multiply censored data $(p = 10, \beta = 6, \theta = 4, \tau = 3\&CL = 0.4)$**.**

Sample size	Parameter	Estimated value	Bias	RMSE	95% limits Lower	95% limits Upper	Coverage rate
20	P	9.9871	0.4105	9.3877	9.3877	10.5788	93.9%
	θ	3.9215	0.1416	0.4717	3.7100	3.9879	
	β	5.8879	0.2642	0.4945	5.3845	5.8967	
50	P	1 9.9633	0.0043	0.3591	9.5433	10.0967	96.7%
	θ	4.0156	0.1023	0.6289	3.6122	4.1305	
	β	5.7125	0.3189	0.1945	5.3087	6.2562	
100	P	9.9378	0.0334	0.2744	9.1002	11.0811	98.1%
	θ	3.9344	0.0689	0.2688	3.4560	4.0123	
	β	5.6673	0.0944	0.2255	5.3001	6.4073	

Table 3: The simulation results for the Power Function distribution under multiply censored data $(p = 15, \beta = 8, \theta = 5, \tau = 4 \& CL = 0.2)$**.**

Sample size	Parameter	Estimated value	Bias	RMSE	95% limits		Coverage rate
					Lower	Upper	
20	P	15.3761	0.1115	1.1187	13.8977	16.4533	92.5%
	θ	5.1216	0.0916	0.2718	4.7880	5.4223	
	β	7.8571	0.1642	0.4965	7.3870	8.4499	
50	P	15.3124	0.0941	0.7591	13.7866	16.3477	91.3%
	θ	5.0156	0.1227	0.2259	4.5639	5.3987	
	β	7.7175	0.1180	0.2940	7.2017	8.5112	
100	P	15.1371	0.0924	0.5744	13.8547	16.1090	98.4%
	θ	5.0044	0.0644	0.1624	4.6988	5.2770	
	β	7.6323	0.0849	0.3034	7.2133	8.0173	

Table 4: The simulation results for the Power Function distribution under multiply censored data $(p = 15, \beta = 8, \theta = 5, \tau = 4 \& CL = 0.4)$**.**

Sample size	Parameter	Estimated value	Bias	RMSE	95% limits		Coverage rate
					Lower	Upper	
20	P	14.8746	0.2110	1.0187	13.5917	15.4563	89.5%
	θ	4.9978	0.1906	0.2718	0.3798	4.6788	
	β	7.7911	0.1655	0.3045	7.2843	8.0455	
50	P	14.9980	0.2100	0.1745	13.9860	16.2247	90.0%
	θ	4.9101	0.1221	0.2478	4.3679	5.7787	
	β	8.0229	0.1855	0.5908	7.1227	8.2811	
100	P	15.0188	0.0911	0.5045	14.8564	16.1109	89.7%
	θ	5.0910	0.0646	0.1655	4.5908	5.1773	
	β	7.9321	0.1167	0.3795	7.3193	7.6717	

5 Conclusion

The present paper presented the processes and simulated procedure for estimating failure time data under step-stress partially accelerated life tests based on multiply censored data. The lifetime distribution of the test units is assumed to follow Power Function distribution. It was observed that ML estimates could not be obtained in closed form and so the Newton-Raphson technique was used as an alternative method. The results showed that the MLE method performed well in most of the cases in terms of bias, RMSE and the coverage rate. Also, the coverage rate in most of the cases close to the nominal value for large sample sizes. Thus, the MLE method is a good approach to estimate the parameters of Power Function distribution and the acceleration factor in step-stress partially accelerated life tests under multiply censored data. As future work, the same can be considered under Bayesian environment.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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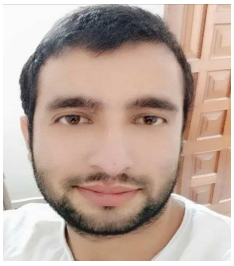

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