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New Method to Optimize the Fully Neutrosophic Linear Programming Problems

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Abstract: The present paper investigates the neutrosophic linear programming (LP) models in line with their advantages, where their parameters are represented with a trapezoidal neutrosophic number, and a method to solve them has been introduced. With some mathematical examples, the introduced method has been represented and demonstrated its predominance with the state of the art by comparison. Finally, we conclude that the proposed approach is easier, more effective, and suitable for solving the neutrosophic LP models compared to other approaches.

Keywords: Fuzzy sets, Linear programming, Neutrosophic set, Trapezoidal neutrosophic number, Triangle neutrosophic number.

1 Introduction

Smarandache [1] and [2] described a neutrosophic set that can absorb information that is indeterminate and inconsistent. Wang et al. [3] defined the Single Valued Neutrosophic Set (SVNS) as an example of a neutrosophic set that simply represents uncertain, imprecise, incomplete, indeterminate, and inconsistent information. However, the domain of SVNSs is a discrete set where the degrees of truth, indeterminacy and falsity memberships are represented only with fuzzy terms. The domain of SVNSs, however, is a discrete set where the degrees of truth, indeterminacy, and falsity memberships are represented only with fuzzy terms such as ”very good,” ”good,” ”evil,” . . . . We can establish the definition of the single-valued neutrosophic number (SVNN) whose domain is viewed as a consecutive sequence, taking the universe as a real line. Therefore, as a special case of single-valued neutrosophic sets, we can consider SVNNs. In decision-making problems, these numbers can express ill-known quantities with uncertain numerical values. The essence of SVNNs truth, indeterminacy, and falsity memberships functions which have distinct shapes such as triangular-shaped, trapezoidal-shaped, bell-shaped, etc. In the present paper, only the trapezoidal-shaped case is presented and triangular-shaped can be presented as a special case from trapezoidal-shaped, where its truth, indeterminacy, and falsity memberships functions can be represented as trapezoidal fuzzy numbers, incomplete . Recently, little focus has been provided to studies on SVNNs, and many descriptions of SVNNs and their operating laws have been proposed. Ye [4] analyzed some attribute decision-making problems by presenting a trapezoidal fuzzy neutrosophic set. He [4] also described score function, accuracy function, and certain operational rules for trapezoidal fuzzy neutrosophic sets in his research. The trapezoidal fuzzy neutrosophic number and membership functions were described by Biswas et al., [5]. To solve multi-attribute decision making problems, Biswas et al., [5] have suggested relative expected value and cosine similarity measure.

In decision-making problems concerning indeterminate information outside the range of fuzzy numbers, intuitionist fuzzy numbers, the ranking system of SVTrNNs may play an important role. The literature review showed that little consideration was given to the researchers about the SVTrNNs ranking method. Deli and Subas [6] have recently suggested a ranking system for generalized SVTrNNs and given a numerical example for solving the problem of multi-attribute decision-making in the neutrosophic environment. In the present paper, normalized SVTrNNs are established, an SVTrNNs

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ranking method is developed to solve the problem of multi-attribute decision-making in the neutrosophic environment, numerical examples are given and comparison with other studies is to show the importance of the present paper.

2 Preliminaries

In this section, some of the fundamental concepts in neutrosophic set theory are introduced:

Definition 1 [7] The trapezoidal neutrosophic number is defined as a set in R with the following truth, indeterminacy, and falsity memberships functions:

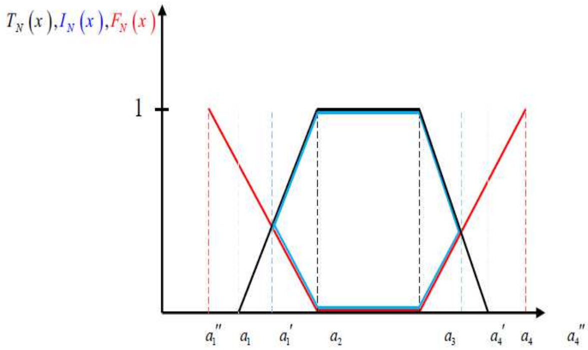

Fig. 1: Truth, indeterminacy and falsity memberships functions of trapezoidal neutrosophic numbers

$$T_A(x)=\begin{cases}\frac{\alpha_A(x-a_1)}{a_2-a_1} & : a_1\le x\le a_2\\ \alpha_A & : a_2\le x\le a_3\\ \alpha_A\left(\frac{a_4-x}{a_4-a_3}\right) & : a_3\le x\le a_4\\ 0 & otherwise\end{cases}$$

$$I_A(x)=\begin{cases}\frac{a_2-x+\theta_A(x-a_1')}{a_2-a_1'} & : a_1'\le x\le a_2\\ \theta_A & : a_2\le x\le a_3\\ \frac{x-a_3+\theta_A(a_4'-x)}{a_4'-a_3} & : a_3\le x\le a_4'\\ 1 & otherwise\end{cases}$$

$$F_A(x)=\begin{cases}\frac{a_2-x+\beta_A(x-a''_1)}{a_2-a''_1} & : a''_1\le x\le a_2\\ \beta_A & : a_2\le x\le a_3\\ \frac{x-a_3+\beta_A(a''_4-x)}{a''_4-a_3} & : a_3\le x\le a''_4\\ 1 & otherwise\end{cases}$$

Definition 2 [8] A single-valued trapezoidal neutrosophic number (SVTN-number), denoted by $\widetilde{A}=\langle(a,b,c,d),\alpha_A,\theta_A,\beta_A\rangle$ is a special neutrosophic set on the real number set R, whose truth, indeterminacy, and falsity memberships functions are given as follows:

$$T_A(x)=\begin{cases}\frac{\alpha_A(x-a)}{b-a} & : a\le x\le b\\ \alpha_A & : b\le x\le c\\ \alpha_A\left(\frac{d-x}{d-c}\right) & : c\le x\le d\\ 0 & otherwise\end{cases}\quad (1)$$

$$I_A(x)=\begin{cases}\frac{b-x+\theta_A(x-a)}{a_2-a_1'} & : a\le x\le b\\ \theta_A & : b\le x\le c\\ \frac{x-c+\theta_A(d-x)}{d-c} & : c\le x\le d\\ 1 & otherwise\end{cases}\quad (2)$$

$$F_A(x)=\begin{cases}\frac{b-x+\beta_A(x-a)}{b-a} & : a\le x\le b\\ \beta_A & : b\le x\le c\\ \frac{x-c+\beta_A(d-x)}{d-c} & : c\le x\le d\\ 1 & otherwise\end{cases}\quad (3)$$

Definition 3 [9] The algebraic operations on two trapezoidal neutrosophic numbers

$\widetilde{A}=\langle(a_1,b_1,c_1,d_1),\alpha_A,\theta_A,\beta_A\rangle$ and

$\widetilde{B}=\langle(a_2,b_2,c_2,d_2),\alpha_B,\theta_B,\beta_B\rangle$ are as follows:

$$\widetilde{A}+\widetilde{B}=\langle(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\rangle$$

$$\widetilde{A}-\widetilde{B}=\langle(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\rangle$$

$$\widetilde{A}^{-1}=\left\langle\left(\frac{1}{d_1},\frac{1}{c_1},\frac{1}{b_1},\frac{1}{a_1}\right),\alpha_A,\theta_A,\beta_A\right\rangle\quad where\ \widetilde{A}\neq 0$$

$$\gamma\widetilde{A}=\begin{cases}\langle(\gamma a_1,\gamma b_1,\gamma c_1,\gamma d_1),\alpha_A,\theta_A,\beta_A\rangle & if\ \gamma>0\\ \langle(\gamma d_1,\gamma c_1,\gamma b_1,\gamma a_1),\alpha_A,\theta_A,\beta_A\rangle & if\ \gamma<0\end{cases}$$

$$\frac{\widetilde{A}}{\widetilde{B}}=\begin{cases}\left\langle\left(\frac{a_1}{d_2},\frac{b_1}{c_2},\frac{c_1}{b_2},\frac{d_1}{a_2}\right),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\right\rangle & if\ d_1>0,d_2>0\\ \left\langle\left(\frac{d_1}{d_2},\frac{c_1}{c_2},\frac{b_1}{b_2},\frac{a_1}{a_2}\right),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\right\rangle & if\ d_1<0,d_2>0\\ \left\langle\left(\frac{d_1}{a_2},\frac{c_1}{b_2},\frac{b_1}{c_2},\frac{a_1}{d_2}\right),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\right\rangle & if\ d_1<0,d_2<0\end{cases}$$

$$\widetilde{A}\widetilde{B}=\begin{cases}\langle(a_1a_2,b_1b_2,c_1c_2,d_1d_2),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\rangle & if\ d_1>0,d_2>0\\ \langle(a_1d_2,b_1c_2,c_1b_2,d_1a_2),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\rangle & if\ d_1<0,d_2>0\\ \langle(d_1d_2,c_1c_2,b_1b_2,a_1a_2),\alpha_A\wedge\alpha_B,\theta_A\vee\theta_B,\beta_A\vee\beta_B\rangle & if\ d_1<0,d_2<0\end{cases}$$

Definition 4 [10] A ranking function of neutrosophic numbers is a function $R:N(R)\to R$, where $N(R)$ is a set of neutrosophic numbers specified on a set of real numbers, which transforms each neutrosophic number into the real line.

Let $\widetilde{A}=\langle(a_1,b_1,c_1,d_1),\alpha_A,\theta_A,\beta_A\rangle$ and $\widetilde{B}=\langle(a_2,b_2,c_2,d_2),\alpha_B,\theta_B,\beta_B\rangle$ are two trapezoidal neutrosophic numbers, then

1.If $R\left(\widetilde{A}\right)>R\left(\widetilde{B}\right)$ then $\widetilde{A}>\widetilde{B}$
2.If $R\left(\widetilde{A}\right)<R\left(\widetilde{B}\right)$ then $\widetilde{A}<\widetilde{B}$
3.If $R\left(\widetilde{A}\right)=R\left(\widetilde{B}\right)$ then $\widetilde{A}=\widetilde{B}$

3 Neutrosophic linear programming problem (NLP) [7]

In this section, different kinds of NLP concerns are presented.

1. The first kind of problem with NLP is the problem in which coefficients of the variables of the objective function are trapezoidal neutrosophic numbers, but all other parameters are represented by real numbers.

$$\begin{aligned}&Maximize/minimize\ \ \widetilde{Z}\approx\sum_{j=1}^{n}\widetilde{c}_jx_j\\&Subject\ to\\&\sum_{j=1}^{n}a_{ij}x_j\le,=,\ge b_i;\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,x_j\ge o.\end{aligned}\tag{4}$$

In this type of problem, $\widetilde{c}_j$is trapezoidal neutrosophic number.

2. The second kind of NLP problem is the problem in which coefficients of constraints variables and right-hand side are represented by trapezoidal neutrosophic numbers but objective function variables and coefficients are exemplified by real values.

$$\begin{aligned}&Maximize/minimize\ \ Z\approx\sum_{j=1}^{n}c_jx_j\\&Subject\ to\\&\sum_{j=1}^{n}\widetilde{a}_{ij}x_j\lesssim,\approx,\gtrsim\widetilde{b}_i;\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,x_j\ge o.\end{aligned}\tag{5}$$

where $\widetilde{a}_{ij}$ and $\widetilde{b}_i$are trapezoidal neutrosophic numbers.

3. In this type of NLP problem, the problems in all parameters are represented by trapezoidal neutrosophic numbers, except variables are considered by real values.

$$\begin{aligned}&Maximize/minimize\ \ \widetilde{Z}\approx\sum_{j=1}^{n}\widetilde{c}_jx_j\\&subject\ to\\&\sum_{j=1}^{n}\widetilde{a}_{ij}x_j\lesssim,\approx,\gtrsim\widetilde{b}_i;\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ x_j\ge o\end{aligned}\tag{6}$$

Here, $\widetilde{c}_j$; $\widetilde{a}_{ij}$ and $\widetilde{b}_i$ are trapezoidal neutrosophic numbers.

4. The NLP problem may be also a problem with neutrosophic values for variables, coefficients in goal function, and right-hand side of constraints.

$$\begin{aligned}&Maximize/minimize\ \ \widetilde{Z}\approx\sum_{j=1}^{n}\widetilde{c}_j\widetilde{x}_j\\&subject\ to\\&\sum_{j=1}^{n}a_{ij}\widetilde{x}_j\lesssim,\approx,\gtrsim\widetilde{b}_i;\ i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ x_j\ge o.\end{aligned}\tag{7}$$

Here, $\widetilde{c}_j$; $\widetilde{x}_j$ and $\widetilde{b}_i$ are trapezoidal neutrosophic numbers. Here, $\widetilde{x}_j$is defined as trapezoidal neutrosophic numbers, if authors choose results in the form of neutrosophic numbers. However, in reality, every manager or decision-maker needs to achieve the optimal solution to the problem by considering ambiguous, imprecise and inconsistent information when describing the problem. Therefore, this problem can be considered as another NLP formulation if we obtain the crisp value of decision variables (6). In the next section we will introduce our method.

4 Proposed NLP Method

Decision-makers with trapezoidal neutrosophic numbers insert their NLP problem. We always want to maximize / minimize the objective function with maximum truth degree, minimum indeterminacy, and falsity degree of information. Thus, we maximize / minimize x_j, with maximum truth, minimum indeterminacy, and falsity degrees of information, and then notify decision-makers when entering trapezoidal neutrosophic numbers of NLP model to apply this concept. We consider a linear ranking function in objective function on $\widetilde{A}=\langle(a,b,c,d),\alpha_A,\theta_A,\beta_A\rangle$ as: $R\left(\widetilde{A}\right)=c$ / $R\left(\widetilde{A}\right)=a$ if NLP problem is a maximization / minimization problem respectively, but take $R\left(\widetilde{A}\right)=d$in right-hand side of constrain and $R\left(\widetilde{A}\right)=a$, on left-hand side of the constrain, if the inequality is less incomplete part.

Proof:

We will prove that for maximum NLP, we will replace $\widetilde{A}$by $R\left(\widetilde{A}\right)=c$. To maximize x_j with maximum truth degree; minimum indeterminacy and falsity degree of information, firstly we will normalize $x_j$and the conformation degrees for the neutrosophic numbers to be both with equal weight of effective in objective, i.e. $\max x_j, \max T_A, \min I_A, \min F_A \Rightarrow \max z = \left(\frac{x_j-a}{d-a}+\frac{1}{3}(T_A-I_A-F_A)\right)$, which from definitions of truth, indeterminacy and falsity degrees, to explain, the maximum point is located in the border of the lines which describe the objective function that we want to maximize. Then, the intervals we will discuss will be $[a,b]$, $[b,c]$, $[c,d]$, i.e. at the points $a,b,c$and d.

At $x=a \Rightarrow z_1=-\frac{2}{3}$, at $x=b \Rightarrow z_2=\frac{b-a}{d-a}+\frac{1}{3}(\alpha_A-\beta_A-\theta_A)$, at $x=c \Rightarrow z_3=\frac{c-a}{d-a}+\frac{1}{3}(\alpha_A-\beta_A-\theta_A)$ and at $x=d \Rightarrow z_4=\frac{1}{3}$.

Form z_1, z_2, z_3 and z_4, it is clear that the maximum is at all z_3 (i.e. at $x=c$) but the minimum is also at z_1(i.e. at $x=a$).

5 Comparison Among the Proposed Model and Other Existing Models

In this section, we aim to show the applicability and efficiency of the new method by solving the problem presented by (Abdel-Basset et al., 2018 [7]; Ganesan & Veermani 2006 [11]; Ebrahimnejad & Tavana 2014 [12]; Elsayed Badr et al., 2019 [13]). The neutrosophic set takes the degrees of truth, indeterminacy and falsity into consideration, but only the degree of truth is considered by the fuzzy set. Decision makers often aim to increase the degree of truth and minimise the degree of indeterminacy and falsity. We considered this fact when we discussed our method.

Example 1: Consider the following fuzzy linear programming problem

$$
\begin{aligned}
&\max\ \widetilde{Z} \approx (13,15,2,2)x_1+(12,14,3,3)x_2+(15,17,2,2)x_3\\
&S.t.\ \ 12x_1+13x_2+12x_3 \le (475,505,6,6)\\
&\qquad 14x_1+13x_3 \le (460,480,8,8)\\
&\qquad 12x_1+15x_2 \le (465,495,5,5)\\
&\qquad x_1,x_2,x_3 \ge 0
\end{aligned}
\tag{8}
$$

Here, problem (8) transforms from the fuzzy representation into the neutrosophic representation as follows:

$$
\begin{aligned}
&\max\ \widetilde{Z} \approx (11,13,15,17)x_1+(9,12,14,17)x_2\\
&\qquad\qquad +(13,15,17,19)x_3\\
&S.t.\ \ 12x_1+13x_2+12x_3 \le (469,475,505,511)\\
&\qquad 14x_1+13x_3 \le (452,460,480,488)\\
&\qquad 12x_1+15x_2 \le (460,465,495,500)\\
&\qquad x_1,x_2,x_3 \ge 0
\end{aligned}
$$

Now, we can use the ranking function to convert each neutrosophic trapezoidal number to its equivalent crisp number with $(T_a, I_a, F_a)=(1,0,0)$. Then, the crisp model of the aforementioned problem will be, as follows:

$$
\begin{aligned}
&\max\ \ \widetilde{Z}=15x_1+14x_2+17x_3\\
&S.t.\ \ 12x_1+13x_2+12x_3 \le 511\\
&\qquad 14x_1+13x_3 \le 488\\
&\qquad 12x_1+15x_2 \le 500\\
&\qquad x_1,x_2,x_3 \ge 0
\end{aligned}
$$

Since the conditions of the simplex method are available, any solver for linear programming, such as maple (v12), will solve the crisp model. The following table shows the comparison between the proposed approach and other methods (Abdel-Basset et al, 2018 [7]; Ganesan & Veermani, 2006 [11]; Ebrahimnejad & Tavana 2014 [12], Elsayed Badr et al., 2019 [13]).

Table 1: A comparison between the proposed approach (Abdel-Basset et al., 2018; Ganesan & Veermani, 2006; Ebrahimnejad & Tavana 2014; Elsayed Badr et al., 2019)

Example 1					
	Fuzzy Methods		Neutrosophic Method		
	Ganesan & Veermani [11]	*Ebrahimnejad & Tavana* [12]	*Abdel-Basset* [7]	*Elsayed Badr et al.* [13]	*The proposed method*
Z	634.6	634.6	868 (out of the range from [11])	2622.1 (out of the range from [11])	118866/ 169
x_1	0	0	0	0	0
x_2	730/169	730/169	695/169	1729/200 (out of the range from [11])	805/169
x_3	470/169	470/169	487/13	36193/ 500 (out of the range from [11])	480/13

Table 1 show that the proposed method is better than the other methods. The objective function of the proposed method is 703.35, but that of the fuzzy method was 634. Also Table 1 shows that the results which in [7] was 868 and in [13] was 2622.1. These results are out of the range (from [11]). The maximum value of the objective function (from [11]) must be less than or equal 140084/169.

Example 2: Consider the following NLP:

$$
\begin{aligned}
&\max\ \ \widetilde{Z} \approx 2x_1-x_2+x_3\\
&S.t.\ \ 2x_1+x_2-2x_3 \le (17,35,53,55)\\
&\qquad 4x_1-x_2+2x_3 = (7,9,11,13)\\
&\qquad 2x_1+3x_2-x_3 \ge (7,17,27,29)\\
&\qquad x_1,x_2,x_3 \ge 0
\end{aligned}
\tag{9}
$$

Again, we can use the ranking function to convert each neutrosophic trapezoidal number to its equivalent crisp number that we introduced above. We get the crisp model of the above-mentioned problem, as follows:

$$\begin{aligned} &\max \quad \widetilde{Z} \approx 2x_1 - x_2 + x_3 \\ &S.t. \quad 2x_1 + x_2 - 2x_3 \leq 55 \\ &\qquad 4x_1 - x_2 + 2x_3 = 13 \\ &\qquad 2x_1 + 3x_2 - x_3 \geq 7 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Also, we can solve the crisp model by maple (v12). The following table shows the comparison between the proposed approach and fuzzy approach, (Abdel-Basset et al., 2018 [11]; Elsayed Badr et al., 2019 [2]).

Table 2: A comparison between the proposed approach and fuzzy approach (Abdel-Basset et al., 2018; Elsayed Badr et al., 2019)

Example 2				
	Fuzzy Methods	Neutrosophic Method		
	Fuzzy approach	*Abdel-Basset* [7]	*Elsayed Badr et al.* [13]	*The proposed method*
Z	5.7143	3.5714	6.1429	6.4286
x_1	7.1429	5.7143	7.4286	3.2857
x_2	8.5714	7.8571	8.7143	0.1428
x_3	0	0	0	0

6 Conclusion

This paper suggested new neutrosophic linear programming models and proposed a ranking function for both maximization neutrosophic linear programs and minimization neutrosophic linear programs. On the other hand, where there is a comparison between the fuzzy approach and the neutrosophic approach that resolves the same example, this paper used the same ranking function such that this comparison is fair. Finally, to obtain a neutrosophic basic possible optimal solution to a somewhat modified set of constraints, the suggested technique proposed a neutrosophic approach involving neutrosophic artificial variables. The neutrosophic simplex technique was then used to remove the artificial neutrosophic variables and to resolve the original problem. We plan to extend this approach to fully neutrosophic fractional programming problems in the next work.

Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] F. Smarandache. *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, (1998).

[2] F. Smarandache, Neutrosophic set a generalization of the intuitionistic fuzzy set, *International Journal of Pure and Applied Mathematics*, 24: 287-297, (2005).

[3] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, Single valued neutrosophic sets, *Multispace and Multistructure*, 4:410-413, (2010).

[4] J. Ye, Trapezoidal neutrosophic set and its application to multiple attribute decision-making, *Neural Computing and, Applications*, (2014), doi:10.1007/s00521-014-1787-6.

[5] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-attribute decision making with trapezoidal fuzzy neutrosophic numbers, *Neutrosophic Sets and System*, 8:47-57, (2015).

[6] I. Deli and Y. Subas, A ranking method of single-valued neutrosophic numbers and its applications to multi-attribute decision-making problems, *International Journal of Machine Learning & Cybernetics*, (2016), doi: 10.1007/s13042-016-0505-3.

[7] M. Abdel-Basset, Gunasekaran, M. Mai Mohamed & F. Smarandache *A novel method for solving the fully neutrosophic linear programming problems*, Neural Computing and Applications, Springer, (2018), https://doi.org/10.1007/s00521-018-3404-6.

[8] F. Liu, W. Zhang, & L. Zhang. Consistency analysis of triangular fuzzy reciprocal preference relations. *European Journal of Operational Research*, 235(3), 718-726, (2014).

[9] M. Abdel-Baset, M. Mai, A. Hessian, F. Smarandache, *Neutrosophic integer programming problem*, Neutrosophic Sets Syst, vol. 15, pp. 3-7, (2017).

[10] P. Biswas, S. Pramanik, Bibhas C. Giri, *Value and ambiguity index-based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making*, Neutrosophic Sets and Systems, vol. 12, pp. 127-138, (2016).

[11] K. Ganesan & P. Veeramani. Fuzzy linear programs with trapezoidal fuzzy numbers, *Annals of Opertions Research*, 143:305-315, (2006).

[12] A. Ebrahimnejad & M. Tavana. A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers, *Applied Mathematical Modeling*, 38:4388-4395, (2014).

[13] Elsayed Badr, M. Abdul-Salam and F. Smarandache, "Solving Neutrosophic Linear Programming Problems with Two-Phase Approach," in *Neutrosophic Sets in Decision Analysis and Operations Research*, New Mexico, USA, University of New Mexico, USA, pp. 22-38, (2019).

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