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Linear Fractional Programming Based on Trapezoidal Neutrosophic Numbers

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Abstract: In this paper, we introduce a new solution technique for resolving the neutrosophic linear fractional programming problem, where the objective function's coefficients are neutrosophic trapezoidal-numbers. We convert the problem into a crisp multi-objective linear fractional programming. The transformed multi-objective linear fractional programming problem is reduced to a single-objective linear programming problem (LP) by using the proposed approach, which can be easily solved by suitable algorithm. A numerical example is given to verify the proposed method.

Keywords: Multi Objective Programming, Trapezoidal Neutrosophic Number, Linear Fractional programming, Neutrosophic Set.

1 Introduction

Linear Fractional Programming (LFP) is a generalization of LP, whereas in a linear-fractional programming the objective function is a ratio of two linear functions. The highest profit/cost ratio, inventory/sale, real costs/standard costs, performance/employee, etc., is used in linear fractional programming. Decision makers may be unable, due to incomplete and imprecise information tending to be provided in real-life situations to indicate the coefficients (some or all), of the LFP problem. Often, objective function aspiration level and parameter of the problems are uncertain decision makers. The neutrosophical environment can effectively model these situations. Neutrosophy is an exploration of dialectic neutrality as an extension.
Neutrosophic is the neutrosophy derivative which contains neutrosophic set, neutrosophic probability, and neutrosophic logic. Neutrosophical theory means neutrosophy used in many areas of science to solve indeterminacy-related issues. While intuitionistic fuzzy sets can process incomplete information not indeterminate, information can be treated by the neutrosophical set both incomplete and indeterminate. Smarandache, Florentin. [1,2] introduce Neutrosophic sets characterized by three separate degrees namely truth (T), indeterminacy (I), and falsity memberships degrees (F), in which the T,I,F are standard or non-standard subsets of $]^{-}0,1^{+}[$. The neutrosophical decision makers want to maximize the degree of truthiness and to minimize the degree of indeterminacy and falsity membership. The article has the following structure: in the next section we discuss a preliminary; in the third section presents neutrosophic linear fractional programming problem with solution procedure; the fourth section provides a numerical example to put on view how the approach can be applied; finally, the fifth section provides the conclusion. Also A. M. ElHadidi, O. E. Emam, A. M. Abdelfadel [3] use ranking to optimize the fully neutrosophic.

2 Preliminaries

In this section, some of the fundamental concepts in neutrosophic set theory and Neutrosophic linear fractional programming (NLFP) problem are introduced:

Definition 1 [4] The neutrosophical trapezoidal number is defined as a set in R with the following truth, indeterminacy and falsity memberships functions:

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$$T_A(x) = \begin{cases} \frac{\alpha_A(x-a_1)}{a_2-a_1} & : a_1 \le x \le a_2 \\ \alpha_A & : a_2 \le x \le a_3 \\ \alpha_A\left(\frac{a_4-x}{a_4-a_3}\right) & : a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

$$I_A(x) = \begin{cases} \frac{a_2-x+\theta_A(x-a'_1)}{a_2-a'_1} & : a'_1 \le x \le a_2 \\ \theta_A & : a_2 \le x \le a_3 \\ \frac{x-a_3+\theta_A(a'_4-x)}{a'_4-a_3} & : a_3 \le x \le a'_4 \\ 1 & otherwise \end{cases}$$

$$F_A(x) = \begin{cases} \frac{a_2-x+\beta_A(x-a"_1)}{a_2-a"_1} & : a"_1 \le x \le a_2 \\ \beta_A & : a_2 \le x \le a_3 \\ \frac{x-a_3+\beta_A(a"_4-x)}{a"_4-a_3} & : a_3 \le x \le a"_4 \\ 1 & otherwise \end{cases}$$

where $\alpha_A, \theta_A, \beta_A \in [0,1]$ are represent the maximum degree of truthiness, minimum degree of indeterminacy and minimum degree of falsity, respectively. The membership functions of trapezoidal neutrosophic number are presented in Fig. 1. It is clear that $a"_1 < a_1 < a'_1 < a_2 < a_3 < a'_4 < a_4 < a"_4$.

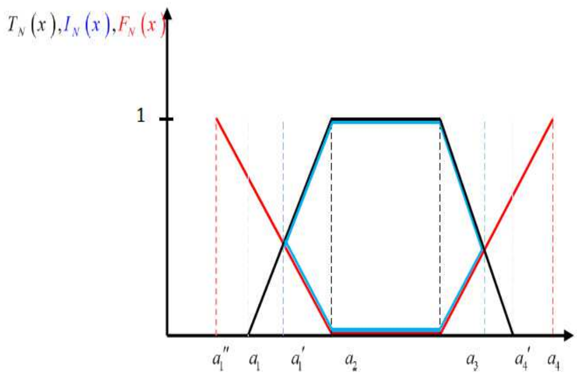

Fig. 1: Truth, indeterminacy and falsity memberships functions of trapezoidal neutrosophic numbers

Definition 2 [5] A single valued trapezoidal neutrosophic number (SVTN-number), denoted by $A^\sim = \langle (a,b,c,d), \alpha_A, \theta_A, \beta_A \rangle$ is a special neutrosophic set on the real number set R, whose truth, indeterminacy, and falsity memberships functions are given as follows:

$$T_A(x) = \begin{cases} \frac{\alpha_A(x-a)}{b-a} & : a \le x \le b \\ \alpha_A & : b \le x \le c \\ \alpha_A\left(\frac{d-x}{d-c}\right) & : c \le x \le d \\ 0 & otherwise \end{cases} \tag{1}$$

$$I_A(x) = \begin{cases} \frac{b-x+\theta_A(x-a)}{b-a} & : a \le x \le b \\ \theta_A & : b \le x \le c \\ \frac{x-c+\theta_A(d-x)}{d-c} & : c \le x \le d \\ 1 & otherwise \end{cases} \tag{2}$$

$$F_A(x) = \begin{cases} \frac{b-x+\beta_A(x-a)}{b-a} & : a \le x \le b \\ \beta_A & : b \le x \le c \\ \frac{x-c+\beta_A(d-x)}{d-c} & : c \le x \le d \\ 1 & otherwise \end{cases} \tag{3}$$

Definition 3 [6] The algebraic operations on two trapezoidal neutrosophic numbers
$A^\sim = \langle (a_1,b_1,c_1,d_1), \alpha_A, \theta_A, \beta_A \rangle$ and
$B^\sim = \langle (a_2,b_2,c_2,d_2), \alpha_B, \theta_B, \beta_B \rangle$ are as follows:

$$A^\sim + B^\sim = \langle (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \rangle$$

$$A^\sim - B^\sim = \langle (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \rangle$$

$$(A^\sim)^{-1} = \left\langle \left(\frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right), \alpha_A, \theta_A, \beta_A \right\rangle \quad where\ A^\sim \ne 0$$

$$\gamma A^\sim = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1), \alpha_A, \theta_A, \beta_A \rangle & if\ \gamma > 0 \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1), \alpha_A, \theta_A, \beta_A \rangle & if\ \gamma < 0 \end{cases}$$

$$\frac{A^\sim}{B^\sim} = \begin{cases} \left\langle \left(\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}\right), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \right\rangle \\ \qquad if\ d_1 > 0, d_2 > 0 \\ \left\langle \left(\frac{d_1}{d_2}, \frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \right\rangle \\ \qquad if\ d_1 < 0, d_2 > 0 \\ \left\langle \left(\frac{d_1}{a_2}, \frac{c_1}{b_2}, \frac{b_1}{c_2}, \frac{a_1}{d_2}\right), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \right\rangle \\ \qquad if\ d_1 < 0, d_2 < 0 \end{cases}$$

$$A^\sim B^\sim = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \rangle \\ \qquad if\ d_1 > 0, d_2 > 0 \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \rangle \\ \qquad if\ d_1 < 0, d_2 > 0 \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2), \alpha_A \wedge \alpha_B, \theta_A \vee \theta_B, \beta_A \vee \beta_B \rangle \\ \qquad if\ d_1 < 0, d_2 < 0 \end{cases}$$

Definition 4 [6] The Neutrosophic linear fractional programming (NLFP) problem can be written as:

$$\begin{aligned} &Maximize\ z^{\sim} \approx \frac{\sum_{j=1}^{n} c_j^{\sim} x_j + p^{\sim}}{\sum_{j=1}^{n} d_j^{\sim} x_j + q^{\sim}} = \frac{N(x)}{D(x)} \\ &Subject\ to \\ &\sum_{j=1}^{n} a_{ij}^{\sim} x_j \le b_i^{\sim};\ i = 1,2,\dots,m,\ x_j \ge o. \end{aligned} \tag{4}$$

In this type of problem, $c_j^{\sim}, p^{\sim}, d_j^{\sim}, q^{\sim}, a_{ij}^{\sim}$ and $b_i^{\sim}$ are trapezoidal neutrosophic numbers. For convenience here, we consider $D(x) > 0$. In reality, every manager or decision-maker needs to achieve the best solution to the problem by considering ambiguous, imprecise and inconsistent information when describing the problem.

2.1 Proposed MOLFP Method

In this section, the general form of MOLFP problem is discussed and the procedure for converting MOLFP problem into MOLP problem is illustrated.

The MOLFP problem can be written as follows:

$$\begin{aligned} &Max\ z(x) = [z_1(x), z_2(x), \dots, z_k(x)] \\ &Subject\ to \\ &x \in \Omega = \{x : Ax \le b, x \ge 0\} \end{aligned} \tag{5}$$

With $b \in R^m, A \in R^{m\times n}$, and $z_i = \frac{c_i x + p_i}{d_i x + q_i} = \frac{N_i(x)}{D_i(x)}, c_i, d_i \in R^n$ and $p_i, q_i \in R, i = 1,2,\dots,k$.

Let I be the index set such that $I = \{i : N_i(x) \ge 0 \text{ for } x \in \Omega\}$ and $I^c = \{i : N_i(x) < 0 \text{ for } x \in \Omega\}$, where $I \bigcup I^c = \{1,2,\dots,k\}$, $D_i(x)$ is positive on Ω, which is non-empty and bounded. For simplicity, let $t = \frac{1}{d_i x + q_i}$ for $i \in I$ and $t = \frac{1}{-(c_i x + p_i)}$ for $i \in I^c$.

To illustrate the method, we propose a procedure for solving neutrosophic linear fractional programming problem where the cost of the objective function, the resources, and the technological coefficients are trapezoidal neutrosophic numbers.

Let us consider the NLFP problem:

$$\begin{aligned} &Max\ z(x^{\sim n}) = \frac{\sum c_j^{\sim n} x_j + p^{\sim n}}{\sum d_j^{\sim n} x_j + q^{\sim n}} \\ &Subject\ to \\ &\sum a_{ij}^{\sim n} x_j \le b_i^{\sim n}, i = 1,2,\dots,m, \\ &\qquad x_j \ge 0,\ j = 1,2,\dots,n. \end{aligned} \tag{6}$$

We assume that $c_j^{\sim n}, p^{\sim n}, d_j^{\sim n}, q^{\sim n}, a_{ij}^{\sim n}$ and $b_i^{\sim n}$ are trapezoidal neutrosophic numbers for each $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$ therefore, the problem (6) can be written as:

$$Max\ z(x^{\sim n}) = \frac{\sum(c_{j1}, c_{j2}, c_{j3}, c_{j4}; \alpha_c, \theta_c, \beta_c)x_j + (p_1, p_2, p_3, p_4; \alpha_p, \theta_p, \beta_p)}{\sum(d_{j1}, d_{j2}, d_{j3}, d_{j4}; \alpha_d, \theta_d, \beta_d)x_j + (q_1, q_2, q_3, q_4; \alpha_q, \theta_q, \beta_q)} \tag{7}$$

Subject to

$\sum(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}; \alpha_a, \theta_a, \beta_a)x_j \le (b_{j1}, b_{j2}, b_{j3}, b_{j4}; \alpha_b, \theta_b, \beta_b), i = 1,2,\dots,m,$

$x_j \le 0, j = 1,2,\dots,n.$

Where $\alpha, \theta, \beta \in [0,1]$ and stand for truth-membership, indeterminacy and falsity-membership function of each neutrosophic number.

In this situation the decision maker desires to improve the degree of truthfulness and to reduce the degree of indeterminacy and falsity membership.

Using the concept of component wise optimization, the problem (7) reduces to an equivalent MOLFP as follows:

$Maxz_1 = \frac{\sum c_{j1} x_j + p_1}{\sum d_{j4} x_j + q_4}$

$Maxz_2 = \frac{\sum c_{j2} x_j + p_2}{\sum d_{j3} x_j + q_3}$

$Maxz_3 = \frac{\sum c_{j3} x_j + p_3}{\sum d_{j2} x_j + q_2}$

$Maxz_4 = \frac{\sum c_{j4} x_j + p_4}{\sum d_{j1} x_j + q_1}$

$Maxz_5 = \frac{\sum \alpha_c x_j + \alpha_p}{\sum \beta_c x_j + \beta_q}$

$Maxz_6 = 1 - \frac{\sum \theta_c x_j + \theta_p}{\sum \theta_c x_j + \theta_q}$

$Maxz_7 = 1 - \frac{\sum \beta_c x_j + \beta_p}{\sum \alpha_c x_j + \alpha_q}$

Subject to

$\sum a_{ij1} x_j \le b_{j1},$

$\sum a_{ij2} x_j \le b_{j2},$

$\sum a_{ij3} x_j \le b_{j3},$

$\sum a_{ij4} x_j \le b_{j4},$

$\sum \alpha_a x_j \le \alpha_b,$

$\sum \theta_a x_j \le \theta_b,$

$\sum \beta_a x_j \le \beta_b,$

$x_j \ge 0, i = 1,2,\dots,m j = 1,2,\dots,n.$

Using Charnes and Cooper's linear transformation $y = tx$ [7], where $t = \frac{1}{D(x)}, D(x) > 0$, the previous MOLFP problem is equivalent to the following MOLP problem:

$Maxz_1 = \sum c_{j1} y_j + P_1 t$

$Maxz_2 = \sum c_{j2} y_j + P_2 t$

$Maxz_3 = \sum c_{j3} y_j + P_3 t$

$Maxz_4 = \sum c_{j4} y_j + P_4 t$

$Maxz_5 = \sum \alpha_c y_j + \alpha_p t$

$Maxz_6 = 1 - \sum \theta_c y_j + \theta_p t$

$Maxz_7 = 1 - \sum \beta_c y_j + \beta_p t$

Subject to

$\sum d_{j4} y_j + q_4 t \le 1$

$\sum d_{j3} y_j + q_3 t \le 1$

$\sum d_{j2} y_j + q_2 t \le 1$

$\sum d_{j1} y_j + q_1 t \le 1$

$\sum \beta_c y_j + \beta_q t \le 1$

$\sum \theta_c y_j + \theta_q t \le 1$

$\sum \alpha_c y_j + \alpha_q t \leq 1$
$\sum a_{ij1} y_j + b_{j1} t \leq 0,$
$\sum a_{ij2} y_j + b_{j2} t \leq 0,$
$\sum a_{ij3} y_j + b_{j3} t \leq 0,$
$\sum a_{ij4} y_j + b_{j4} t \leq 0,$
$\sum \alpha_a y_j + \alpha_b t \leq 0,$
$\sum \theta_a y_j + \theta_b t \leq 0,$
$\sum \beta_a y_j + \beta_b t \leq 0,$
$t, y_j \geq 0, i = 1,2,\ldots,m, j = 1,2,\ldots,n.$

Solving the transformed MOLP problem for each objective function, we obtain $z_1^*, z_2^*, z_3^*, z_4^*, z_5^*, z_6^*$ and z_7^*. Using Zimmermann's min operator [8], the above model transformed to the following crisp model as:

$Max\ \lambda,$

Subject to

$\sum c_{j1} y_j + p_1 t - z_1^* \lambda \geq 0$
$\sum c_{j2} y_j + p_2 t - z_2^* \lambda \geq 0$
$\sum c_{j3} y_j + p_3 t - z_z^* \lambda \geq 0$
$\sum c_{j4} y_j + p_4 t - z_4^* \lambda \geq 0$
$\sum \alpha_c y_j + \alpha_p t - z_5^* \lambda \leq 0$
$1 - (\sum \theta_c y_j + \theta_p t) - z_6^* \lambda \leq 0$
$1 - (\sum \beta_c y_j + \beta_p t) - z_7^* \lambda \leq 0$
$\sum d_{j4} y_j + q_4 t \leq 1$
$\sum d_{j3} y_j + q_3 t \leq 1$
$\sum d_{j2} y_j + q_2 t \leq 1$
$\sum d_{j1} y_j + q_1 t \leq 1$
$\sum \beta_c y_j + \beta_q t \leq 1$
$\sum \theta_c y_j + \theta_q t \leq 1$
$\sum \alpha_c y_j + \alpha_q t \leq 1$
$\sum a_{ij1} y_j - b_{j1} t \leq 0,$
$\sum a_{ij2} y_j - b_{j2} t \leq 0,$
$\sum a_{ij3} y_j - b_{j3} t \leq 0,$
$\sum a_{ij4} y_j - b_{j4} t \leq 0,$
$\sum \alpha_a y_j - \alpha_b t \leq 0,$
$\sum \theta_a y_j - \theta_b t \leq 0,$
$\sum \beta_a y_j - \beta_b t \leq 0,$
$t, y_j \geq 0, i = 1,2,\ldots,m, j = 1,2,\ldots,n.$

2.2 Algorithm

The proposed approach for solving NLFP problem can be summarized as follows:

Step 1. The NLFP problem is transformed into MOLFP problem using component wise optimization.

Step 2. The MOLFP problem is converted into MOLP problem using Charnes and Cooper method.

Step 3. Solve each objective function subject to the given set of constraints.

Step 4. Use Zimmermann's operator to obtain crisp model, then solve this crisp model.

3 Numerical Example

In this section, we try to prove the applicability of the proposed method, we solved the same problem which introduced by (M. Mohamed, M. Abdel-Baset, F. Smarandache) [9].

Let x_1, x_2 and x_3 units be the amount of I, II and III, respectively to be produced. After prediction of estimated parameters, the above problem can be formulated as the following NLFPP:

$$Maxz(x^{\sim n}) = \frac{8^{\sim n}x_1 + 7^{\sim n}x_2 + 9^{\sim n}x_3}{8^{\sim n}x_1 + 9^{\sim n}x_2 + 6^{\sim n}x_3 + 1.5^{\sim n}}$$

Subject to

$$\begin{aligned} &4^{\sim n}x_1 + 3^{\sim n}x_2 + 5^{\sim n}x_3 \leq 28^{\sim n}, \\ &5^{\sim n}x_1 + 3^{\sim n}x_2 + 3^{\sim n}x_3 \leq 20^{\sim n}, \\ &x_1, x_2, x_3 \geq 0 \end{aligned} \tag{8}$$

with

$8^{\sim n} = (7,8,9,10;0.5,0.8,0.3)$
$7^{\sim n} = (6,7,8,9;0.2,0.6,0.5)$
$9^{\sim n} = (8,9,10,11;0.8,0.1,0.4)$
$6^{\sim n} = (4,5,6,7;0.75,0.25,0.1)$
$1.5^{\sim n} = (0.5,1,1.5,2;0.75,0.5,0.25)$
$4^{\sim n} = (3,4,5,6;0.4,0.6,0.5)$
$3^{\sim n} = (2,3,4,5;1,0.25,0.3)$
$5^{\sim n} = (4,5,6,7;0.3,0.4,0.8)$
$28^{\sim n} = (24,26,28,30;04,0.25,0.5)$
$20^{\sim n} = (18,19,20,21;0.9,0.2,0.6)$

This problem is equivalent to the following MOLFPP:

$\max z_1 = \frac{7x_1+6x_2+8x_3}{10x_1+11x_2+7x_3+2}$
$\max z_2 = \frac{8x_1+7x_2+9x_3}{9x_1+10x_2+6x_3+1.5}$
$\max z_3 = \frac{9x_1+8x_2+10x_3}{8x_1+9x_2+5x_3+1}$
$\max z_4 = \frac{10x_1+9x_2+11x_3}{7x_1+8x_2+4x_3+1.5}$
$\max z_5 = \frac{0.5x_1+0.2x_2+0.8x_3}{0.3x_1+0.4x_2+0.1x_3+0.25}$
$\max z_6 = \frac{0.8x_1+0.6x_2+0.1x_3}{0.8x_1+0.1x_2+0.25x_3+0.5}$
$\max z_7 = \frac{0.3x_1+0.5x_2+0.4x_3}{0.5x_1+0.8x_2+0.75x_3+0.75}$

Subject to

$3x_1 + 2x_2 + 4x_3 \leq 24,$
$4x_1 + 3x_2 + 5x_3 \leq 26,$
$5x_1 + 4x_2 + 6x_3 \leq 28,$
$6x_1 + 5x_2 + 7x_3 \leq 30,$
$4x_1 + 2x_2 + 2x_3 \leq 18,$
$5x_1 + 3x_2 + 3x_3 \leq 19,$
$6x_1 + 4x_2 + 4x_3 \leq 20,$
$7x_1 + 5x_2 + 5x_3 \leq 21,$

$0.4x_1 + x_2 + 0.3x_3 \le 0.4,$
$0.6x_1 + 0.25x_2 + 0.4x_3 \le 0.25,$
$0.5x_1 + 0.3x_2 + 0.8x_3 \le 0.5,$
$0.3x_1 + x_2 + x_3 \le 0.9,$
$0.4x_1 + 0.25x_2 + 0.25x_3 \le 0.2,$
$0.8x_1 + 0.3x_2 + 0.3x_3 \le 0.6,$
$x_1, x_2, x_3 \ge 0$

Using the transformation, the problem is equivalent to the following MOLPP:

$Max\ z_1 = 7y_1 + 6y_2 + 8y_3$
$Max\ z_2 = 8y_1 + 7y_2 + 9y_3$
$Max\ z_3 = 9y_1 + 8y_2 + 10y_3$
$Max\ z_4 = 10y_1 + 9y_2 + 11y_3$
$Max\ z_5 = 0.5y_1 + 0.2y_2 + 0.8y_3$
$Max\ z_6 = 0.5y_2 + 0.15y_3 + 0.5$
$Max\ z_7 = 0.2y_1 + 0.3y_2 + 0.35y_3 + 0.75$

Subject to

$10y_1 + 11y_2 + 7y_3 + 2t \le 1,$
$9y_1 + 10y_2 + 6y_3 + 1.5t \le 1,$
$8y_1 + 9y_2 + 5y_3 + t \le 1,$
$7y_1 + 8y_2 + 4y_3 + 0.5t \le 1,$
$0.3y_1 + 0.4y_2 + 0.1y_3 + 0.25t \le 1,$
$0.8y_1 + 0.1y_2 + 0.25y_3 + 0.5t \le 1,$
$0.5y_1 + 0.8y_2 + 0.75y_3 + 0.75t \le 1,$
$3y_1 + 2y_2 + 4y_3 - 24t \le 0,$
$4y_1 + 3y_2 + 5y_3 - 26t \le 0,$
$5y_1 + 4y_2 + 6y_3 - 28t \le 0,$
$6y_1 + 5y_2 + 7y_3 - 30t \le 0,$
$4y_1 + 2y_2 + 2y_3 - 18t \le 0,$
$5y_1 + 3y_2 + 3y_3 - 19t \le 0,$
$6y_1 + 4y_2 + 4y_3 - 20t \le 0,$
$7y_1 + 5y_2 + 5y_3 - 21t \le 0,$
$0.4y_1 + y_2 + 0.3y_3 - 0.4t \le 0,$
$0.6y_1 + 0.25y_2 + 0.4y_3 - 0.25t \le 0,$
$0.5y_1 + 0.3y_2 + 0.8y_3 - 0.5t \le 0,$
$0.3y_1 + y_2 + y_3 - 0.9t \le 0,$
$0.4y_1 + 0.25y_2 + 0.25y_3 - 0.2t \le 0,$
$0.8y_1 + 0.3y_2 + 0.3y_3 - 0.6t \le 0,$
$t, y_1, y_2, y_3 \ge 0$

Solving each objective at a time we get

$z1 = 0.7843, z_2 = 0.8824, z_3 = 0.9804, z_4 = 1.0784, z_5 = 0.0784, z_6 = 0.5147, z_7 = 0.7843$

Now the previous problem can be reduced to the following LPP:

$Max\ \lambda,$

Subject to

$7y_1 + 6y_2 + 8y_3 - 0.7843\lambda \ge 0$
$8y_1 + 7y_2 + 9y_3 - 0.8824\lambda \ge 0$
$9y_1 + 8y_2 + 10y_3 - 0.9804\lambda \ge 0$
$10y_1 + 9y_2 + 11y_3 - 1.0784\lambda \ge 0$
$0.5y_1 + 0.2y_2 + 0.8y_3 - 0.0784\lambda \ge 0$
$-0.5y_2 + 0.15y_3 + 0.5 - 0.5147\lambda \ge 0$
$0.2y_1 + 0.3y_2 + 0.35y_3 + 0.75 - 0.7843\lambda \ge 0$
$10y_1 + 11y_2 + 7y_3 + 2t \le 1,$
$9y_1 + 10y_2 + 6y_3 + 1.5t \le 1,$
$8y_1 + 9y_2 + 5y_3 + t \le 1,$
$7y_1 + 8y_2 + 4y_3 + 0.5t \le 1,$
$0.3y_1 + 0.4y_2 + 0.1y_3 + 0.25t \le 1,$
$0.8y_1 + 0.1y_2 + 0.25y_3 + 0.75t \le 1,$
$3y_1 + 2y_2 + 4y_3 - 24t \le 0,$
$4y_1 + 3y_2 + 5y_3 - 26t \le 0,$
$5y_1 + 4y_2 + 6y_3 - 28t \le 0,$
$6y_1 + 5y_2 + 7y_3 - 30t \le 0,$
$4y_1 + 2y_2 + 2y_3 - 18t \le 0,$
$5y_1 + 3y_2 + 3y_3 - 19t \le 0,$
$6y_1 + 4y_2 + 4y_3 - 20t \le 0,$
$7y_1 + 5y_2 + 5y_3 - 21t \le 0,$
$0.4y_1 + y_2 + 0.3y_3 - 0.4t \le 0,$
$0.6y_1 + 0.25y_2 + 0.4y_3 - 0.25t \le 0,$
$0.5y_1 + 0.3y_2 + 0.8y_3 - 0.5t \le 0,$
$0.3y_1 + y_2 + y_3 - 0.9t \le 0,$
$0.4y_1 + 0.25y_2 + 0.25y_3 - 0.2t \le 0,$
$0.8y_1 + 0.3y_2 + 0.3y_3 - 0.6t \le 0,$
$\lambda, t, y_1, y_2, y_3 \ge 0$

Solving by Maple we have

$\lambda = 1, t = 0.1569, y_1 = 0, y_2 = 0, y_3 = 0.09804$
$\Rightarrow x_1 = 0, x_2 = 0, x_3 = 0.625$

$$z(x^{\sim n}) = \frac{(0.625)9^{\sim n}}{(0.625)6^{\sim n} + 1.5^{\sim n}} = (0.7843, 1.0714, 1.5152, 2.2917; 0.75, 0.5, 0.4)$$

4 Conclusion

We introduced a new approach to solve trapezoidal-neutrosophic linear fractional programming models. A comparison between the fuzzy approach and the neutrosophic approach is given. We illustrated the importance of neutrosophic approach than used fuzzy approach. Finally, to obtain a neutrosophic basic possible optimal solution to a somewhat modified set of constraints, the suggested technique proposed a neutrosophic approach involving neutrosophic artificial variables. The neutrosophic simplex technique is then used to remove the artificial neutrosophic variables and to resolve the original problem. We plan to extend this approach to solve multi-level multi-objective fully neutrosophic linear fractional programming problems in the next work.

Conflict of Interest

The authors declare that they have no conflict of interest.

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