Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/150509

Deriving Higher Order Rogue Waves Solutions in the Nonlinear System Using a Symbolic Computation Approach

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Received: 7 Jan. 2021, Revised: 21 Apr. 2021, Accepted: 03 Jun. 2021
Published online: 1 Sep. 2021

Abstract: In this paper, we consider a novel multiple rogue wave solutions of a (3 +1)-dimensional Kudryashov-Sinelshchikov equation by using a symbolic computation approach.Some higher order rogue wave solutions of a (3 +1)-dimensional Kudryashov-Sinelshchikov equation are presented. Some properties of the multi-rogue waves and their collision structures are given through numerical examples. Finally, some expected applications and extension have been mentioned.

Keywords: $(3+1)$-dimensional Kudryashov-Sinelshchikov equation, Bilinear form, Rogue waves

1 Introduction

Rogue waves observed in the water tanks [1,2],deep ocean [3,4], and optical fibers [5,6].To construct rogue wave solutions, many several methods have been proposed, including the inverse scattering method [7], the Hirota bilinear method [8], Darboux transformation method [9] and Backlund transformation method [10] and so on. Recently [11,12,13], the first order rogue wave and rational solutions to some (3+1) and (2+1)-dimensional systems are constructed by the symbolic computation approach.

In this paper, we mainly focus on a $(3+1)$-dimensional Kudryashov-Sinelshchikov equation.The description of the proposed method in [14] is given in section 2, the bilinear form and multi rogue waves for (3+1)-dimensional Kudryashov-Sinelshchikov equation derived in section 3, the first-order, second-order and third-order rogue wave solutions for (3+1)-dimensional Kudryashov-Sinelshchikov equation derived in section 4-6. Finally, conclusions are given in Section 7.

2 Analytical method

Suppose the nonlinear partial differential equations (NLEEs), take the form

$$H(u,u_t,u_x,u_y,u_z,u_{xt},u_{xy},u_{xz}\ldots)=0, \qquad (1)$$

where H is a polynomial in unknown function $u(x,y,z,t)$ and its derivatives.

The main steps for the used method are consists of the following steps

Step 1. According the transformation to the Painlève analysis

$$u(x,y,z,t)=u(f), \qquad (2)$$

is made by a dependent variable function f.

Step 2. The NLEEs(1) using (2) converted into Hirota's bilinear form

$$F(D_\zeta,D_z)=0, \qquad (3)$$

where $\zeta=x+y-et$, e is a real parameter and e is a wave speed D-operator [15] is defined by

$$D_x^kD_y^mD_z^nD_t^lf(x,y,z,t)\cdot g(x,y,z,t)=$$

$$(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'})^k(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'})^m(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'})^n$$

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$$(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'})^l[f(x,y,z,t)g(x',y',z',t')]\,|_{x'=x,y'=y,z'=z,t'=t} \quad (4)$$

. **Step 3.** Suppose

$$F=G_{n+1}(\zeta,z;\alpha,\beta)=F_{n+1}(\zeta,z)+2\alpha zP_n(\zeta,z) \\ +2\beta(\zeta)Q_n(\zeta,z)+\left(\alpha^2+\beta^2\right)F_{n-1}(\zeta,z), \quad (5)$$

$$F_n(\zeta,z;\alpha,\beta)= \\ \sum_{k=0}^{\frac{1}{2}n(n+1)}\left(\sum_{i=0}^{k}a_{n(n+1)-2k,2i}z^{2i}\zeta^{n(n+1)-2k}\right), \\ P_n(\zeta,z)= \\ \sum_{k=0}^{\frac{1}{2}n(n+1)}\left(\sum_{i=0}^{k}b_{n(n+1)-2k,2i}z^{2i}\zeta^{n(n+1)-2k}\right), \\ Q_n(\zeta,z)= \\ \sum_{k=0}^{\frac{1}{2}n(n+1)}\left(\sum_{i=0}^{k}c_{n(n+1)-2k,2i}z^{2i}\zeta^{n(n+1)-2k}\right), \quad (6)$$

where $F_0=1, F_{-1}=P_0=Q_0=0$, $\alpha,\beta,a_{m,l},b_{m,l}$ and $c_{m.l},(m,l=0,2,4,....,n(n+1))$ are real numbers and α,β are used to control the rogue-wave center.

Step 4. The systems obtained Solved by inserting (5) into (2) and equating all the coefficients of ζ and z to zero we can obtain a system of polynomials and using the symbolic software Maple to solve the system.

Step 5. Substituting the values of $a_{m,l},b_{m,l}$ and $c_{m,l}$ into (1) to get the multi rogue wave solutions in terms of ζ and z.

3 The bilinear form for $(3+1)$-dimensional Kudryashov-Sinelshchikov equation

In this section, we consider the $(3+1)$-dimensional Kudryashov-Sinelshchikov equation [16]

$$(u_t+f_1uu_x+f_2u_{xxx})_x+f_3u_{yy}+f_4u_{zz}=0, \quad (7)$$

where $u(x,y,z,t)$ is a differentiable function and f_1,f_2,f_3,f_4 are arbitrary constants.

To derive the rogue-waves solutions for (7), we find the Hirota bilinear form by setting $\zeta=x+y-et$, to obtain the ordinary differential equation (ODE) for (7) as follows

$$f_2u_{\zeta\zeta\zeta\zeta}+(f_1u+f_3-e)u_{\zeta\zeta}+f_1u_\zeta^2+f_4u_{zz}=0. \quad (8)$$

Assume that the variable transformation has the following form

$$u=u_0+\frac{12f_2}{f_1}(\ln F)_{\zeta\zeta}. \quad (9)$$

Then the Hirota bilinear form for (7) can be obtained by inserting (9) into (8) as

$$(f_2D_\zeta^4+(f_1u_0+f_3-e)D_\zeta^2+f_4D_z^2)F.F=0. \quad (10)$$

The multi rogue wave solutions for the for a (3 + 1)-dimensional Kudryashov-Sinelshchikov equation (7) can be obtained as follows

4 First-order rogue waves $n=0$

In this case, we select

$$F=G_1=a_{2,0}\zeta^2+a_{0,2}z^2+a_{0,0}. \quad (11)$$

Let $a_{2,0}=1$ without loss of generality. Substituting (11) into (10) and setting the coefficients of all powers of ζ and z to zero, we can obtain the coefficients $a_{0,0}$ and $a_{0,2}$ as follows

$$a_{0,0}=\frac{3f_2}{-u_0f_1+e-f_3},a_{0,2}=-\frac{-u_0f_1+e-f_3}{f_4} \quad (12)$$

Inserting (12) in (11), then the first-order rogue waves for Eq. (7) takes the form

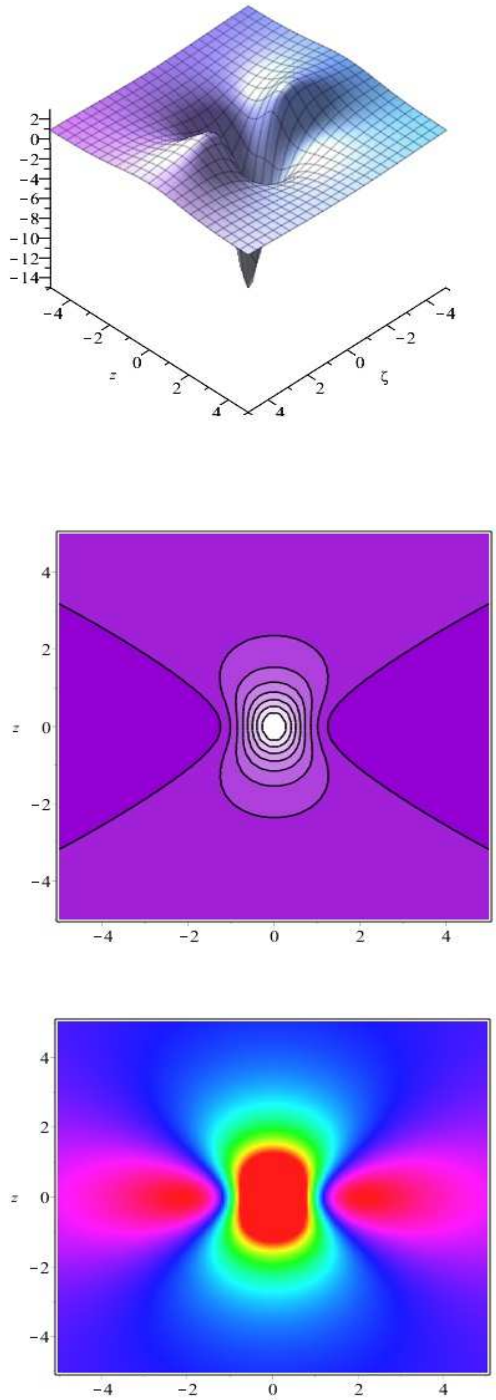
$$u=u_0+\frac{12f_2}{f_1}(\ln F)_{\zeta\zeta}. \quad (13)$$

where

$$F=-\frac{(-u_0f_1+e-f_3)(z-\alpha)^2}{f_4}+ \\ (\zeta-\beta)^2-\frac{3f_2}{u_0f_1-e+f_3}. \quad (14)$$

The first-order rogue wave solutions (13) when $\alpha=\beta=0$ are shown in Fig.1, has three center $(0,0)$ and $(\mp3\sqrt{\frac{f_2}{-U_0f_1+e-f_3}},0)$ in three-dimensional, contour plot and the corresponding density plot, we notice that in this case,we have one peaks, the first-order rogue wave has the minimum amplitude $u_0+\frac{-8u_0f_1+8e-8f_3}{f_1}$ at $(0,0)$ and maximal amplitude $u_0-\frac{-u_0f_1+e-f_3}{f_1}$ at $(\mp3\sqrt{\frac{f_2}{-u_0f_1+e-f_3}},0)$ when $f_2>0, f_1u_0+f_3<e$. But the first-order rogue wave solutions (14) at $\alpha=-2,\beta=-2$ the center of rogue wave will be $(-2,-2)$ and $(\frac{2U_0f_1+3\sqrt{-u_0f_1f_2+ef_2-f_2f_3}-2e-2f_3}{-u_0f_1+e-f_3},-2)$ as shown in Fig.2, moreover, the minimal and maximal amplitudes also change into $u_0+\frac{-8u_0f_1+8e-8f_3}{f_1}$ and $-\frac{-2u_0f_1+e-f_3}{f_1}$, respectively.

Fig.1 The first-order rogue wave solution (13) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha=\beta=0$

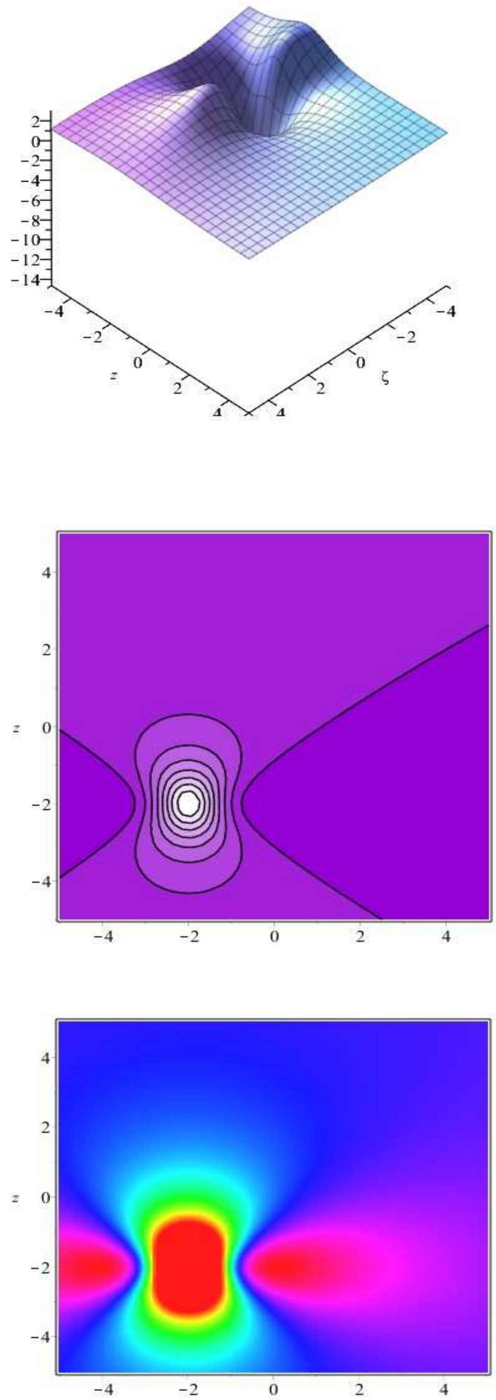


(a)

(b)

(c)

Fig. 1: The first-order rogue wave solution (13) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = 0$



(a)

(b)

(c)

Fig. 2: The first-order rogue wave solution (13) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = -2$

5 Second-order rogue waves n=1

In this section, we will find the second-order rogue wave of Eq. (7) by setting $n=1$ in Eq. (5) as follows

$$\begin{cases} F=G_2(\zeta,z;\alpha,\beta)=F_2(\zeta,z)+2\alpha zP_1(\zeta,z)+ \\ 2\beta\zeta Q_1(\zeta,z)+\left(\alpha^2+\beta^2\right)F_0(\zeta,z)= \\ a_{6,0}\zeta^6+\left(z^2a_{4,2}+a_{4,0}\right)\zeta^4+2\zeta^3\beta c_{2,0}+ \\ \left(z^4a_{2,4}+2\alpha zb_{2,0}+z^2a_{2,2}+a_{2,0}\right)\zeta^2+2\beta \\ \left(c_{0,2}z^2+c_{0,0}\right)\zeta+a_{0,6}z^6+a_{0,4}z^4+2\alpha z^3b_{0,2} \\ +a_{0,2}z^2+2\alpha zb_{0,0}+a_{0,0}\left(\alpha^2+\beta^2+1\right). \end{cases} \tag{15}$$

Substituting (14) into (10) and setting all coefficients of all powers for ζ and z to zero, we can get a set of parameters $a_{m,l},b_{m,l},c_{m.l},(m,l=0,2,4,6)$ as follows

$$\begin{aligned} a_{0,0}=&\frac{1}{(\alpha^2+\beta^2+1)(-u_0f_1+e-f_3)^3} \\ &(-9\beta^2(-u_0f_1+e-f_3)^3{c_{2,0}}^2+ \\ &\alpha^2{u_0}^2{b_{2,0}}^2{f_1}^2f_4-2{b_{2,0}}^2\alpha^2u_0f_4(e-f_3)f_1 \\ &+\alpha^2e^2{b_{2,0}}^2f_4-2\alpha^2e{b_{2,0}}^2f_3f_4+ \\ &\alpha^2{b_{2,0}}^2f_3^2f_4-16875\,{f_2}^3), \\ a_{0,2}=&-475\frac{{f_2}^2}{f_4(-u_0f_1+e-f_3)}, \\ a_{0,4}=&17\frac{(-U_0f_1+e-f_3)f_2}{{f_4}^2}, \\ a_{0,6}=&-\frac{(-u_0f_1+e-f_3)^3}{{f_4}^3}, \\ a_{2,0}=&-125\frac{{f_2}^2}{(-u_0f_1+e-f_3)^2}, \\ a_{2,4}=&3\frac{(-u_0f_1+e-f_3)^2}{{f_4}^2}, \end{aligned}$$

$$\begin{aligned} a_{4,0}&=25\frac{f_2}{-u_0f_1+e-f_3}, \\ a_{4,2}&=-3\frac{-u_0f_1+e-f_3}{f_4}, \\ b_{0,0}&=\frac{f_4b_{2,0}f_2}{-3u_0f_1+3e-3f_3}, \\ b_{0,2}&=\frac{b_{2,0}(-u_0f_1+e-f_3)}{3f_4}, \\ c_{0,0}&=-\frac{c_{2,0}f_2}{-u_0f_1+e-f_3}, \\ c_{0,2}&=3\frac{(-u_0f_1+e-f_3)c_{2,0}}{f_4}, \\ a_{2,2}&=-90\frac{f_2}{f_4} \end{aligned} \tag{16}$$

where $b_{2,0}$ and $c_{2,0}$ is an arbitrary constant. The second-order rogue wave for Eq. (7) takes the form

$$u=u_0+\frac{12f_2}{f_1}(\ln G_2(\zeta,z;\alpha,\beta))_{\zeta\zeta}. \tag{17}$$

In Fig 3.,4 show the second-order rogue waves for (13) for different values of α,β, the second-order peak breaks apart and for sufficiently big parameters, where the set of three first order rogue waves forms and these centers is formed a triangle, this is called a rogue wave triplet.

6 Third-order rogue waves $n=2$

The third-order rogue wave of Eq. (7) is obtained by taking $n=2$ in Eq.(5) as follows

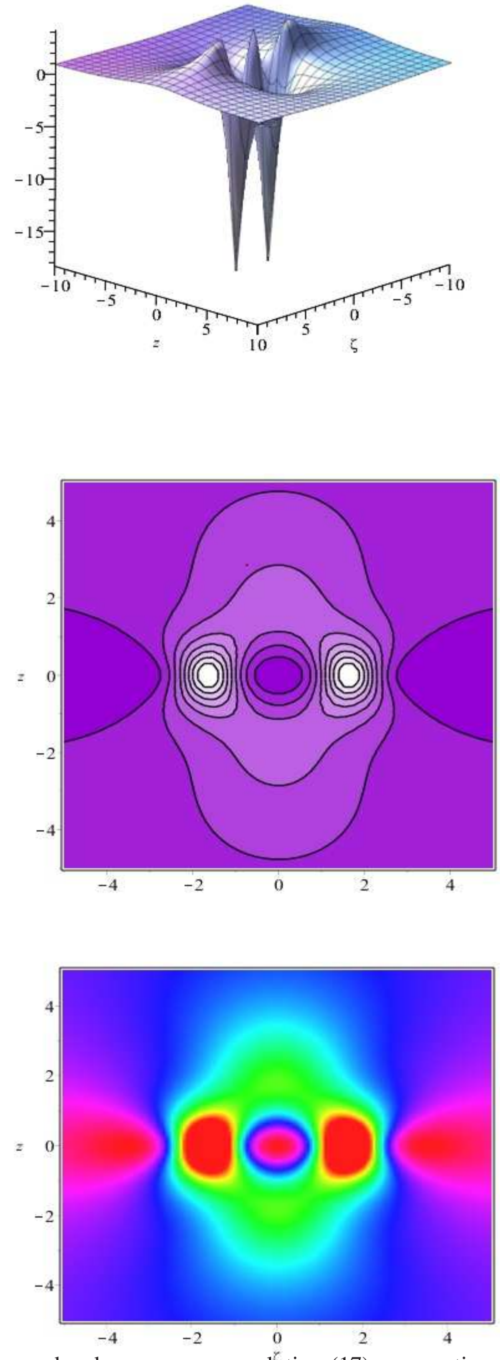
$$\begin{cases} F=G_3(\zeta,z;\alpha,\beta) \\ =F_3(\zeta,z)+2\alpha zP_2(\zeta,z)+2\beta\zeta Q_2(\zeta,z) \\ +\left(\alpha^2+\beta^2\right)F_1(\zeta,z) \\ =a_{12,0}\zeta^{12}+a_{10,0}\zeta^{10}+a_{10,2}z^2\zeta^{10} \\ +a_{8,0}\zeta^8+a_{8,2}z^2\zeta^8+a_{8,4}z^4\zeta^8 \\ +\zeta^6+a_{6,2}z^2\zeta^6+a_{6,4}z^4\zeta^6 \\ +a_{6,6}z^6\zeta^6+a_{4,0}\zeta^4+a_{4,2}z^2\zeta^4 \\ +a_{4,4}z^4\zeta^4+a_{4,6}z^6\zeta^4+a_{4,8}z^8\zeta^4 \\ +a_{2,0}\zeta^2+a_{2,2}z^2\zeta^2+a_{2,4}z^4\zeta^2+a_{2,6}z^6\zeta^2 \\ +a_{2,8}z^8\zeta^2+a_{2,10}z^{10}\zeta^2+2\beta(c_{6,0}\zeta^6+c_{4,2}z^2\zeta^4+c_{4,0}\zeta^4 \\ +c_{2,4}z^4\zeta^2+c_{2,2}z^2\zeta^2+c_{2,0}\zeta^2 \\ +c_{0,6}z^6+c_{0,4}z^4+c_{0,2}z^2+c_{0,0})(\zeta)+(\alpha^2+\beta^2) \\ \times(a_{2,0}\zeta^2+a_{0,2}z^2+a_{0,0})+a_{0,0}+2\alpha z(b_{6,0}\zeta^6 \\ +b_{4,2}z^2\zeta^4+b_{4,0}\zeta^4+b_{2,4}z^4\zeta^2 \\ +b_{2,2}z^2\zeta^2+b_{2,0}\zeta^2+b_{0,6}z^6 \\ +b_{0,4}z^4+b_{0,2}z^2+b_{0,0})+a_{0,2}z^2+a_{0,4}z^4+a_{0,6}z^6 \\ +a_{0,8}z^8+a_{0,10}z^{10}+a_{0,12}z^{12}. \end{cases} \tag{18}$$

Inserting (18) into (10), then setting all coefficients of all powers of ζ and z to zero, we can get a set of parameters $a_{m,l},b_{m,l},c_{m.l},(m,l=0,2,4,6)$where $b_{2,0}$ and $c_{4,0}$ are arbitrary constants.Then the third-order rogue wave solution for Eq. (7) takes the form

$$u=u_0+\frac{12f_2}{f_1}(\ln G_3(\zeta,z;\alpha,\beta))_{\zeta\zeta}. \tag{19}$$

In Fig. 5,6. show the third-order rogue waves for (13) for big values of α,β, the third-order rogue waves consists of five first-order rogue waves are located in the corners of a pentagon and other one sites in the center

Fig.6. The third-order rogue wave solution (20) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha=\beta=10^8$. It is shown that multiple rogue waves solutions for a (3+1)-dimensional Kudryashov-Sinelshchikov equation can be used in different application such as matter-field interaction and Schrödinger equation [16]-[21].

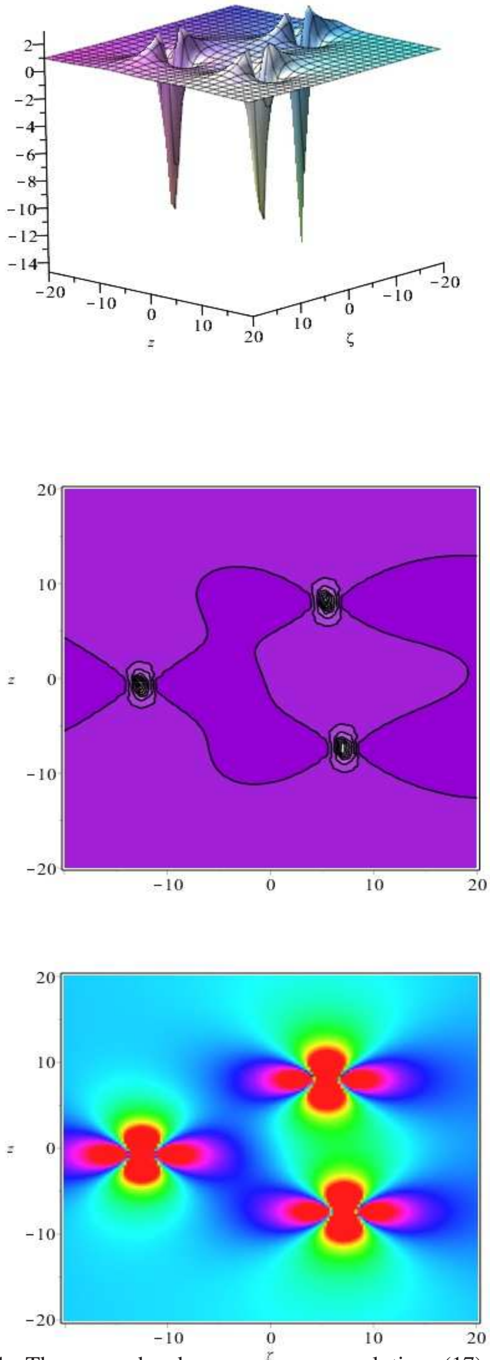


(a)

(b)

(c)

Fig. 3: The second-order rogue wave solution (17) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = 0$

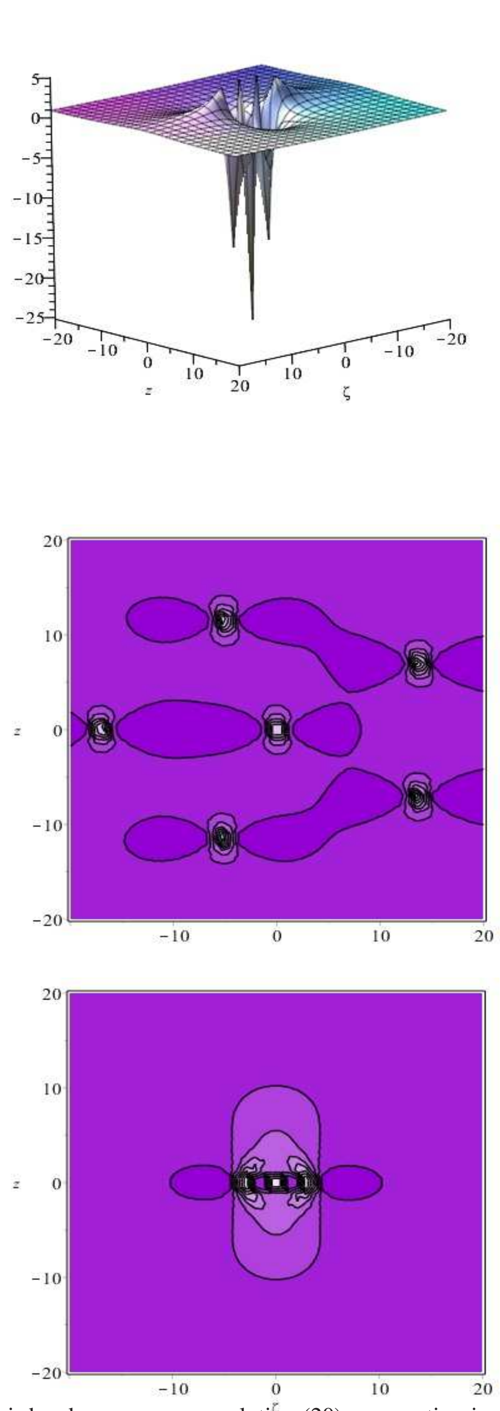


(a)

(b)

(c)

Fig. 4: Fig.4. The second-order rogue wave solution (17) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = 2000$

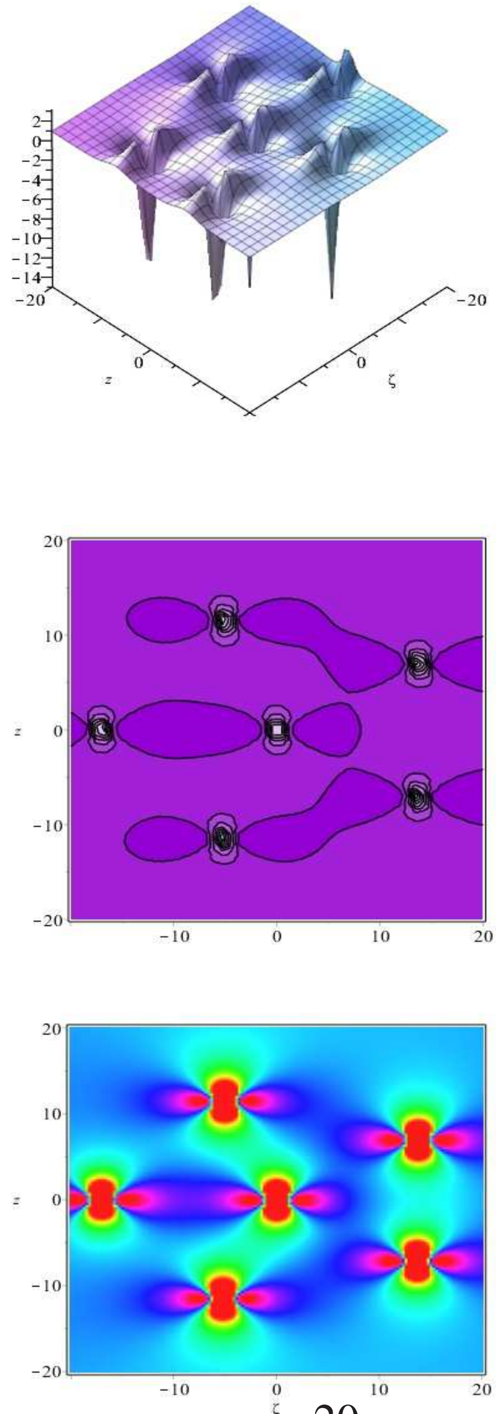


(a)

(b)

(c)

Fig. 5: The third-order rogue wave solution (20) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = 0$



(a)

(b)

(c)

Fig. 6: The third-order rogue wave solution (20) propagation in three-dimensional plot (a), contour plot (b) and density plot (c) with $\alpha = \beta = 10^8$

7 Conclusion

In this paper, we have discussed a novel multiple rogue wave solutions of a (3 +1)-dimensional Kudryashov-Sinelshchikov equation contain two free parameters α and β, which are used to control the center of the rogue waves.This exact solutions include higher order rogue wave solutions of a (3 +1)-dimensional Kudryashov-Sinelshchikov equation.

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