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Computational Modeling of Detecting a Randomly Target in a Bounded Known Region

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Abstract: This paper formulates a novel quartile coordinated search technique that finds a 3-*D* randomly located landmine. In addition we calculated the expected time to detect the landmine in Symmetric Coordinated search Technique (SCST). We introduce the optimal search strategy that minimizes the expected time for detecting the landmine assuming trivarite standard normal distribution in the cases SCST. An approximation algorithm has been introduced to facilitate searching procedures. The stochastic Euclidean distance between a known satellite equations and a randomly moving meteor in the 3 dimensional has been introduced. Also, we illustrate a technique known as coordinated search that completely characterizes the search for a randomly located target on the plane. The idea is to avoid wasting time looking for a missing target. Four searchers or robots start from the center of circle to search out a lost target, an illustrative application from real life has been introduced not only to demonstrate the applicability of this search technique but also to facilitate the using of constrained approaches and expectations.

Keywords: Coordinate search; Lost target; Search theory; Trivarite standard normal distribution; 3-*D* search algorithm.

1 Introduction

Concepts in search theory have been introduced since 1975 with the World War II. Many of important applications of search strategies have emerged since then, search plans or moving targets have become solutions for complex problems [1–4]. Our study on a special technique of search plans it called coordinated search, to mention some important models of search plans, we necessary illustrate the coordinated linear search, see And El-Moneim A. M. et al. [5, 6]. In earlier works by Abd El-Moneim A. M. and his collaborators [8, 10], the authors illustrated the coordinated technique in the Plane where the target was located. S. N. Al-Aziz illustrated a novel coordinated search by two searchers [9]. Recently, due to the technological development A. H. El-Bagoury et al. [11] introduced a modern search technique in the 3-Dimensional space that calculated the expected time of detecting a randomly located lost black box in a 3-Dimensional known zone by a single search. More recently, A. H. El-Bagoury et al. [7, 12, 13] introduced search technique that find a 3-Dimensional randomly located lost target by two searchers. Rather than calculating the expected time of detecting in the case of symmetric and asymmetric trivariate known distribution and obtained the optimal search strategy that minimize the expected time of the detection, assuming trivarite standard normal and skew standard normal distributions. It is known that the more important of the target makes the research team increases the number of searchers (robots), Tomas Caraballo et al. [14] calculated the optimal search path that minimizes the expected time of detecting randomly located black box of aircraft hanged between two rocks in the bottom of seas using four robots in different four regions without coordinating search technique. Now our main purpose to develop the techniques in [12] and [14] to quartile coordinated search technique, for more different search plans and applications, see [15–20].

World War II caused many losses to Egypt; the Western desert region suffers from the existence of nearly 18 million mines. In Al-Alamein area of the northern coast of Western desert, many battles occurred between the British and German armies, Landmines have Landmines have been placed in an unprecedented manner, "Gardens of the Devil", an unusual minefield planed vertically on three layers, if the first mine was removed in the same

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time the second one explodes. The disruption of agriculture and economic development in many places in Western desert due to anti-personnel landmines and tanks. Inn addition to the huge financial costs required by the demining process and the lack of sufficient experts, therefore, we should use coordinated search to solve this prob- lem by using four searchers (robots) in a known region. Wee present a quartile coordinated search algorithm, with the important advantage that it avoids wasting time searching for the target. The mission is carried out by four searchers in both two and 3-dimensional space to facilitate the search process in the two cases. The four searchers starting from the origin, we use modern means of communication and sign language to save much effort and time.

2 The Framework for the 3-Dimensional Search

2.1 The Space

A 3-dimensional search space zone (Ξ), divided by two planes, plane I is horizontal and the vertial will be plane II, intersected on $(0,0,0)$.

2.2 The Means of Search

Searching technique will carry put using a quartile coordinated search using four searchers.

2.3 The target

The landmine is randomly located Ξ.

3 Material and Methods

3.1 The Searching Technique

Symmetric Coordinated Search Technique (SCST): Technique in which the four searchers start from the same point $(0,0,0)$ and move the same distances, the movement directions may differ to prevent any physical effect on each other as illustrated in fig. (1).

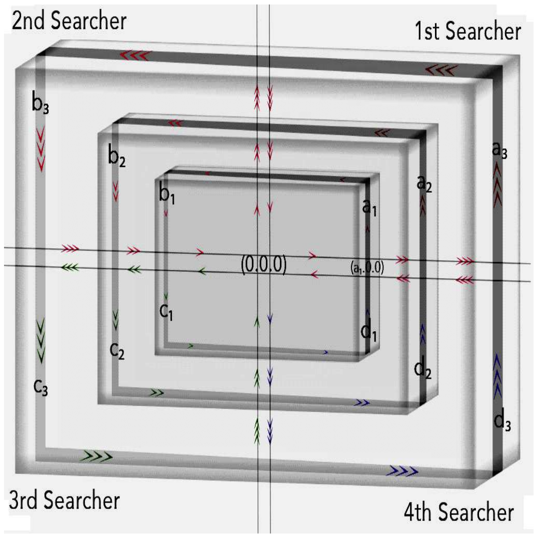

Fig. 1: (SCST), the destances $a_1 = b_1 = c_1 = d_1$

3.2 Finite the expected time for (SCST)

Theorem 1. By considering the landmine has symmetric trivariate known distribution, then the expected time of the search to detect the landmine is given by:

$$16\sum_{i=1}^{k}[(a_i)\{\xi_i - \xi_{i-1}]. \tag{1}$$

Where

$$\xi_{i-1} = \int_{-a_{i-1}}^{a_{i-1}}\int_{-a_{i-1}}^{a_{i-1}}\int_{-a_{i-1}}^{a_{i-1}} f(x,y,z)dxdydz,$$

$$\xi_i = \int_{-a_i}^{a_i}\int_{-a_i}^{a_i}\int_{-a_i}^{a_i} f(x,y,z)dxdydz.$$

Proof. Let the search path is $\Gamma = \{\Gamma_i, i = 1,2,\cdots,K\}$ and the landmine has symmetric trivariate known distribution, then,

For the first robot:

The searching in the first cubic C_1 and its track will be Ð $(\Gamma_1) = 4a_1$ and if the landmine lies in the sapce between C_1 and C_1 then Ð $(\Gamma_2) = 4a_1 + 4a_2$, if the landmine lies in the space between C_2 and C_3 then Ð $(\Gamma_3) = 4a_1 + 4a_2 + 4a_3$ and etc.

For the second robot:

The searching in the first cubic C_1' and its track then Ð $(\Gamma_1') = 4a_1$ and if the landmine lies in the space between C_1' and C_2' then Ð $(\Gamma_2') = 4a_1 + 4a_2$, if the landmine lies in the space between C_2' and C_3' then Ð $(\Gamma') = 4a_1 + 4a_2 + 4a_3$ and etc.

For the third robot:

The searching in the first cubic C_1'' and its track then Ð $(\gamma_2'') = 4a_1$ and if the landmine lies in the space between C_1'' and C_2'' then Ð $(\Gamma_2'') = 4a_1 + 4a_2$, if the landmine lies in the space between C_2'' and then Ð $(\Gamma_3'') = 4a_1 + 4a_2 + 4a_3$ and etc.

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For the fourth robot:

The searching in the first cubic C_1''' and its track then $Ð\left(\Gamma_2'''\right) = 4a_1$ and if the landmine lies in the space between C_1''' and C_2''' then $Ð\left(\Gamma_2'''\right) = 4a_1 + 4a_2$, if the landmine lies in the space between C_2''' and C_3''' then $Ð\left(\Gamma_3'''\right) = 4a_1 + 4a_2 + 4a_3$ and etc.

$$\begin{aligned} Ð\left(\Gamma, \wedge, F\right) =& \{4a_1 + [\xi_1]\} + \{4(a_1+a_2)[\xi_2 - \xi_1]\} \\ &+ \{4(a_1+a_2+a_3)[\xi_3 - \xi_2]\} + \cdots \\ &+ \{4a_1[\xi_1]\} + \{4(a_1+a_2)[\xi_2 - \xi_1]\} \\ &+ \{4(a_1+a_2+a_3)[\xi_3 - \xi_2]\} + \cdots \\ &+ \{4a_1[\xi_1]\} + \{4(a_1+a_2)[\xi_2 - \xi_1]\} \\ &\{4(a_1+a_2+a_3\}[\xi_3 - \xi_2]\} + \cdots \\ &+ \{4a_1[\xi_1]\} + \{4(a_1+a_2)[\xi_2 - \xi_1]\} \\ &+ \{4(a_1+a_2+a_3\}[\xi_3 - \xi_2]\} + \cdots \end{aligned}$$

It will be,

$$Ð\left(\Gamma, \wedge, F\right) = 16 \sum_{i=1}^{k} [(a_i)\{\xi_i - \xi_{i-1}\}].$$

3.3 Optimal search path

Mine search conducted in the wild environment require accurate search, that helping us to minimize the time of detecting the landmine so. the main purpose is to find the optimal distances in case of SCST.

Theorem 2. If $F(x,y,z)$ be a standard trivarite normal distribution function with density function $f(x,y,z)$ and let $\Gamma, \Lambda \in Q$ are optimal search path, then the necessary conditions that determines the optimal value of a_i by solving the following equation:

$$[a_i - a_{i+1}] \frac{\partial}{\partial a_i} \xi_i + \xi_1 = 0. \tag{2}$$

Proof.

From (1) we have,

$$Ð\left(\Gamma, \wedge, F\right) = 16 \sum_{i=1}^{k} [(a_i)\{\xi_i - \xi_{i-1}\}].$$

Then,

$$\frac{\partial Ð\left(\Gamma, \Lambda, F\right)}{\partial a_1} = 6a_1 \left[\frac{\partial}{\partial a_1} \xi_1\right] + 6[\xi_1] - 6a_2 \left[\frac{\partial}{\partial a_1} \xi_1\right] = 0.$$

So,

$$a_1 \left[\frac{\partial}{\partial a_1} \xi_1\right] + \xi_1 - a_1 \left[\frac{\partial}{\partial a_1} \xi_1\right] = 0.$$

Also,

$$\frac{\partial Ð\left(\Gamma, \Lambda, F\right)}{\partial a_2} = a_2 \left[\frac{\partial}{\partial a_2} \xi_2\right] + \xi_2 - a_3 \left[\frac{\partial}{\partial a_2} \xi_2\right] = 0.$$

By mathematical induction, we get:

$$\begin{aligned} \frac{\partial Ð\left(\Gamma, \Lambda, F\right)}{\partial a_i} &= a_i \left[\frac{\partial}{\partial a_i} \xi_i\right] + \xi_i - a_{i+1} \left[\frac{\partial}{\partial a_i} \xi_i\right] = 0. \\ &= [a_i - a_{i+1}] \frac{\partial}{\partial a_i} \xi_i + \xi_i. \end{aligned}$$

3.4 The case of trivarite standard normal distribution

Let X, Y, Z are three independent random variables that represent the position of the target and they have a trivariate standard normal distribution with means $\mu_1 = \mu_2 = \mu_3 = 0$ and variances $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$. Thus, the joint probability density function of the trivarite standard normal distribution will become:

$$f(x,y,z) = \left(\frac{1}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}(x^2+y^2+z^2)},$$

where $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < z < \infty$. By using spherical coordinates, $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, and $z = \rho \cos\phi$ where,

$$\begin{aligned} Ð\left(\Gamma, \Lambda, F\right) =& 16 \sum_{i=1}^{\infty} \left[(a_i) \left\{ \int_{-a_i}^{a_i} \int_{-a_i}^{a_i} \int_{-a_i}^{a_i} f(x,y,z)\, dx dy dz \right.\right. \\ &\left.\left. - \int_{-a_{i-1}}^{a_{i-1}} \int_{-a_{i-1}}^{a_{i-1}} \int_{-a_{i-1}}^{a_{i-1}} f(x,y,z)\, dx dy dz \right\} \right]. \end{aligned}$$

We get,

$$\begin{aligned} Ð\left(\Gamma, \Lambda, F\right) =& \sum_{i=1}^{k} \left[16(a_i) \left\{ \int_0^{\pi} \int_0^{2\pi} \int_0^{a_i} g(\rho,\phi,\theta) \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta \right.\right. \\ &\left.\left. - \int_0^{\pi} \int_0^{2\pi} \int_0^{a_{i-1}} g(\rho,\phi,\theta) \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta \right\} \right]. \\ =& \sum_{i=1}^{k} \left[16(a_i) \left\{ \int_0^{\pi} \int_0^{2\pi} \int_0^{a_i} \left(\frac{1}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}\rho^2} \rho^2 \sin\phi \right.\right. \\ & d\rho\, d\phi\, d\theta - \int_0^{\pi} \int_0^{2\pi} \int_0^{a_{i-1}} \left(\frac{1}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}\rho^2} \rho^2 \sin\phi \\ &\left.\left. d\rho\, d\phi\, d\theta \right\} \right] \\ =& 4 \sqrt{\frac{2}{\pi}} \sum_{i=1}^{k} [a_i] \left\{ \int_0^{a_i} \rho^2 e^{-\frac{1}{2}\rho^2} d\rho - \int_0^{a_{i-1}} \rho^2 e^{-\frac{1}{2}\rho^2} \right. \\ & \left. d\rho \right\} \end{aligned}$$

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$$= 4\sqrt{\frac{2}{\pi}}\sum_{i=1}^{k}[a_i]\left\{\sqrt{\frac{\pi}{2}}ERF\left(\frac{a_i}{\sqrt{2}}\right)a_i e^{\frac{1}{2}a_i^2}\right.$$
$$\left. + \sqrt{\frac{\pi}{2}}Erf\left(\frac{a_{i-1}}{\sqrt{2}}\right) - a_{i-1}e^{-\frac{1}{2}a_{i-1}^2}\right\}$$

Where Erf = Error function

$$Erf = \frac{2}{\pi}\int_0^z e^{-t^2}dt.$$

By using conventional approach, the distances a_i^*, $i = 1,2,\cdots,k$ are changed to the probabiility of the target. The above symmetric nonlinear optimization problem (SNL-OP) will be:

$$\min_{a_i}$$

subject to

$$a_i a_{i-1} \geq 0, \quad \forall\, i = 1,2,\cdots,k$$
$$\sqrt{\frac{\pi}{2}}Erf\left(\frac{a_i}{\sqrt{2}}\right) - a_i e^{-\frac{1}{2}a_i^2} - \frac{\sqrt{2}}{\sqrt{\pi}} \leq 0.$$

Let Đ (γ,λ,F) is convexity, then the necessary conditions of Kuhn-Tucker will becomes

$$4\sqrt{\frac{2}{\pi}}\sum_{i=1}^{k}\left\{\sqrt{\frac{\pi}{2}}Erf\left(\frac{a_i}{\sqrt{2}}\right) - a_i e^{-\frac{1}{2}a_i^2} + \sqrt{\frac{\pi}{2}}Erf\left(\frac{a_{i-1}}{\sqrt{2}}\right)\right.$$
$$\left. - a_{i-1}e^{-\frac{1}{2}a_{i-1}^2} + a_i\left(e^{-a_i^2} - e^{-\frac{1}{2}a_i^2} + a_i^2 e^{-\frac{1}{2}a_i^2}\right)\right\} + \mu_1(1)$$
$$+ \mu_2\left(e^{-a_i^2} - e^{-\frac{1}{2}a_i^2} + a_i^2 e^{-\frac{1}{2}a_i^2}\right) = 0, \tag{3}$$
$$\mu_1(a_i - a_{i-1}) = 0,$$
$$\mu_2\left(\sqrt{\frac{\pi}{2}}Erf\left(\frac{a_i}{\sqrt{2}}\right) - a_i e^{-\frac{1}{2}a_i^2} - \frac{\sqrt{2}}{\sqrt{\pi}}\right). \tag{4}$$

We have many cases to solve Equation (3), but the best case to find the optimal values is $\mu_\hbar = 0$, $\hbar = 1,2$.

Using the iteration method, where $a_0 \cong 0$, we get the optimal values of a_i^*, $i = 1,2,\cdots,k$, after solving the following equation,

$$\sum_{i=1}^{k}\left\{\sqrt{\frac{\pi}{2}}Erf\left(\frac{a_i}{\sqrt{2}}\right) - a_i e^{-\frac{1}{2}a_i^2} + \sqrt{\frac{a_{i-1}}{\sqrt{2}}} - a_{i-1}e^{-\frac{1}{2}a_{i-1}^2}\right.$$
$$\left. + a_i\left(a_i^2 e^{-\frac{1}{2}a_i^2}\right)\right\} = 0. \tag{5}$$

For more optimality of our search, in the case of SCST, our main interesting involved in calculating the optimal values by solving (5) numerically using Mathematica and Maple, it will facilitate the search process. The optimal values will be:

$a_1^* = 0.132\times 10^{-5}$, $a_2^* = 0.133\times 10^{-5}$, $a_3^* = 0.245\times 10^{-4}$, $a_4^* = 0.3017\times 10^{-4}$, $a_5^* = 0.3019\times 10^{-4}$.

3.5 Three dimensional randomly moving target

In space there is a large cloud of waste in Earth's orbit due to the spread of idle satellites and fragmented missile parts, that threatens to collide and spread chaos outside the planet, according to NASA, there are millions of objects that cause this orbital problem. The shrapnel can move to approximately $18,000$ miles per hour ($19,000$ km/h), about seven times the speed of a bullet, and about $20,000$ of them are the size of a ball or larger.

Scientists believe that meteorites graduated from belt meteorites (asteroid belt), because the heat of the sun heats the meteorites in an irregular manner. This is what makes meteorites slowly deviate from the original tracks to spin oceans. The atmosphere plays an important role were protecting us from these small and medium-speed objects. But it unable to protect us at high speeds of objects that we met, it also fails at large objects sized. What happened to Jupiter in 1992, it can happen at any time for our land, so we have to be ready to protect the earth and protect ourselves. Norwegian Space Centre was announced in late 2005 that a large asteroid called APOPHIS will be shocked with the Earth in 2029.Where will rotate in its own orbit around the Earth to reach Earth Friday, April 13^{th} 2029. It will fall to Russia and destroys a large part of it and as a result will see the earth in the summer for three years due to dust as a result of this collision is likely to happen. The meteor APOPHIS is the largest meteorite known where it has a large rock diameter of 500 meters and because of its rotation around the earth and being it near to the land may it will destroy satellites in the space. Physicists expected that space the bumping force of APOPHIS to the earth would be equivalent to the explosion of 2000 Mega tons of explosives, TNT. As a result of this collision, it will be a hole its diameter four kilometers.

Now supposing the satellite motion follows the equations:

$$x(t) = X_0 + t^2\cos t; \tag{6}$$

$$y(t) = Y_0 + t^2\sin t; \tag{7}$$

$$z(t) = Z_0. \tag{8}$$

Supposing $X_1(t), X_2(t), X_3(t)$ are 3-D independent variables, the stochastic process $\{X(t); t \in R^+\}$, where $X(t) = \{X_1(t), X_2(t), X_3(t)\}$.

Then, the collision will occur when $SED = 0$ where "SED" stochastic Euclidean, distance [17]

$$SED = \left((X_0 + t^2\cos t - X_1(t))^2 + (Y_0 + t^2\sin t - X_2(t))^2 + (Z_0 - X_3(t))^2\right)^{\frac{1}{2}}.$$

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3.6 Searching technique of four searchers in the plane

The searchers S_1, S_2, S_3 and S_4 follow coordinated search path to find the target. Let E_i is the search path for the first searcher S_1, $i = 1, 2, \cdots$, and F_i is the search path of the second searcher S_2, and is the, $i = 1, 2, \cdots$. Also, let A_i, B_i, C_i, and D_i, $i = 1, 2, \cdots$, are the sectors which the searchers S_1, S_2, S_3 and S_4 searches it respectively. Let t_1 is the time which the first searcher S_1 takes it to search each sector in the path E_i and t_2 is the time which the second searcher S_2 takes it to search each sector in the path F_i.

The search process will be done according to the following procedures:

Step 1: The four searchers move from $(0,0)$ to detect the target. The first searcher follows the search path u_1 as follows: The searcher S_1 go to $(0, r_1)$ with distance $|r_1|$ through $a + ve$ part in x-axis if the target not found complete searching on the first sector A_1, and it's tracks, until the point $(r_1, 0)$,in the same time the searcher s_2 go to $(-r_1, 0)$ with distance $|r_1|$ through a $-ve$ part in x-axis if the target not found complete searching on the first sector B_1, and it's tracks. Until the point $(0, r_1)$, the searcher s_3 go to $(0, r_1)$ with distance $|r_1|$ through a $+ve$ part in y-axis, if the target not found complete searching on the first sector C_1, and it's tracks until the point $(r_1, 0)$, the searcher s_4 go to $(0, -r_1)$ with distance $|-r_1|$ through $a - ve$ part in y-axis, if the target not found complete searching on the first sector D_1, and it's tracks until the point $(r_1, 0)$. In this time, the four searchers send the signs to a marine ship signals reception center, if one of them send positive sign then the search is end but if the four searchers send a negative signs the searches will move to the following step.

Step 2: The searcher S_1 completes the search for the lost target and moves to the point $(r_2, 0)$ with distance $|r_2 - r_1|$, the first searcher follows the search path u_2 as follows: The searcher S_1 go to $(0, -r_2)$ with distance $|r_2 - r_1|$ through $a - ve$ part in y-axis if the target not found complete searching on the second sector A_2, and it's tracks, until the point $(0, r_2)$, in the same time the searcher s_2 go to $(0, r_2)$ with distance $|r_2 - r_1|$ through $a - ve$ part in x-axis if the target not found complete searching on the second sector B_2, and it's tracks, until the point $(-r_2, 0)$, the searcher s_3 go to $(r_2, 0)$ with distance $|r_2 - r_1|$ through $a + ve$ part in x-axis if the target not found complete searching on the second sector C_2,and it's tracks until the point, $(0, r_2)$. The searcher s_4 go to $(-r_2, 0)$ with distance through $a - ve$ part in y-axis if the target not found complete searching on the second sector D_2 and it's tracks until the point $(0, -r_2)$. In this time the four searchers send the signs to a marine ship signals reception center, if one of them send positive sign then the search is end but if the four searchers send a negative signs the searches will move to the following step.

Step 3: The searcher S_1 completes the search for the lost target and moves to the point $(r_3, 0)$ with distance $|r_3 - r_2|$. The first searcher follows the search path u_3 as follows: The searcher S_1 go to $(r_3, 0)$ with distance $|r_3 - r_2|$ through $a - ve$ part in y-axis if the target not found complete searching on the third sector A_3, and it's tracks, until the point $(0, -r_3)$, in the same time the searcher s_2 go to $(-r_3, 0)$ with distance $|r_3 - r_2|$ through $a - ve$ part in x-axis if the target not found complete searching on the third sector B_3 , and it's tracks, until the point $(0, r_3)$, the third searcher s_3 go to $(0, r_3)$ with distance $|r_3 - r_2|$ through $a + ve$ part in x-axis if the target not found complete searching on the third sector C_3, and it's tracks until the point $(r_3, 0)$ and the fourth searcher s_4 go to $(0, -r_3)$ with distance $|r_3 - r_2|$ through $a - ve$ part in y-axis if the target not found complete searching on the third sector D_3 and it's tracks until the point $(-r_3, 0)$. In this time the four searchers send the signs to a marine ship signals reception center, if one of them send positive sign then the search is end but if the four searchers send a negative signs the searches will move to the next sectors and so on until one of the four searchers detect the lost ship.

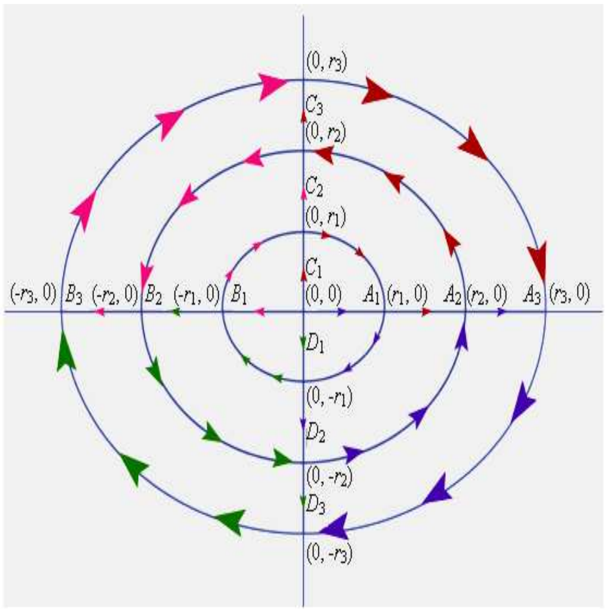

Fig. 2: The search path of detecting the lost ship.

3.7 Expected time value to detect the target in the plane

Theorem 3. The expected value of the time for detecting the target given by:

$$E(t_\phi) = \left[\left[\left(|r_1| + \frac{\pi}{w_1}\right) \int_0^{\frac{\pi}{4}} \int_0^{r_1} g_1(r,\theta) r dr d\theta\right] + \left[\left(|r_2 - r_1|\right.\right.\right.$$

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$$+\frac{\pi}{w_2}\Big)\int_0^{\frac{\pi}{4}}\int_{r_1}^{r_2} g_2(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_3-r_2|+\frac{\pi}{w_3}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_2}^{r_3} g_3(r,\theta)rdrd\theta\Big]+\cdots+\Big[\Big(|r_n-r_{n-1}|+\frac{\pi}{w_n}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_{n-1}}^{r_n} g_n(r,\theta)rdrd\theta\Big]\Big]$$

$$=4\Big[\Big[\Big(|r_i-r_{i-1}|+\frac{\pi}{w_i}\Big)\int_0^{\frac{\pi}{4}}\int_{r_{i-1}}^{r_i} g_i(r,\theta)rdrd\theta\Big]\Big], \quad (9)$$

where, $i=1,2,3,\cdots,n$, $r_0=0$.

Proof.

(1) For the first searcher S_1:
If the target is located in any place in the first sector A_1, then $t_1=|r_1|+\frac{\pi}{w_1}$.
If the target is located in any place in the second sector A_2, then $t_1=|r_2-r_1|+\frac{\pi}{w_2}$.
If the target is located in any place in the third sector A_3, then $t_1=|r_3-r_2|+\frac{\pi}{w_3}$.

(2) For the second searcher s_2. And so on.
If the target is located in any place in the first sector B_1, then $t_2=|r_1|+\frac{\pi}{w_1}$.
If the target is located in any place in the second sector B_2, then $t_2=|r_2-r_1|+\frac{\pi}{w_2}$.
If the target is located in any place in the third sector B_3, then $t_2=|r_3-r_2|+\frac{\pi}{w_3}$. And so on.

(3) For the third searcher s_3. And so on.
If the target is located in any place in the first sector C_1, then $t_3=|r_1|+\frac{\pi}{w_1}$.
If the target is located in any place in the second sector C_2, then $t_3=|r_2-r_1|+\frac{\pi}{w_2}$.
If the target is located in any place in the third sector C_3, then $t_3=|r_3-r_2|+\frac{\pi}{w_3}$. And so on.

(4) For the forth searcher s_2. And so on.
If the target is located in any place in the first sector D_1, then $t_4=|r_1|+\frac{\pi}{w_1}$.
If the target is located in any place in the second sector D_2, then $t_4=|r_2-r_1|+\frac{\pi}{w_2}$.
If the target is located in any place in the third sector D_3, then $t_4=|r_3-r_2|+\frac{\pi}{w_3}$. And so on

Then

$$E(t_\phi)=\Big[\Big(|r_1|+\frac{\pi}{w_1}\Big)\int_0^{\frac{\pi}{4}}\int_0^{r_1} g_1(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_2-r_1|$$

$$+\frac{\pi}{w_2}\Big)\int_0^{\frac{\pi}{4}}\int_{r_1}^{r_2} g_2(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_3-r_2|+\frac{\pi}{w_3}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_2}^{r_3} g_3(r,\theta)rdrd\theta\Big]+\cdots+\Big[\Big(|r_n-r_{n-1}|+\frac{\pi}{w_n}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_{n-1}}^{r_n} g_n(r,\theta)rdrd\theta\Big]+[\Big(|r_1|+\frac{\pi}{w_1}\Big)\int_0^{\frac{\pi}{4}}\int_0^{r_1} g_1(r,\theta)$$

$$rdrd\theta\Big]+\Big[\Big(|r_2-r_1|+\frac{\pi}{w_2}\Big)\int_0^{\frac{\pi}{4}}\int_{r_1}^{r_2} g_2(r,\theta)rdrd\theta\Big]$$

$$+\Big[\Big(|r_3-r_2|+\frac{\pi}{w_3}\Big)\int_0^{\frac{\pi}{4}}\int_{r_2}^{r_3} g_3(r,\theta)rdrd\theta\Big]+\cdots$$

$$+\Big[\Big(|r_n-r_{n-1}|+\frac{\pi}{w_n}\Big)\int_0^{\frac{\pi}{4}}\int_{r_{n-1}}^{r_n} g_n(r,\theta)rdrd\theta\Big]$$

$$+\Big[\Big(|r_1|+\frac{\pi}{w_1}\Big)\int_0^{\frac{\pi}{4}}\int_0^{r_1} g_1(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_2-r_1|$$

$$+\frac{\pi}{w_2}\Big)\int_0^{\frac{\pi}{4}}\int_{r_1}^{r_2} g_2(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_3-r_2|+\frac{\pi}{w_3}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_2}^{r_3} g_3(r,\theta)rdrd\theta\Big]+\cdots+\Big[\Big(|r_n-r_{n-1}|+\frac{\pi}{w_n}\Big)$$

$$\int_0^{\frac{\pi}{4}}\int_{r_{n-1}}^{r_n} g_n(r,\theta)rdrd\theta\Big]+\Big[\Big(|r_1|+\frac{\pi}{w_1}\Big)\int_0^{\frac{\pi}{4}}\int_0^{r_1} g_1(r,\theta)$$

$$rdrd\theta\Big]+\Big[\Big(|r_2-r_1|+\frac{\pi}{w_2}\Big)\int_0^{\frac{\pi}{4}}\int_{r_1}^{r_2} g_2(r,\theta)rdrd\theta\Big]$$

$$+\Big[\Big(|r_3-r_2|+\frac{\pi}{w_3}\Big)\int_0^{\frac{\pi}{4}}\int_{r_2}^{r_3} g_3(r,\theta)rdrd\theta\Big]+\cdots$$

$$\Big[\Big(|r_n-r_{n-1}|+\frac{\pi}{w_n}\Big)\int_0^{\frac{\pi}{4}}\int_{r_{n-1}}^{r_n} g_n(r,\theta)rdrd\theta\Big]\Big]$$

$$=4\Big[\Big[\Big(|r_i-r_{i-1}|+\frac{\pi}{w_i}\Big)\int_0^{\frac{\pi}{4}}\int_{r_{i-1}}^{r_i} g_i(r,\theta)rdrd\theta\Big]\Big].$$

3.8 Algorithm and Flowchart

Main algorithm

initialize the Searcher S_1, Searcher S_2, Searcher S_3 and Searcher S_4 to his start point $(0,0)$
Velocities $V_1 =$ Velocities, $V_2 =$ Velocities $V_3 =$ Velocities $V_4 = 1$
$i = 1$
$r = 0$
for $(i > 0)$
$j = r$
task user to input r and w
Đ$= |r - j|$
if ($i =$ odd number) then
$X + 1$
if ($X =$ odd number) then
moves S_1 towards the point $(0,r)$ with distance Đ && moves S_2 towards the point $(-r,0)$ with distance Đ && move S_3 towards the point $(0,r)$ with distance Đ && move S_4 towards the point $(0,-r)$ with distance Đ
If (target found) then Break
S_1 complete searching in the sector (A_i) with same distance Đ and tracks until the point $(r,0)$ && S_2 complete searching in the sector B_i with same distance Đ and tracks until the point $(0,r)$ && S_3 complete searching in the sector C_i with same distance Đ and tracks until the point $(r,0)$ && S_4 complete searching in the sector D_i with same distance Đ and tracks until the point $(-r,0)$
else ◦
the searcher S_1 completes the search for the lost target and moves to the point $(r,0)$ with distance Đ
moves S_1 towards the point $(0,-r)$ with distance Đ && moves S_2 towards the point $(-r,0)$ with distance Đ && moves S_3 towards the point $(0,r)$ with distance Đ && moves S_4 towards the point $(0,-r)$ with distance Đ
If (target found) then Break
S_1 complete searching in the sector (A_i) with same distance Đ and tracks until the point $(0,-r)$ && S_2 complete searching in the sector B_i with same distance Đ and tracks until the point $(0,r)$ && S_3 complete searching in the sector C_i with same distance Đ and tracks until the point $(r,0)$ && S_4 complete searching in the sector D_i with same distance Đ and tracks until the point $(-r,0)$
end if
$X++$
else
moves S_1 towards the point $(r,0)$ with distance Đ && moves S_2 towards the point $(0,r_i)$ with distance Đ && move S_3 towards the point $(r_i,0)$ with distance Đ && move S_4 towards the point $(-r_i,0)$ with distance Đ
If (target found) then Break
S_1 complete searching in the sector (A_i) with same distance Đ and tracks until the point $(0,r)$ && S_2 complete searching in the sector B_i with same distance Đ and tracks until the point $(-r_i,0)$ && S_3 complete searching in the sector C_i with same distance Đ and tracks until the point $(0,r_i)$. S_4 complete searching in the sector D_i with same distance Đ and tracks until the point $(0,-r_i)$
end if
calculate time for the step $T =$ distance Đ $+(3.14/w)$;
if (target not found in S_1) then
print Searchers + "not found target in" + Sectors)
else
if (target found in S_1) then
print green signal $+S_1+$ "found the target in" $+A_i+$ "time:" $+T$
else
print green signal $+S_2+$ "found the target in" $+B_i+$ "time:" $+T$
else
print green signal $+S_3+$ "found the target in" $+C_i+$ "time:" $+T$
else
print green signal $+S_4+$ "found the target in" $+D_i+$ "time:" $+T$
exit a for loop
End if
End if
$i{+}{+}$
End for
End Main.

4 Application and Results

The location of the lost ship follows the standard bivariate normal distribution for two independent random variables x,y are defined as $f(x,y) = \frac{1}{2\pi}e^{-\frac{1}{2}(x^2+y^2)}$. By changing to polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, then $g(r,\theta) = \frac{1}{2\pi}e^{\frac{-r^2}{2}}$.

The Jacobin matrix defined as:

$$det\begin{pmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{pmatrix} = r\cos^2\theta + r\sin^2\theta$$
$$= r(cos^2\theta + sin^2\theta) = r.$$

So, substituting in (1), the expected value will be

$$= 4\left[\left[\left(|r_i - r_{i-1}| + \frac{\pi}{w_i}\right)\int_0^{\frac{\pi}{4}}\int_{r_{i-1}}^{r_i} g_i(r,\theta)rdrd\theta\right]\right.$$

The first search if $r: 0 \to r_1$, then

$$E(t_\phi) = \frac{\pi}{2}\left[\frac{e^{-\frac{r_1^2}{2}}\left(\sqrt{\pi}e^{\frac{r_1^2}{2}}Erf\frac{r_1}{\sqrt{2}} - \sqrt{2}r_1\right)}{\sqrt{2}}\right].$$

The second search if $r: r_1 \to r_2$, then the expected value will b as a following:

$$= \pi\left[\frac{e^{-\frac{r_2^2}{2}}\left(\sqrt{\pi}e^{\frac{r_2^2}{2}}Erf\frac{r_2}{\sqrt{2}} - \sqrt{2}r_2\right)}{\sqrt{2}}\right.$$
$$\left. - \frac{e^{-\frac{r_1^2}{2}}\left(\sqrt{\pi}e^{\frac{r_1^2}{2}}Erf\frac{r_1}{\sqrt{2}} - \sqrt{2}r_1\right)}{\sqrt{2}}\right],$$

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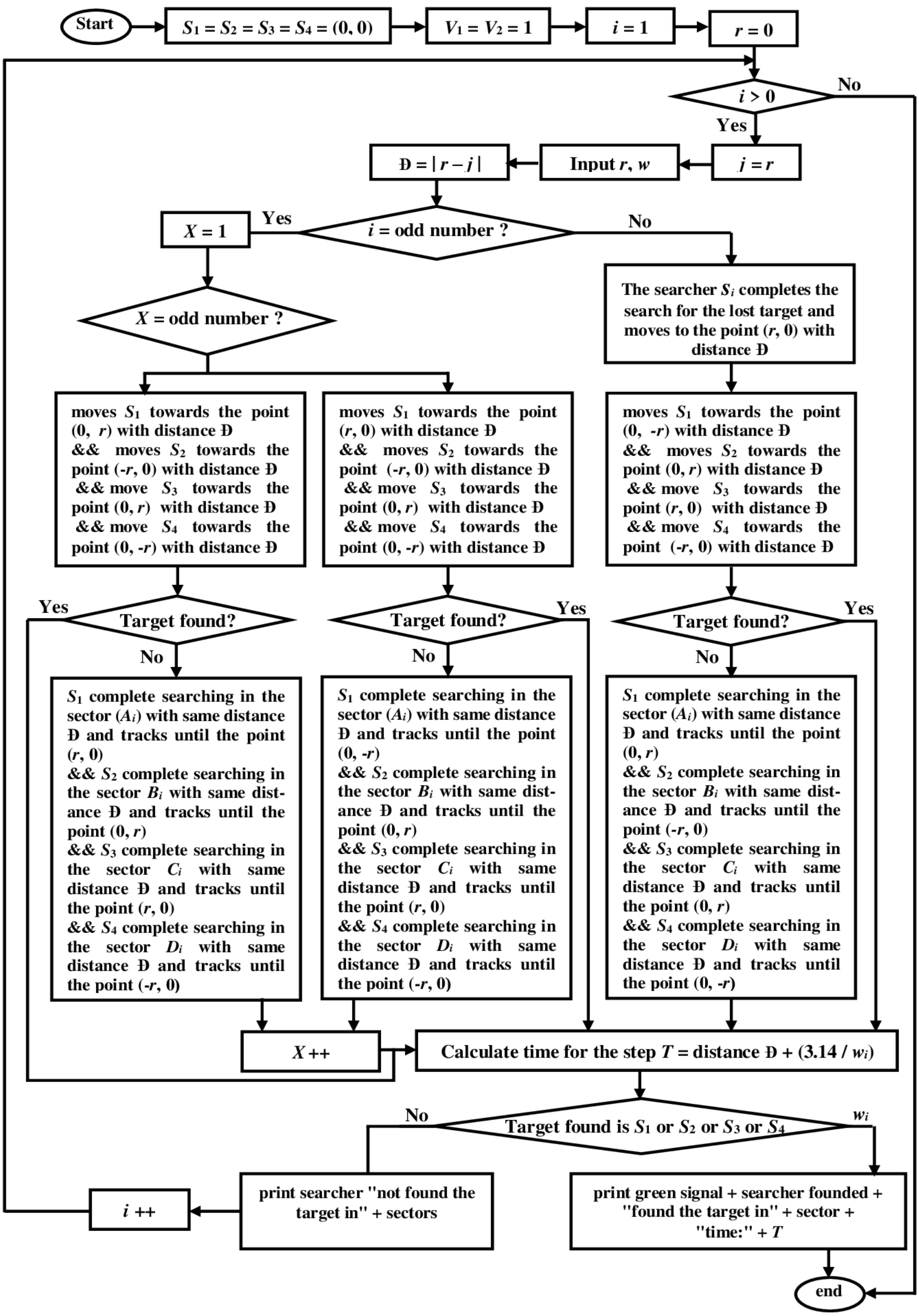

Fig. 3: Flowchart for the quartile symmetric coordinate technique

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$$E(t_\phi) = 2 \cdot \frac{1}{2\pi} \cdot \frac{\pi}{\sqrt{2}} \left[\left(|r_i - r_{i-1}| + \frac{\pi}{w_i} \right) \left(-r_i e^{-\frac{r_i^2}{2}} + \sqrt{\frac{\pi}{2}} Erf \frac{r_i}{\sqrt{2}} + r_{i-1} e^{-\frac{r_1^2}{2}} - \sqrt{\frac{\pi}{2}} Erf \frac{r_{i-1}}{\sqrt{2}} \right) \right],$$

$$E(t_\phi) = \frac{1}{\sqrt{2}} \left[|r_i - r_{i-1}| + \frac{\pi}{w_i} \right] \left[-r_i e^{-\frac{r_i^2}{2}} + r_{i-1} e^{-\frac{r_{i-1}^2}{2}} + \sqrt{\frac{\pi}{2}} \left(Erf \frac{r_i}{\sqrt{2}} - Erf \frac{r_{i-1}}{\sqrt{2}} \right) \right]. \quad (10)$$

Special Case

If

$$r_1 - r_0 = r \quad (11)$$

where $r_0 = 0$, then

$$r_2 - r_1 = r \quad \text{and} \quad r_2 = 2r. \quad (12)$$

Also, $r_3 - r_2 = r \Rightarrow r_3 = 3r$. Hence,

$$E(t_\phi) = \frac{1}{\sqrt{2}} \left[r + \frac{\pi}{w_i} \right] \left[-r_i e^{-\frac{r_i^2}{2}} + (r_i - r) e^{-\frac{(r_i-1)^2}{2}} + \sqrt{\frac{\pi}{2}} \left(Erf \frac{r_i}{\sqrt{2}} - Erf \frac{(r_i - r)}{\sqrt{2}} \right) \right],$$

$$E(t_\phi) = \frac{1}{\sqrt{2}} \left[r + \frac{\pi}{w_i} \right] \left[-ire^{-\frac{(ir)^2}{2}} + r(i-1) e^{-\frac{((r(i-r)))^2}{2}} + \sqrt{\frac{\pi}{2}} \left(Erf \frac{ir}{\sqrt{2}} - Erf \frac{r(i-1)}{\sqrt{2}} \right) \right], \quad (13)$$

By considering the values of i, r and w_i in the Table (1) using the Mathematica program, and substituting in relation (13) we can get the different values of $E(t_\phi)$ as following:

Table 1: Expected Time Value to Decting the Target

i	X	w_i	E_t	i	r	w_i	E_t
1	0	10	0	1	0	30	0
1	2	10	1.51454	1	2	30	1.37753
1	4	10	3.81885	1	4	30	3.63354
1	6	10	5.59564	1	6	30	5.41012
1	8	10	7.36809	1	8	30	7.18257
1	10	10	9.14054	1	10	30	8.95503
1	12	10	10.9130	1	12	30	10.7275
1	14	10	12.6855	1	14	30	12.4999
1	16	10	14.4579	1	16	30	14.2724
2	0	10	0	2	0	30	0
2	2	10	3.36347	2	2	30	3.0592
2	4	10	0.000242	2	4	30	0.00023
2	6	10	0	2	6	30	0
2	8	10	0	2	8	30	0
2	10	10	0	2	10	30	0

Table 1: Continue.

i	X	w_i	E_t	i	r	w_i	E_t
2	12	10	0	2	12	30	0
2	14	10	0	2	14	30	0
2	16	10	0	2	16	30	0
3	0	10	0	3	0	30	0
3	2	10	0.885897	3	2	30	0.885755
3	4	10	0.008187	3	4	30	0.007789
3	6	10	0	3	6	30	0
3	8	10	0	3	8	30	0
3	10	10	0	3	10	30	0
3	12	10	0	3	12	30	0
3	14	10	0	3	14	30	0
3	16	10	0	3	16	30	0
4	0	10	0	4	0	30	0
4	2	10	0.003293	4	2	30	0.002995
4	4	10	36.6055	4	4	30	34.8293
4	6	10	0	4	6	30	0
4	8	10	0	4	8	30	0
4	10	10	0	4	10	30	0
4	12	10	0	4	12	30	0
4	14	10	0	4	14	30	0
4	16	10	0	4	16	30	0
5	0	10	0	5	0	30	0
5	2	10	0	5	2	30	0
5	4	10	0.016373	5	4	30	0.015579
5	6	10	0	5	6	30	0
5	8	10	0	5	8	30	0
5	10	10	0	5	10	30	0
5	12	10	0	5	12	30	0
5	14	10	0	5	14	30	0
5	16	10	0	5	16	30	0
6	0	10	0	6	0	30	0
6	2	10	0	6	2	30	0
6	4	10	0	6	4	30	0
6	6	10	133.94	6	6	30	129.50
6	8	10	0	6	8	30	0
6	10	10	0	6	10	30	0
6	12	10	0	6	12	30	0
6	14	10	0	6	14	30	0
6	16	10	0	6	16	30	0

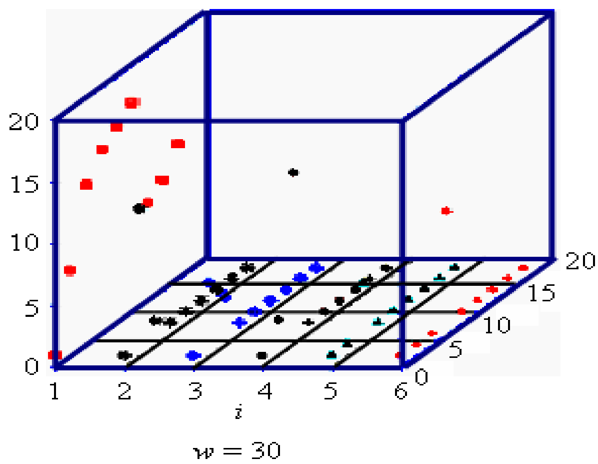

Fig. 4: Expected Time Value to Decting the Target.

5 Conclusion

This research proposes a new quartile coordinated search approach for locating a 3-D landmine that is randomly positioned. In addition, for the lost target in general, we computed the expected time to find the landmine using the Symmetric Coordinated Search technique (SCST). Under the assumption of trivarite standard normality distribution in the cases SCST, we present the best search approach for identifying the landmine. To make searching easier, an approximation technique has been implemented. In three dimensions, the stochastic Euclidian distance between known satellite equations and a randomly traveling metro has been introduced. We also determine coordinated search, a strategy that completely describes the search for a randomly located object on the plane. The goal is to prevent wasting time seeking for a target that has vanished. To find a missing target, four searchers or robots start from the center of a circle.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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