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A New Non-parametric Statistical Test for Testing Exponentiality Based on Kernel Method with Applications on Hypothesis Testing

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Abstract: In this paper, the intention of determining the non-parametric test is to assess the methodology of studying the failure behavior of the observed life data. This methodology represents a new test statistic for testing the exponentiality versus the class of life distribution used better than aged in increasing concave ordering (UBAC(2)) based on the kernel method. Using simulation, the percentiles for complete and right censored data and the power of our test statistic are tabulated. The pitman's asymptotic efficiency is calculated to assess the performance of our proposed test with respect to other tests. Finally, medical applications for real data are presented for complete and censored data using our proposed test.

Keywords: Kernel method, Non-parametric test, Pitman's asymptotic efficiency, Power of the test, UBAC(2) class of life distribution.

1 Introduction and Motivation

In the reliability analysis, the failure behavior of the life data set includes determining if these data have a constant or decreasing (increasing) failure rate. The exponential distribution has two features: constant failure rate and memory-less property. The two features make the exponential distribution the main member in the classes of life distribution. Now, the data set is collected and two claims are considered; the first is H_0: the data are exponential property, and the second is H_1: the data belong to the class of life distribution property and not exponential. Statistical test statistics are necessary to show which of the two claims is right. The classification of the life distributions has contributed to set up a new statistical test with high efficiency. The main aim of constructing new tests is to gain higher efficiencies. The problem of testing exponentiality against various classes of life distributions, such as (IFR, IFRA, DMRL, NBU, NBUE, UBA, UBAE, UBAC and UBACT), has got more attention in the literature [1–7].

Using the kernel method in reliability appeared in the early work of Watson and Leadbetter [8]. It is used in some general goodness of fit problems for testing exponentiality versus the unknown age classes of life distributions successfully, see, among others, [9–11]. Testing the exponentiality against the classes of life distributions DVRL (IVRL), UBAC and UBACT based on the kernel method is introduced in [12–14].

Accordingly, the present paper aims to formulate and address a new test statistic technique for testing exponentiality against UBAC(2) class based on the kernel method.

Suppose a unit with lifetime X having a continuous life distribution $F(x)$, survival function $\bar{F}(x) = 1 - F(x)$ and finite mean $\mu = \int_0^\infty \bar{F}(x)dx$.

Definition (1.1): The idea of the kernel function is introduced in [15] such that:

$$k(x) = \frac{1}{a}k(\frac{x}{a}), \qquad a > 0. \tag{1}$$

Where the kernel function is a probability density function, so

$$\int k(x)dx = 1$$

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Definition (1.2) [16]: The distribution function F is said to be UBAC(2) if:

$$\int_t^{\infty} \bar{F}(u)du - \int_{x+t}^{\infty} \bar{F}(u)du \geq (1-e^{-x})\bar{F}(t). \quad (2)$$

which can be written as :

$$\nu(t) - \nu(x+t) \geq (1-e^{-x})\bar{F}(t). \quad (3)$$

The motivation of the proposed UBAC(2) class of life distributions includes the well-known classes of life distribution as increasing failure rate (IFR), decreasing mean residual life (DMRL) and used better than aged (UBA). Previous researches proved that the UBA class includes the DMRL class [17] and the used better than aged in expectation (UBAE) class is contained in the harmonic used better than aged in expectation class (HUBAE) [18]. From [19], we have:

$$IFR \Rightarrow UBA \Rightarrow UBAC(2)$$

Thus, we have:

$$\begin{array}{ccccccc} IFR & \subset & DMRL & \subset & UBA & \subset & UBAC(2) \\ & & & & \cup & & \\ & & & & UBAE & \subset & HUBAE \end{array}$$

For definitions and properties of these classes, you can check [20–22].

The present paper is organized, as follows: In section 2, a new test statistic based on the kernel method for testing H_0 : F is exponential against H_1 : F is UBAC(2) class of life distribution and not exponential is studied. Pitman's asymptotic efficiency (PAE) of the test for several common distributions are discussed and the power of the test is estimated in section 3. In section 4, testing for censored data is proposed. Finally, medical applications for real data are proposed for complete and censored data using our proposed test in section 5.

2 Testing hypotheses

In this section, a new test statistic based on kernel method for testing:

$$H_0 : \overline{F} is exponential$$

against

$$H_1 : \overline{F} is UBAC(2) and isn't exponential$$

is studied for a random sample $X_1, X_2, ..., X_n$ from a population with distribution function F is proposed.

We proposed the following measure of departure from H_0

$$\delta_{U_K} = \int_0^{\infty}\int_0^{\infty} f(x)[\nu(t) - \nu(x+t) - (1-e^{-x})\bar{F}(t)] \times dF(x)dF(t). \quad (4)$$

Remarks :-

(i) It is easy to see that if F is exponential, then $\delta_{U_K} = 0$,

(ii) Under H_1 , we have $\delta_{U_K} > 0$. Then, to estimate δ_{U_K} by $\hat{\delta}_{U_K}$, let

$\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^{n}(x_j > x)$ denotes the empirical distribution of $\overline{F}(x)$,

$\nu_n(x) = \frac{1}{n}\sum\limits_{j=1}^{n}(x_j - x)I(x_j > x)$ denotes the empirical distribution of $\nu(x)$,

$dF_n(x) = \frac{1}{n}$ denotes the empirical distribution of $dF(x)$, and pdf $f(x)$ is estimated by $\hat{f}_n(x) = \frac{1}{na_n}\sum_{i=1}^{n} k(\frac{x-x_i}{a_n})$, where k(.) is a known pdf. Then,

$$\hat{\delta}_{U_K} = \int_0^{\infty}\int_0^{\infty} \hat{f}_n(x)[\nu_n(t) - \nu_n(x+t) - (1-e^{-x})\bar{F}_n(t)] \times dF_n(x)dF_n(t)$$

$$= \frac{1}{n^4 a}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{m=1}^{n}\sum_{p=1}^{n} k(\frac{X_i - X_p}{a_n})[(X_m - X_j)I(X_m > X_j) - (X_m - X_i - X_j)I(X_m > X_i + X_j) - \frac{1}{2}(1-e^{-X_i})]. \quad (5)$$

where,

$$I(y > t) = \begin{cases} 1 & if\ y > t \\ 0, & O.W \end{cases}$$

let us rewrite (5), as follows:

$$\hat{\delta}_{U_K} = \frac{1}{n^4 a}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{m=1}^{n}\sum_{p=1}^{n} \phi(X_i, X_j, X_m, X_p),$$

where,

$$\phi(X_i, X_j, X_m, X_p) = k(\frac{X_i - X_p}{a_n})[(X_m - X_j)I(X_m > X_j) - (X_m - X_i - X_j)I(X_m > X_i + X_j) - \frac{1}{2}(1-e^{-X_i})]. (6)$$

To make the test scale invariant, we take,

$$\hat{\Delta}_{U_K} = \frac{\hat{\delta}_{U_K}}{\bar{x}} \quad (7)$$

Then, $\hat{\Delta}_{U_K}$ in (7) is equivalent to the U-statistics and The following theorem summarizes the large sample properties of $\hat{\Delta}_{U_K}$.

To use the U-statistic procedure, we set

$$\phi(X_1,X_2,X_3,X_4) = k(\frac{X_1-X_4}{a_n})[(X_3-X_2)I(X_3>X_2) - (X_3-X_1-X_2)I(X_3>X_1+X_2) - \frac{1}{2}(1-e^{-X_1})]. \quad (8)$$

Here, X_1, X_2, X_3 and X_4 are four independent lifetimes, and each with distribution function F. We define the symmetric kernel as :

$$\phi(X_1,X_2,X_3,X_4) = \frac{1}{4!}\sum_{\Re}\phi(X_1,X_2,X_3,X_4) \quad (9)$$

Where the sum is over all permutations of X_1, X_2, X_3 and X_4 .Then, $\hat{\delta}_{U_K}$ is equivalent to the U-statistic

$$U_n = \frac{1}{\binom{n}{4!}}\sum_{i<j<m<p}\phi(X_1,X_2,X_3,X_4) \quad (10)$$

Theorem 1 If $n^4 a_n \to \infty$ as $n\to\infty$, $\sqrt{n}(\hat{\delta}_{U_k}-\delta_{U_k})/\sigma$ is convergence asymptotically normal with mean 0 and variance $\sigma^2 = var[\phi(X_i,X_j,X_m,X_p)]$, where $\phi(X_i,X_j,X_m,X_p)$ is given in (6).

Under H_0

$$\sigma^2 = var[2f(x)[\int_0^\infty\int_0^v (v-u)f(u)f(v)dudv - \int_X^\infty\int_0^{v-X}(v-X-u)f(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-X})f(u)f(v)dudv] + \int_X^\infty\int_0^\infty(v-X)f^2(u)f(v)dudv - \int_X^\infty\int_0^{v-X}(v-X-u)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv + \int_0^X\int_0^\infty(X-v)f^2(u)f(v)dudv - \int_0^x\int_0^{x-v}(X-u-v)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv]. \quad (11)$$

Proof:
To compute σ^2, we must compute :

$$\phi_1(X_1) = E[\phi(X_1,X_2,X_3,X_4|X_1)] = f(x_1)[\int_0^\infty\int_0^v(v-u)f(u)f(v)dudv - \int_{X_1}^\infty\int_0^{v-X_1}(v-X_1-u)f(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-X_1})f(u)f(v)dudv]. \quad (12)$$

$$\phi_2(X_2) = E[\phi(X_1,X_2,X_3,X_4|X_2)] = \int_{X_2}^\infty\int_0^\infty(v-X_2)f^2(u)f(v)dudv - \int_{X_2}^\infty\int_0^{v-X_2}(v-X_2-u)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv. \quad (13)$$

$$\phi_3(X_3) = E[\phi(X_1,X_2,X_3,X_4|X_3)] = \int_0^{X_3}\int_0^\infty(X_3-v)f^2(u)f(v)dudv - \int_0^{X_3}\int_0^{X_3-v}(X_3-u-v)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv. \quad (14)$$

$$\phi_4(X_4) = E[\phi(X_1,X_2,X_3,X_4|X_4)] = f(x_4)[\int_0^\infty\int_0^v(v-u)f(u)f(v)dudv - \int_{X_4}^\infty\int_0^{v-X_4}(v-X_4-u)f(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-X_4})f(u)f(v)dudv]. \quad (15)$$

Due to the fact that the variables X_1, X_2, X_3 and X_4 are independent and identical, they can be written in short X instead of X_1, X_2, X_3 and X_4.
set :

$$\zeta(X) = \phi_1(X)+\phi_2(X)+\phi_3(X)+\phi_4(X) = 2f(x)[\int_0^\infty\int_0^v(v-u)f(u)f(v)dudv - \int_X^\infty\int_0^{v-X}(v-X-u)f(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-X})f(u)f(v)dudv] + \int_X^\infty\int_0^\infty(v-X)f^2(u)f(v)dudv - \int_X^\infty\int_0^{v-X}(v-X-u)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv + \int_0^X\int_0^\infty(X-v)f^2(u)f(v)dudv - \int_0^x\int_0^{x-v}(X-u-v)f^2(u)f(v)dudv - \frac{1}{2}\int_0^\infty\int_0^\infty(1-e^{-u})f^2(u)f(v)dudv. \quad (16)$$

By direct calculation,, under H_0: $f(u) = e^{-u}$, $f(v) = e^{-v}$, $f(x) = e^{-x}$, then

$$E[\zeta(X)] = E[\phi_1(X) + \phi_2(X) + \phi_3(X) + \phi_4(X)] = 0. \quad (17)$$

$$\sigma^2 = var[\zeta(X)] = \frac{1}{1080}. \quad (18)$$

To use the above test, calculate $\sqrt{n}\hat{\Delta}_{U_K}/\sigma$ and reject H_0 if this exceeds the normal variate value $Z_{1-\alpha}$. To illustrate the test, we calculate the empirical critical values of $\hat{\Delta}_{U_K}$ in (7) for sample sizes 5(5)100 by using monte carlo method. Table 1 presents the percentile points for 1%, 5%, 10%, 90%, 95%, 99%. The calculations are based on 10000 simulated samples of sizes n = 5(5)100. These values will be the criteria for dividing the samples space into acceptance or rejection region for H_0.

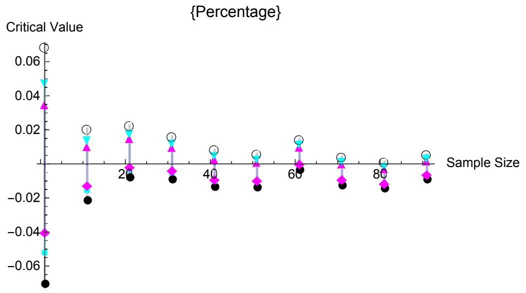

Fig. 1: The relation between sample size and critical values

Table 1 and Fig. 1 indicate that the values of the percentiles change slowly as n increases.

3 Pitman's asymptotic efficiency

In this section, we calculate PAE for UBAC(2) class of life distributions and compare our proposed test with tests of other well-known classes of life distribution based on PAE.

Here, we choose K^* for DMRL [23], $\hat{\delta}_2$ for UBAE [21], $\hat{\Delta}_{UK}$ for UBACT [14] and Λ_n for overall decreasing life in Laplace transform (ODL) [24].

PAE of $\hat{\delta}_{U_K}$ is given by :

$$PAE(\delta_{U_K}(\theta)) = \frac{1}{\sigma}\left|\frac{d}{d\theta}\delta_{U_K}(\theta)\right|_{\theta\to\theta_0}$$
$$= \frac{1}{\sigma}\left|\int_0^\infty\int_0^\infty \grave{f}_\theta(x)[\grave{\nu}_\theta(t) - \grave{\nu}_\theta(x+t) - (1-e^{-x})\grave{\bar{F}}_\theta(t)]d\grave{F}_\theta(x)d\grave{F}_\theta(t)\right|_{\theta\to\theta_0}. \quad (19)$$

Two of the most commonly used alternatives (see [26]) are:

(i) LFR : $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}$, $\quad x > 0, \theta > 0$
(ii) Makeham : $\bar{F}_\theta = e^{-x-\theta(x+e^{-x}-1)}$, $\quad x \geq 0, \theta > 0$

For LFR family :
Under H_0, $\theta = 0$, then :

$$\grave{\nu}_\theta(t) = -e^{-t} - te^{-t} - \frac{t^2}{2}e^{-t}. \quad (20)$$

$$\grave{\nu}_\theta(x+t) = -e^{-(x+t)} - (x+t)e^{-(x+t)} - \frac{(x+t)^2}{2}e^{-(x+t)}. \quad (21)$$

$$\grave{\bar{F}}_\theta(t) = -\frac{t^2}{2}e^{-t}. \quad (22)$$

$$\grave{f}_\theta(x) = e^{-x}. \quad (23)$$

Then, by direct calculation from equation (20), (21), (22), (23) in (19),

$$PAE(\delta_{U_K}(\theta)) \cong 1.29 \quad (24)$$

For Makeham family :
Under H_0, $\theta = 0$, then :

$$\grave{\nu}_\theta(t) = -te^{-t} - \frac{1}{2}e^{-2t}. \quad (25)$$

$$\grave{\nu}_\theta(x+t) = -(x+t)e^{-(x+t)} - \frac{1}{2}e^{-2(x+t)}. \quad (26)$$

$$\grave{\bar{F}}_\theta(t) = (1-t-e^{-t})e^{-t}. \quad (27)$$

$$\grave{f}_\theta(x) = e^{-x}. \quad (28)$$

Then, by direct calculation from equation (25), (26), (27), (28) in (19),

$$PAE(\delta_{U_K}(\theta)) \cong 0.456 \quad (29)$$

The null hypothesis is at $\theta = 0$ for LFR and makeham distributions, respectively. Direct calculations of PAE for $\hat{\delta}_2$, K^*,$\hat{\Delta}_{Uk}$, Λ_n and $\hat{\delta}_{U_K}$ are summarized in Table 2.

These calculations indicate that our proposed test is more efficient compared with K^*, $\hat{\delta}_2$, $\hat{\Delta}_{Uk}$ and Λ_n for both LFR and makeham families.

Finally, the power of the test statistic $\hat{\delta}_{U_K}$ is considered for 95% percentiles in Table 3 for three of the most widely used alternatives (see [25]) as follows:

(i) LFR : $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}$, $\quad x > 0, \theta > 0$
(ii) Makeham : $\bar{F}_\theta = e^{-x-\theta(x+e^{-x}-1)}$, $\quad x \geq 0, \theta > 0$
(iii) Weibull : $\bar{F}_\theta = e^{-x^\theta}$, $\quad x \geq 0, \theta > 0$

These distributions are reduced to the exponential distribution for appropriate values of θ .

Table 3 shows that power estimates increased when θ increased for LFR, makeham and weibull families.

Table 1: Critical values of $\hat{\Delta}_{U_K}$

n	1%	5%	10%	90%	95%	99%
5	-0.03596	-0.0266384	-0.0216713	0.0131659	0.0181329	0.0274545
10	-0.0144965	-0.00790512	-0.00439291	0.0202407	0.0237529	0.0303443
15	-0.0212876	-0.0159057	-0.013038	0.00707527	0.00994298	0.0153249
20	-0.0208338	-0.016173	-0.0136895	0.00372913	0.00621264	0.0108735
25	-0.00585595	-0.00168717	0.000534147	0.0161138	0.0183351	0.0225039
30	-0.0100158	-0.0062102	-0.00418242	0.0100398	0.0120676	0.0158731
35	-0.0163574	-0.0128341	-0.0109568	0.00221043	0.00408779	0.00761105
40	-0.0103138	-0.00701814	-0.00526203	0.00705477	0.00881088	0.0121066
45	-0.0128623	-0.00975503	-0.00809935	0.00351304	0.00516872	0.00827594
50	-0.0090716	-0.00612383	-0.00455312	0.00646336	0.00803407	0.0109818
55	-0.00596529	-0.0031547	-0.00165709	0.00884672	0.0103443	0.0131549
60	-0.00259273	0.0000982023	0.00153206	0.0115887	0.0130225	0.0157135
65	-0.013207	-0.0106217	-0.00924405	0.00041805	0.00179565	0.00438102
70	-0.0160994	-0.0136081	-0.0122806	-0.00296996	-0.00164247	0.000848853
75	-0.00563782	-0.00323097	-0.00194849	0.00704643	0.00832891	0.0107358
80	-0.00337443	-0.00104401	0.000197747	0.00890704	0.0101488	0.0124792
90	-0.00569157	-0.00349443	-0.0023237	0.00588751	0.00705825	0.00925539
95	0.000270603	0.00240914	0.00354865	0.0115409	0.0126804	0.0148189
100	-0.000852038	0.00123235	0.00234301	0.0101328	0.0112435	0.0133279

Table 2: PAE of $\hat{\delta}_{U_K}$

Distribution	K^*	$\hat{\delta}_2$	$\hat{\Delta}_{Uk}$	Λ_n	$\hat{\delta}_{U_K}$
F_1 LFR	0.81	0.63	0.776	0.982	1.29
F_2 Makeham	0.29	0.385	0.245	0.218	0.456

Table 3: Power estimate of $\hat{\delta}_{U_K}$

Distribution	θ	Sample Size		
		n=10	n=20	n=30
F_1 LFR	2	1	1	1
	3	1	1	1
	4	1	1	1
F_2 Makham	2	0.99987	1	1
	3	1	1	1
	4	1	1	1
F_3 Weibull	2	1	1	1
	3	1	1	1
	4	1	1	1

4 Testing for Censored Data

Censored data are usually the only information available in a life testing model or in a clinical study where patients may be lost (censored) before the completion of the study. This experimental situation can be modeled, as follows: Suppose n items are put on test, and $X_1, X_2, \ldots, X_n$ are independent and identically distributed (i.i.d) random variables according to a continuous life distribution F which denotes their true lifetime. Let $Y_1, Y_2, \ldots, Y_n$ be (i.i.d) according to a continuous life distribution G and assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_i, δ_i), $i = 1, \ldots, n$, where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1, & if\ Z_i = X_i \quad (i-th\ observation\ is\ uncensored) \\ 0, & if\ Z_i = Y_i \quad (i-th\ observation\ is\ censored) \end{cases}$$

Let $Z_{(0)} < Z_{(1)} < \ldots < Z_{(n)}$ denoting the ordered of Z's and δ_i be the δ corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier estimator [26] in the case of censored data (Z_i, δ_i), $i = 1, 2, \ldots, n$, then the proposed test statistic given by (5) can be written using right-censored data as

$$\delta^{\hat{c}}_{U_K} = \sum_{k=1}^{n} \sum_{k=1}^{n} \hat{f}_\theta(x)[\hat{v}_\theta(t) - \hat{v}_\theta(x+t) - (1 - e^{-Z_{(j)}})\hat{\bar{F}}_\theta(t)] \times [\prod_{p=1}^{i-2} C_m^{\delta(i)} - \prod_{p=1}^{i-1} C_m^{\delta(i)}][\prod_{q=1}^{j-2} C_q^{\delta(j)} - \prod_{q=1}^{j-1} C_q^{\delta(j)}] \tag{30}$$

where

$$c_k = \frac{n-k}{n-k+1}$$

To make the test invariant, let

$$\Delta^{\hat{c}}_{U_K} = \frac{\delta^{\hat{c}}_{U_K}}{\bar{Z}}, \bar{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_i. \tag{31}$$

Table 4 shows the critical values percentiles of $\Delta^{\hat{c}}_{U_K}$ for sample size n=2(2)20(10)100.

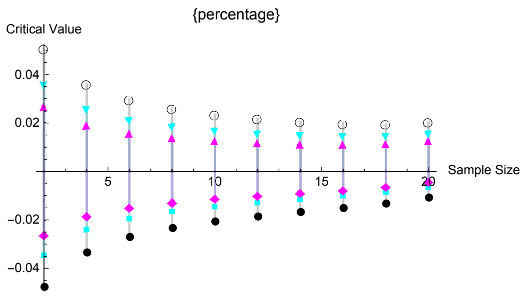

Fig. 2: The relation between sample size and critical values

Table 4: Critical values of $\Delta^{\hat{c}}_{U_K}$

n	90%	95%	99%
2	0.0279112	0.0357647	0.0505036
4	0.0199001	0.0254534	0.0358754
6	0.0163964	0.0209306	0.0294401
8	0.0143558	0.0182826	0.025652
10	0.0130204	0.0165326	0.023124
12	0.0121091	0.0153153	0.0213324
14	0.0115064	0.0144748	0.0200455
16	0.0111866	0.0139632	0.0191742
18	0.0112193	0.0138371	0.0187501
20	0.0118994	0.0143829	0.0190438
30	0.00711111	0.00913889	0.0129444
40	0.0061584	0.00791451	0.0112102
50	0.0127279	0.0142986	0.0172464
60	0.00615441	0.00758826	0.0102792
70	0.0051514	0.00647889	0.00897021
80	0.0046384	0.00588016	0.00821057
90	0.00429032	0.00546106	0.00765819
100	0.00402505	0.0051357	0.00722009

According to Table 4 and Fig. 2, the critical values decrease when the sample size increases. These values will be the criteria for dividing the samples space into acceptance or rejection region for H_0.

5 Applying the test

5.1 Applications for Complete Data

Application 1
The following data represent 39 patients with liver cancer which were taken from El Minia Cancer Center, Ministry of Health Egypt [27]. The ordered life times (in days) are:

107 , 18 , 74 , 20 , 23 , 20 , 23 , 24 , 52 , 105 , 60 , 31 , 75 , 107 , 71 , 107 , 14 , 49 , 10 , 15 , 30 , 26 , 14 , 87 , 51 , 17 , 116 , 67 , 20 , 14 , 40 , 14 , 30 , 96 , 20 , 20 , 61 , 150 , 14.

Using equation (7), the value of test statistic based on the above data is $\hat{\Delta}_{U_K} = -113.039$. The critical value at $\alpha = 0.05$ is 0.0189533, then we accept H_0 at $\alpha = 0.05$. Therefore, the data doesn't have UBAC(2) Property.

Application 2
In an experiment at Florida state university to study the effect of methyl mercury poisoning on the length of life of fish goldfish where subjected to various dosages of methyl mercury [28]. At one dosage level, the ordered times to death in a week are:

$6, 6.143, 7.286, 8.714, 9.429, 9.857, 10.143, 11.571, 11.714, 11.714$

The value of test statistic based on the above data is $\hat{\Delta}_{U_K} = 0.787771$. The critical value at $\alpha = 0.05$ is 0.0237529. Then, H_0 at the significance level $\alpha = 0.05$ is rejected. Thus, the data have UBAC(2) Property.

5.2 Applications for Censored Data

Application 1
Consider the following data in [29] that represent 51 patients with liver cancer taken from the El Minia Cancer Center, Ministry of Health in Egypt. (39) represent whole lifetimes (non-censored data), while the others represent censored data. The ordered lifetimes (in days) are:

(i) Non-censored data
$10, 14, 14, 14, 14, 14, 15, 17, 18, 20, 20, 20, 20, 20, 23, 23, 24, 26, 30, 30, 31, 40, 49, 51, 52, 60, 61, 67, 71, 74, 75, 87, 96, 105, 107, 107, 107, 116, 150.$

(ii) Censored data
$30, 30, 30, 30, 30, 60, 150, 150, 150, 150, 150, 185.$

It was detected that the test statistic for the set of data $\Delta^{\hat{c}}_{U_K} = 0.00700921$. The critical value is 0.00998176, so we accept H_0 which states that the set of data doesn't have UBAC(2) Property under significant level $\alpha = 0.5$.

6 Conclusion

We derived a new test statistic for testing the exponentiality against the UBAC(2) class of life distributions based on the technique of the kernel method. The critical values of this test statistic are calculated and tabulated in Tables 1 and 2 respectively for uncensored and censored data. The PAE of the test statistics is computed and compared with test statistics of other well-known classes of life distribution. Moreover, the power estimates of our test statistic are simulated for most widely used families in reliability. It is shown that our proposed test statistics is more efficient and involves good power. Finally, some sets of data are used to elucidate the applications of the UBAC(2) property to real data.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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