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An Inexact Rough Interval of Normalized Heptagonal Fuzzy Numbers for Solving Vendor Selection Problem

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Abstract: Vendor selection problem is considered very complicated because a variety of unpredictable and uncontrollable factors affected the evaluation and decision making process. In this paper, a vendor selection problem (VSP) where the buyer allocates a quantity order for a commodity among a set of supplier to achieve the requirements of aggregate cost, service and lead time at the maximum equality is studied. One of the best inexact intervals, namely an inexact rough interval of normalized heptagonal fuzzy numbers is proposed. An inexact rough interval of normalized heptagonal fuzzy numbers for solving VSP without converting it into a deterministic (crisp) problem is developed. An inexact rough interval for parameters represents its dual uncertainty. A solution method for solving HFNVSP is introduced. In the end, an example is solved to clarify the proposed method.

Keywords: Vendor selection problem, Heptagonal fuzzy number, Normalized heptagonal fuzzy number, Rough interval approximation

1 Introduction

Vendor selection problem (VSP) is significantly important for the effectiveness of management. This leads to the evaluation of strategic alliances with the vendors. Thereby, the optimization of vendor- base is essential to determine the best performing vendors in a supply chain management [1]. Different methods have been used for VS as:

- Linear weighting method;
- Statistical methods;
- Mathematical programming models.

Linear weighting method is one of the most widely used for VSP. Many authors have used these methods for VSPs (for instance, [2,3,4] and many others). Multiple criteria vendor service factor ratings investigated in [5]. An extensive state-of-the-art literature review related to supplier selection problem (SSP) over the past two decades has been provided by [6]. A comprehensive literature review of SSP with order allocation from year 2000 to 2017 has been introduced by [7]. Some new scientific decision-making layouts for effective selection of clouds vendor for an industry have been investigated by [8] and have compared the obtained results with the classical method based on the sensitivity analysis. A literature survey on reverse logistics SSP in terms of criteria and methods has been presented by [9].

In literature, [10] first proposed the philosophy of fuzzy sets. Decision-making in a fuzzy environment, developed by [11], have been an improvement and a great help in the management decision problems. Fuzzy programming and linear programming with multiple objective functions has introduced (see, [12]). Later, several researchers addressed fuzzy set theory. [13] explored the theory and applications of fuzzy sets and systems. [14] studied several fuzzy mathematical models with their applications to engineering and management sciences. [15] proposed a very effective method for solving linear programming with fuzzy variables based on the comparison of fuzzy numbers. Many authors [16,17, 18,19,20] considered the situations where all the parameters are uncertain.

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In the past few decades, several authors presented their works using z-transform and mathematical programming. [21] presented a supplier selection model and applied z-transformation to solve the model. [22] studied the supplier selection model by considering various empirical cases. [23] formulated a model for the total cost of ownership supplier selection model. They studied their model under the criteria of activity-based costing. In addition, they applied mathematical programming approach to deal with the model. [24] studied the supplier selection model. They considered the multiple criteria assumption in volume discount policy.

In literature, numerous authors considered the stochastic supply selection model. [25] investigated the stochastic vendor selection problem. They applied the chance-constrained approach as well as the genetic algorithm to validate the solutions. VSP has been studied by applying an efficient fuzzy based multi-objective technique [26], and they considered the modified S- curve membership function. An inexact rough-interval fuzzy linear programming technique has been proposed by [27], and a conjunctive water-allocation strategy with an application to the irrigation field has been studied. The multi-objective supplier selection problem has been developed by [28]. Some criteria for strategic decisions on vendor selection model have been investigated by [29].

An interactive fuzzy programming method to solve the vender selection model, where fuzzy parameters have been considered in supply chain model, has been treated by [30]. [31] presented a decision making approach to deal with vendor selection model, and have considered the uncertain inputs in their model formulation. [32] suggested a nadir compromise programming method to deal with the supplier selection model with uncertainty.[33] presented a study on green supplier selection. They considered the dynamic environment in their model development. [34] presented a preference approach to the supplier selection model. They applied the concept of hesitant fuzzy sets in their study. Recently, [35] addressed a supplier selection model with the applications to hospital. They applied a combination of fuzzy VIKOR and neural network. [36] proposed a novel approach based on the fuzzy-stochastic compromise ratio, and applied the approach to the green supplier selection model under stochastic statistical input data. [37] presented a multi-supplier joint replenishment inventory model. They considered the deterioration as well as the quantity discount policy in the supplier selection model.

In this paper, a new method for solving an inexact rough interval is proposed without converting the rough coefficients into its crisp. The method converts the problem into four classical LP problems, and each of them can be easily solved even manually.

The rest of the paper is organized, as follows: **Section 2** introduces some preliminaries needed in this paper. **Section 3** presents an inexact rough interval vendor selection problem formulation. **Section 4** proposes a solution method for solving the problem introduced in Section 3. **Section 5** introduces a numerical example to illustrate the solution method. Finally, some concluding remarks are reported in **Section 6**.

2 Preliminaries

This section presents some basic concepts and results based on fuzzy numbers, heptagonal fuzzy numbers, as well as inexact rough interval approximation and their arithmetic operations are recalled.

Definition 2.1. A fuzzy set $\widetilde{P}$ defined on the set of real numbers $\mathbb{R}$ is said to be fuzzy numbers if its membership function $\mu_{\widetilde{P}}(x) : \mathbb{R} \to [0,1]$ has the following properties:

1. $\mu_{\widetilde{P}}(x)$ is an upper semi-continuous membership function;
2. $\widetilde{P}$ is convex fuzzy set, i. e., $\mu_{\widetilde{P}}(\delta x + (1-\delta)y) \geq \min\{\mu_{\widetilde{P}}(x), \mu_{\widetilde{P}}(y)\}$ for all $x, y \in \mathbb{R}$; $0 \leq \delta \leq 1$;
3. $\widetilde{P}$ is normal, i. e., $\exists x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{P}}(x_0) = 1$;
4. $\text{supp}(\widetilde{P}) = \{x \in \mathbb{R} : \mu_{\widetilde{P}}(x) > 0\}$ is the support of $\widetilde{P}$, and the closure $cl(\text{supp}(\widetilde{P}))$ is a compact.

Definition 2.2. [38]. A fuzzy number $\widetilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is a heptagonal fuzzy number (HFN) whereas $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \in \mathbb{R}$ and its membership function is defined by (Fig. 1)

$$\mu_{\widetilde{A}_H}(x) = \begin{cases} \frac{1}{3}\left(\frac{x-a_1}{a_2-a_1}\right) & \text{for } a_1 \leq x \leq a_2, \\ \frac{1}{3} + \frac{1}{3}\left(\frac{x-a_2}{a_3-a_2}\right) & \text{for } a_2 \leq x \leq a_3, \\ \frac{2}{3} + \frac{1}{3}\left(\frac{x-a_3}{a_4-a_3}\right) & \text{for } a_3 \leq x \leq a_4, \\ 1 - \frac{1}{3}\left(\frac{x-a_4}{a_5-a_4}\right) & \text{for } a_4 \leq x \leq a_5, \\ \frac{2}{3} - \frac{1}{3}\left(\frac{x-a_5}{a_6-a_5}\right) & \text{for } a_5 \leq x \leq a_6, \\ \frac{1}{3}\left(\frac{a_7-x}{a_7-a_6}\right) & \text{for } a_6 \leq x \leq a_7, \\ 0, & \text{for } x < a_1 \text{ and } x > a_7. \end{cases}$$

A HFN can be characterized by the so-called interval of confidence at level α as

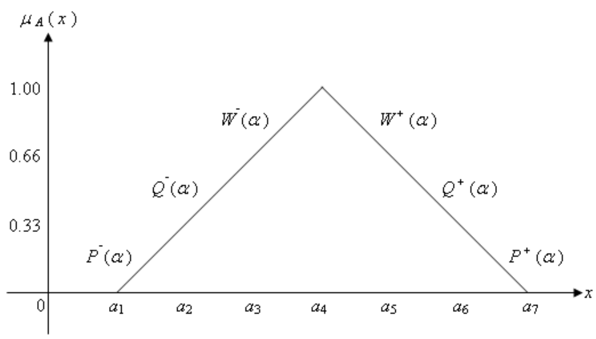

Fig. 1: Heptagonal fuzzy number

$$\widetilde{A}_{H\alpha}(x) = \{x \in X : \mu_{\widetilde{A}_H} \geq \alpha\}$$
$$= \begin{cases} [P^-(\alpha), P^+(\alpha)] & \text{for } \alpha \in \left[0, \frac{1}{3}\right], \\ [Q^-(\alpha), Q^+(\alpha)] & \text{for } \alpha \in \left[\frac{1}{3}, \frac{2}{3}\right], \\ [W^-(\alpha), W^+(\alpha)] & \text{for } \alpha \in \left[\frac{2}{3}, 1\right], \end{cases}$$

Definition 2.3. Let $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ and $\widetilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$. Then,

Addition: $\widetilde{A}_H \oplus \widetilde{B}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7) \oplus (b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$.

Subtraction: $\widetilde{A}_H \ominus \widetilde{B}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7) \ominus (b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$.

Scalar multiplication:

$$k\widetilde{A}_H = \begin{cases} k(a_1, a_2, a_3, a_4, a_5, a_6, a_7), \ k \geq 0, \\ k(a_7, a_6, a_5, a_4, a_3, a_2, a_1), \ k < 0. \end{cases}$$

Definition 2.4. A rough interval approximation x^R of normalized heptagonal fuzzy number $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is defined as an interval with known lower and upper bounds, while the distribution information for x is unknown:

$$A^R = \left[A_\alpha^{(\mathrm{UAI})} : A_\alpha^{(\mathrm{LAI})}\right] \tag{1}$$

where, $A_\alpha^{(\mathrm{LAI})} = \inf\left\{x \in \mathbb{R} : \mu_{\widetilde{A}} \geq \frac{1}{3}\right\}$, and $A_\alpha^{(\mathrm{UAI})} = \sup\left\{x \in \mathbb{R} : \mu_{\widetilde{A}} \geq \frac{1}{3}\right\}$, are the upper and lower approximation intervals of A^R, respectively.

If $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$, the rough interval of $\widetilde{A}_H$ is $A^R = \left[[a_2, a_6] : [a_3, a_5]\right]$.

Definition 2.5. Let $A^R = \left[A_\alpha^{(\mathrm{UAI})} : A_\alpha^{(\mathrm{LAI})}\right]$, and $B^R = \left[B_\alpha^{(\mathrm{UAI})} : B_\alpha^{(\mathrm{LAI})}\right]$ be two rough intervals, $A^R > 0$ and $B^R > 0$. The arithmetic operations $\{+, -, \times, \div\}$ are defined as:

$$1.\ A^R \oplus B^R = \left[A_\alpha^{(\mathrm{UAI})} + B_\alpha^{(\mathrm{UAI})}\right] : \left[A_\alpha^{(\mathrm{LAI})} + B_\alpha^{(\mathrm{LAI})}\right], \tag{2}$$

$$2.\ A^R \ominus B^R = \left[A_\alpha^{(\mathrm{UAI})} - B_\alpha^{(\mathrm{UAI})}\right] : \left[A_\alpha^{(\mathrm{LAI})} - B_\alpha^{(\mathrm{LAI})}\right], \tag{3}$$

$$3.\ A^R \otimes B^R = \left[A_\alpha^{(\mathrm{UAI})} \times -B_\alpha^{(\mathrm{UAI})}\right] : \left[A_\alpha^{(\mathrm{LAI})} \times B_\alpha^{(\mathrm{LAI})}\right], \tag{4}$$

$$4.\ A^R \oslash B^R = \left[A_\alpha^{(\mathrm{UAI})} / B_\alpha^{(\mathrm{UAI})}\right] : \left[A_\alpha^{(\mathrm{LAI})} / B_\alpha^{(\mathrm{LAI})}\right], \tag{5}$$

If $A_\alpha^{(\mathrm{UAI})} = \left[A_\alpha^{-(\mathrm{UAI})}, A_\alpha^{+(\mathrm{UAI})}\right]$, $A_\alpha^{(\mathrm{LAI})} = \left[A_\alpha^{-(\mathrm{LAI})}, A_\alpha^{+(\mathrm{LAI})}\right]$, $B_\alpha^{(\mathrm{UAI})} = \left[B_\alpha^{-(\mathrm{UAI})}, B_\alpha^{+(\mathrm{UAI})}\right]$, $B_\alpha^{(\mathrm{LAI})} = \left[B_\alpha^{-(\mathrm{LAI})}, B_\alpha^{+(\mathrm{LAI})}\right]$, where $A_\alpha^{-(\mathrm{UAI})}$, $A_\alpha^{+(\mathrm{UAI})}$, $A_\alpha^{-(\mathrm{LAI})}$, $A_\alpha^{+(\mathrm{LAI})}$, $B_\alpha^{-(\mathrm{UAI})}$, $B_\alpha^{+(\mathrm{UAI})}$, $B_\alpha^{-(\mathrm{LAI})}$, $B_\alpha^{+(\mathrm{LAI})}$, are the deterministic numbers as well as the lower and upper bounds of $A_\alpha^{(\mathrm{UAI})}$, $A_\alpha^{(\mathrm{LAI})}$, $B_\alpha^{(\mathrm{UAI})}$, and $B_\alpha^{(\mathrm{LAI})}$, respectively. Then, Equations (2)-(4) become:

$$1.\ A^R \oplus B^R = \left[A_\alpha^{-(\mathrm{UAI})} + B_\alpha^{-(\mathrm{UAI})}, A_\alpha^{+(\mathrm{UAI})} + B_\alpha^{+(\mathrm{UAI})}\right] : \left[A_\alpha^{-(\mathrm{LAI})} + B_\alpha^{-(\mathrm{LAI})}, A_\alpha^{+(\mathrm{LAI})} + B_\alpha^{+(\mathrm{LAI})}\right], \tag{2'}$$

$$2.\ A^R \ominus B^R = \left[A_\alpha^{-(\mathrm{UAI})} - B_\alpha^{-(\mathrm{UAI})}, A_\alpha^{+(\mathrm{UAI})} - B_\alpha^{+(\mathrm{UAI})}\right] : \left[A_\alpha^{-(\mathrm{LAI})} - B_\alpha^{-(\mathrm{LAI})}, A_\alpha^{+(\mathrm{LAI})} - B_\alpha^{+(\mathrm{LAI})}\right], \tag{3'}$$

$$3.\ A^R \otimes B^R = \left[A_\alpha^{-(\mathrm{UAI})} \times B_\alpha^{-(\mathrm{UAI})}, A_\alpha^{+(\mathrm{UAI})} \times B_\alpha^{+(\mathrm{UAI})}\right] : \left[A_\alpha^{-(\mathrm{LAI})} \times B_\alpha^{-(\mathrm{LAI})}, A_\alpha^{+(\mathrm{LAI})} \times B_\alpha^{+(\mathrm{LAI})}\right], \tag{4'}$$

$$4.\ A^R \oslash B^R = \left[A_\alpha^{-(\mathrm{UAI})} / B_\alpha^{-(\mathrm{UAI})}, A_\alpha^{+(\mathrm{UAI})} / B_\alpha^{+(\mathrm{UAI})}\right] : \left[A_\alpha^{-(\mathrm{LAI})} / B_\alpha^{-(\mathrm{LAI})}, A_\alpha^{+(\mathrm{LAI})} / B_\alpha^{+(\mathrm{LAI})}\right], \tag{5'}$$

Definition 2.6. For $A^R = \left[A_\alpha^{(\mathrm{UAI})} : A_\alpha^{(\mathrm{LAI})}\right]$, and $B^R = \left[B_\alpha^{(\mathrm{UAI})} : B_\alpha^{(\mathrm{LAI})}\right]$, their order relations are, as follows:

$$1.\ A^R \leq B^R | \Leftrightarrow A_\alpha^{+(\mathrm{UAI})} \leq B_\alpha^{+(\mathrm{UAI})} \text{ and } A_\alpha^{-(\mathrm{UAI})} + B_\alpha^{-(\mathrm{UAI})}, \tag{6}$$

$$2.\ A^R < B^R | \Leftrightarrow A^R \leq B^R \text{ and } A^R \neq B^R., \tag{7}$$

3 Vendor selection problem

A linear programming problem for maximizing quality performance measures subject to the price, services, and lead time is provided as [25]:

$$\max Z(x) = \sum_{i=1}^{n} q_i x_i$$

subject to

$$\begin{aligned} &\sum_{i=1}^{n} p_i x_i \leq P, \\ &\sum_{i=1}^{n} s_i x_i \geq S, \\ &\sum_{i=1}^{n} l_i x_i \leq L, \\ &\sum_{i=1}^{n} x_i = 1, \\ &x_i \geq 0, \ \forall i. \end{aligned} \tag{8}$$

Here x_i is the fraction of demand allocated to vendor i; p_i is the price of the item; q_i is the quality level; l_i is the lead time, s_i the level of service; P, L and S are the required overall price level, lead time level and service level respectively.
In addition A^R and C^R are two sets of rough intervals, $A^R \in \mathbb{R}^{R(m\times n)}$, $B^R \in \mathbb{R}^{R(1\times n)}$ and $C^{R(1\times n)}$; X represents a set of decision variables.

4 Rough vender selection problem

Consider the vendor selection problem (8) with inexact roug intervals

$$\begin{aligned} &\max Z^R(x) = \sum_{i=1}^{n} q_i^{\pm} \oplus x_i^R \\ &\text{subject to} \\ &\quad \sum_{i=1}^{n} p_i^{\pm} \oplus x_i^R \le P^R, \\ &\quad \sum_{i=1}^{n} s_i^{\pm} \oplus x_i^R \ge S^R, \\ &\quad \sum_{i=1}^{n} l_i^{\pm} \oplus x_i^R \le L^R, \\ &\quad \sum_{i=1}^{n} x_i^R = 1, \\ &\quad x_i \ge 0,\ \forall i. \end{aligned} \tag{9}$$

where A^R and C^R are two sets of rough intervals, $A^R \in \mathbb{R}^{R(m\times n)}$, $B^{R(1\times n)}$ and $C^{R(1\times n)}$; x_i^R $(i = 1, 2, \cdots, n)$ represents a set of decision variables.
Definition 2.7. A point x^R which satisfies the conditions in (9) is called rough optimization solution.

5 Solution procedure

In this section, a solution procedure for obtaining the rough optimal solution for problem (9) can be introduced, as follows:
Step 1: Convert problem (9) into two boundary problems, namely, Z^{-R} and Z^{+R}, respectively;
Step 2: Transform each of Z^{-R} and Z^{+R} into two subproblems;
Step 3: Apply the upper bound sub-problem and solve it using GAMS software or any computer package to obtain the optimal solution $X_{\text{opt}}^{+(\text{UAI})}$ with corresponding optimum value $Z_{\text{opt}}^{+(\text{UAI})}$
Step 4: Solve the second upper bound sub-problem after adding the constraints $X^{+(\text{LAI})} \le Z_{\text{opt}}^{+(\text{UAI})}$ to get the solution $X_{\text{opt}}^{+(\text{LAI})}$ and $Z_{\text{opt}}^{+(\text{LAI})}$;
Step 5: Add the constraint $X^{-(\text{LAI})} \le Z_{\text{opt}}^{+(\text{LAI})}$ to the lower bound sub-problem and solve the new problem to obtain the optimal solution $X_{\text{opt}}^{-(\text{LAI})}$ with $Z_{\text{opt}}^{-(\text{LAI})}$;
Step 6: Solve the second lower bound sub-problem with additional constraint $X^{-(\text{UAI})} \le Z_{\text{opt}}^{-(\text{LAI})}$ to obtain the solution $X_{\text{opt}}^{-(\text{UAI})}$ with $Z_{\text{opt}}^{-(\text{UAI})}$;
Step 7: Embed the solutions from steps 3, 4, 5, 6 to obtain the solution of problem (8) as:

$$X^R = \left[[X^{-(\text{UAI})}, X^{+(\text{UAI})}] : [X^{-(\text{LAI})}, X^{+(\text{LAI})}] \right],$$

and

$$Z^R = \left[[Z_{\text{opt}}^{-(\text{UAI})}, Z_{\text{opt}}^{+(\text{UAI})}] : [Z_{\text{opt}}^{-(\text{LAI})}, Z_{\text{opt}}^{+(\text{LAI})}] \right]$$

6 Numerical example

Consider the following problem:

$$\begin{aligned} &\max Z^R = (0, 0.5, 1, 2, 3, 4, 5) \odot x_1^R \oplus (2, 2.5, 2.75, 3, 4, \\ &\qquad 5, 6) \odot x_2^R \\ &\text{subject to} \\ &\quad (0, 0.5, 0.75, 12, 3, 4) x_1^R \oplus (1, 1.5, 1.75, 2, 3, 4, 5) x_2^R \\ &\quad = (7, 8, 9, 10, 12, 15, 18), \\ &\quad (1, 1.5, 1.75, 2, 3, 4, 5) x_1^R \oplus (0, 0.5, 0.75, 1, 2, 3, 4) x_2^R \\ &\quad = (6, 7, 8, 12, 15, 16, 21) \\ &\quad x_1^R \text{ and } x_2^R \ge 0. \end{aligned} \tag{10}$$

According to problem (9), problem (10) can be rewritten, as follows:

$$\begin{aligned} \min Z^R = \Big(&[[0.5, 4] : [1, 3]] \odot \left[[x_1^{-(\text{UAI})}, x_1^{+(\text{UAI})}] : \right. \\ &\left. [x_1^{-(\text{LAI})}, x_1^{+(\text{LAI})}] \right] \oplus [[1.5, 5] : [2, 4]] \\ &\odot \left[[x_2^{-(\text{UAI})}, x_2^{+(\text{UAI})}] : [x_2^{-(\text{LAI})}, x_2^{+(\text{LAI})}] \right] \Big) \end{aligned}$$

subject to

$$\begin{aligned} &\Big([[0.5, 3] : [0.75, 2]] \odot \left[[x_1^{-(\text{UAI})}, x_1^{+(\text{UAI})}] : \right. \\ &\left. [x_1^{-(\text{LAI})}, x_1^{+(\text{LAI})}] \right] \oplus [[1.5, 4] : [1.75, 3]] \\ &\odot \left[[x_2^{-(\text{UAI})}, x_2^{+(\text{UAI})}] : [x_2^{-(\text{LAI})}, x_2^{+(\text{LAI})}] \right] \Big) \\ &= [[8, 15] : [9, 12]], \\ &\Big([[1.5, 4] : [1.75, 3]] \odot \left[[x_1^{-(\text{UAI})}, x_1^{+(\text{UAI})}] : \right. \\ &\left. [x_1^{-(\text{LAI})}, x_1^{+(\text{LAI})}] \right] \oplus [[0.5, 3] : [0.75, 2]] \\ &\odot \left[[x_2^{-(\text{UAI})}, x_2^{+(\text{UAI})}] : [x_2^{-(\text{LAI})}, x_2^{+(\text{LAI})}] \right] \Big) \\ &= [[7, 18] : [8, 15]], \\ &x_1^{-(\text{UAI})}, x_1^{+(\text{UAI})}, x_1^{-(\text{LAI})}, x_1^{+(\text{LAI})}, x_2^{-(\text{UAI})}, x_2^{+(\text{UAI})}, \\ &x_2^{-(\text{LAI})}, x_2^{+(\text{LAI})} \ge 0. \end{aligned} \tag{11}$$

Equivalently,

$$\begin{aligned}
&\max\left[Z^{+(LAI)},Z^{+(UAI)}\right]:\left[Z^{-(UAI)},Z^{-(LAI)}\right]\\
&=\left(\left[\left[3x_1^{+(LAI)},4x_1^{+(UAI)}\right]:\left[0.5x_1^{-(UAI)},x_1^{-(LAI)}\right]\right]\right.\\
&\oplus\left[\left[4x_2^{+(LAI)},5x_2^{+(UAI)}\right]:\left[1.5x_2^{-(UAI)},2x_2^{-(LAI)}\right]\right]\\
&\text{subject to} \qquad (12)\\
&\left(\left[\left[2x_1^{+(LAI)},3x_1^{+(UAI)}\right]:\left[0.5x_1^{-(UAI)},0.75x_1^{-(LAI)}\right]\right]\right.\\
&\oplus\left[\left[3x_2^{+(LAI)},4x_2^{+(UAI)}\right]:\left[1.5x_2^{-(UAI)},1.75x_2^{-(LAI)}\right]\right]\\
&=\left[[12,15]:[8,9]\right]\\
&\left(\left[\left[3x_1^{+(LAI)},4x_1^{+(UAI)}\right]:\left[1.5x_1^{-(UAI)},1.75x_1^{-(LAI)}\right]\right]\right.\\
&\oplus\left[\left[2x_2^{+(UAI)},3x_2^{+(LAI)}\right]:\left[0.5x_2^{-(UAI)},0.75x_2^{-(LAI)}\right]\right]\\
&=\left[[15,18]:[7,8]\right]\\
&x_1^{-(UAI)},x_1^{+(UAI)},x_1^{-(LAI)},x_1^{+(LAI)},x_2^{-(UAI)},x_2^{+(UAI)},\\
&x_2^{-(LAI)},x_2^{+(LAI)}\geq 0.
\end{aligned}$$

Step 1:

$$\begin{aligned}
&\max\left[Z^{+(LAI)},Z^{+(UAI)}\right]=\left[3x_1^{+(LAI)},4x_1^{+(UAI)}\right]\\
&\quad\oplus\left[4x_2^{+(LAI)},5x_2^{+(UAI)}\right]\\
&\text{subject to}\\
&\quad\left[2x_1^{+(LAI)},3x_1^{+(UAI)}\right]\oplus\left[3x_2^{+(LAI)},4x_1^{+(UAI)}\right] \qquad (13)\\
&\quad=[12,15]\\
&\quad\left[3x_1^{+(LAI)},4x_1^{+(UAI)}\right]\oplus\left[2x_2^{+(UAI)},3x_1^{+(LAI)}\right]\\
&\quad=[15,18],\\
&\quad x_1^{+(UAI)},x_1^{+(LAI)},x_2^{+(UAI)},\text{ and } x_2^{+(LAI)}\geq 0,
\end{aligned}$$

$$\begin{aligned}
&\max\left[Z^{-(LAI)},Z^{-(UAI)}\right]=\left[3x_1^{-(UAI)},0.5x_1^{-(LAI)}\right]\\
&\quad\oplus\left[1.5x_2^{-(UAI)},2x_2^{-(LAI)}\right]\\
&\text{subject to} \qquad (14)\\
&\quad\left[0.75x_1^{-(LAI)},2x_1^{+(LAI)}\right]\oplus\left[1.75x_2^{-(LAI)},3x_2^{+(LAI)}\right]\\
&\quad=[9,12]\\
&\quad\left[1.75x_1^{-(LAI)},3x_1^{+(LAI)}\right]\oplus\left[0.75x_2^{-(UAI)},2x_2^{+(LAI)}\right]\\
&\quad=[8,15],\\
&\quad x_1^{-(LAI)},x_1^{+(LAI)},x_2^{-(LAI)},\text{ and } x_2^{+(LAI)}\geq 0,
\end{aligned}$$

$$\begin{aligned}
&\max\left[Z^{+(UAI)}\right]=4x_1^{+(UAI)}+5x_2^{+(UAI)}\\
&\text{subject to}\\
&\quad 3x_1^{+(UAI)}+4x_2^{+(UAI)}=15, \qquad (15)\\
&\quad 4x_1^{+(UAI)}+3x_2^{+(UAI)}=18,\\
&\quad x_1^{+(UAI)}\geq 0\text{ and } x_2^{+(UAI)}\geq 0.
\end{aligned}$$

The solution is, as follows:

$(x_1)_{opt}^{+(UAI)}=3.8571$, $(x_2)_{opt}^{+(UAI)}=0.8571$ and $Z_{opt}^{+(UAI)}=19.7143$

Step 4:

$$\begin{aligned}
&\max\left[Z^{+(LAI)}\right]=3x_1^{+(LAI)}+4x_2^{+(LAI)}\\
&\text{subject to}\\
&\quad 2x_1^{+(LAI)}+3x_2^{+(LAI)}=12, \qquad (16)\\
&\quad 3x_1^{+(LAI)}+2x_2^{+(LAI)}=15,\\
&\quad x_1^{+(LAI)}\leq (x_1)_{opt}^{+(UAI)},\\
&\quad x_2^{+(LAI)}\leq (x_2)_{opt}^{+(UAI)},\\
&\quad x_1^{+(LAI)}\geq 0\text{ and } x_2^{+(LAI)}\geq 0.
\end{aligned}$$

The solution is, as follows:

$(x_1)_{opt}^{+(LAI)}=3.8571$, $(x_2)_{opt}^{+(LAI)}=0.8571$ and $Z_{opt}^{+(LAI)}=5.5713$

Step 5: Solve the following problem:

$$\begin{aligned}
&\max\left[Z^{-(LAI)}\right]=3x_1^{-(LAI)}+2x_2^{-(LAI)}\\
&\text{subject to}\\
&\quad 0.75x_1^{-(LAI)}+1.75x_2^{-(LAI)}=9, \qquad (17)\\
&\quad 1.75x_1^{-(LAI)}+0.75x_2^{-(LAI)}=8,\\
&\quad x_1^{-(LAI)}\leq (x_1)_{opt}^{+(LAI)},\\
&\quad x_2^{-(LAI)}\leq (x_2)_{opt}^{+(LAI)},\\
&\quad x_1^{-(LAI)}\geq 0\text{ and } x_2^{-(LAI)}\geq 0.
\end{aligned}$$

The solution is, as follows:

$(x_1)_{opt}^{-(LAI)}=3.8571$, $(x_2)_{opt}^{-(LAI)}=0.8571$ and $Z_{opt}^{-(LAI)}=5.5713$

Step 6: Solve the following problem:

$$\begin{aligned}
&\max\left[Z^{-(UAI)}\right]=0.5x_1^{-(UAI)}+2.5x_2^{-(UAI)}\\
&\text{subject to}\\
&\quad 0.5x_1^{-(UAI)}+1.5x_2^{-(UAI)}=8, \qquad (18)\\
&\quad 1.5x_1^{-(UAI)}+0.5x_2^{-(UAI)}=7,\\
&\quad x_1^{-(UAI)}\leq (x_2)_{opt}^{-(LAI)},\\
&\quad x_2^{-(UAI)}\leq (x_2)_{opt}^{-(LAI)},\\
&\quad x_1^{-(UAI)}\geq 0\text{ and } x_2^{-(UAI)}\geq 0.
\end{aligned}$$

The solution is, as follows:

$$(x_1)_{\text{opt}}^{-(\text{UAI})} = 3.8571,\ (x_2)_{\text{opt}}^{-(\text{UAI})} = 0.8571 \quad \text{and} \quad Z_{\text{opt}}^{-(\text{UAI})} = 3.2141.$$

Thus, the rough optimal solution of problem (10)
Step 7:

$$(x_1)_{\text{opt}}^{R} = \left[\left[(x_1)_{\text{opt}}^{-(\text{UAI})}, (x_1)_{\text{opt}}^{+(\text{UAI})}\right] : \left[(x_1)_{\text{opt}}^{-(\text{LAI})}, (x_1)_{\text{opt}}^{+(\text{LAI})}\right]\right] = \left[[3.8571, 3.8571] : [3.8571, 3.8571]\right],$$

$$(x_2)_{\text{opt}}^{R} = \left[\left[(x_2)_{\text{opt}}^{-(\text{UAI})}, (x_2)_{\text{opt}}^{+(\text{UAI})}\right] : \left[(x_2)_{\text{opt}}^{-(\text{LAI})}, (x_2)_{\text{opt}}^{+(\text{LAI})}\right]\right] = \left[[0.8571, 0.8571] : [0.8571, 0.8571]\right],$$

and

$$Z^R = [Z^{-R}, Z^{+R}] = \left[\left[Z_{\text{opt}}^{-\text{UAI}}, Z_{\text{opt}}^{+\text{UAI}}\right] : \left[Z_{\text{opt}}^{-\text{LAI}}, Z_{\text{opt}}^{+\text{UAI}}\right]\right] = \left[[3.2142, 19.7143] : [5.5713, 14.9997]\right].$$

Thus, the solution desired for problem (18) is, as follows: $x_1 = 3.8571$, $x_2 = 0.8571$, $Z^{+R} = [14.9997, 19.7143]$, and $\widetilde{Z}_H = (1.7142,\ 4.0713,\ 6.214125,\ 10.2855,\ 14.9997,\ 19.7139, 22.8081)$

7 Conclusion

In this paper, a new method for solving fuzzy rough linear programming problems without converting the fuzzy coefficients into its crisp values was proposed. Although the calculations required more effort, the method is considered more effective than the other methods, where our problem reduces to a four classical linear programming problems, each of them can be easily solved even manually. The proposed approach has several future directions. One can consider the neutrosophic sets to cope with uncertainty in the proposed model. In addition, we can impose more real life constraint to the optimization model.

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