Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/150314

Effect of the Negative Velocity Feedback Control for Reducing the Primary Resonance Vibration of a Magnetic Levitation System using the Harmonic Balance Method

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Received: 4 Nov. 2020, Revised: 2 Feb. 2020, Accepted: 21 Mar. 2021
Published online: 1 May 2021

Abstract: In order to reduce the primary resonance vibration of a magnetic levitation system, this paper investigated the effect of applying negative linear velocity feedback control. External and parametric forces that were linear, quadratic and cubic were exposed to the system under analysis. The control of linear velocity feedback shows that it is better to reduce the resulting vibration than the cubic one. To obtain the response system and analyze the stability of it, the harmonic balance method (HBM) was applied. It is confirmed that it is stable overall. Comparison of the approximate and numerical solutions and the effect of the response system parameters were analyzed using MATLAB 14.0. The response system shows that only the coefficient of the linear force is affected, while the coefficients of the quadratic and cubic forces have no effect on the response system.

Keywords: Harmonic Balance Method, Magnetic Levitation System, Negative Velocity Feedback Controller, Primary Resonance Case, Response Equation

1 Introduction

High-amplitude resonant vibrations which appear in various dynamical systems are always undesirable. Active feedback control is an effective control for reducing vibrations magnitude. The active control action on elastic wave metamaterials is presented by Wang et al. [1]. Under the influence of the active feedback control operation, stop band properties, negative effective mass, and device stability are evaluated numerically. They stated that the stop band width can be reduced and the stable properties can be satisfied by both positive acceleration and negative velocity feedback control behavior. For the negative effective mass, it is investigated that the increase in the negative acceleration control behavior contributes to an increase in the frequency area, while the positive case has a reverse effect on the frequency region. Wang et al. [2] presented a nonlinear convergence algorithm for active dynamic undamped vibration absorber (ADUVA) which is composed of equivalent dynamic modeling equations and frequency estimator. They proposed an active absorber which made up of the displacement and velocity components. The nonlinear ADUVA can simultaneously meet the requirements of fast convergence rate and small steady state frequency error, which leads to better vibration control performances than the linear one. The multi- modal negative acceleration feedback (MMNAF) control was proposed by Yang et al. [3] for an active mass damper (AMD) system. Theoretically, it was found that the stability condition for the negative acceleration feedback (NAF) control is static using a single-degree-of-freedom (SDOF) system. Unlike the positive position feedback (PPF) control the NAF does not cause instability in the low frequency region. On the basis of the theoretical results of the SDOF system, it has been proven, both theoretically and experimentally, that the MMNAF control can suppress the vibrations of the multi-degree-of-freedom (MDOF) systems. Velocity or

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acceleration feedback techniques and disturbance prediction are investigated in the work reviewed by Baader and Fontana [4] for the application of vertical floor vibration mitigation. The simulation results demonstrated that for the mitigation of floor vibrations, the tuned mass damper (TMD) and both of these strategies work well. It is clarified that for both examined systems, the velocity feedback shows a good damping quality. Moreover different resonance frequencies can be reduced using this technique. El-Ganaini [5] applied PPF controller on the magnetic levitation system which described in [5] to reduce the vibration amplitude of periodic primary excitation. The effect of PPF controller showed that it is the best compared to the passive controller proposed by Jo and Yabuno [6] where the controller effectiveness was 2 in the model constructed by Jo and Yabuno [6], while it is 226 in the study of El-Ganaini [5]. Multiple scale perturbation technique is applied in [6] and [5] for obtaining the response system. Sayed et al. [7] investigated the Van der Pol equation subjected to external and parametric excitation forces at the simultaneous resonance case.The controller success in reducing the vibrations was about $\Omega_1 = \omega, \Omega_2 = 2\omega$ which was the worst one.

The effect of a negative acceleration feedback active controller demonstrated that it is the best for reducing the vibration dramatically. Talib et al. [8] extended and revised the MMNAF modal proposed by Yang et al. [3] by exchange the AC servo motor with the linear servo motor which is more effective because it is fast and less noisy. The multi-input multi-output (MIMO) modal designed by Talib et al. [8] was more effective in suppressing the vibrations of the test structure with using the linear servo motors. Furthermore, the model was validated both theoretically and numerically. Negative velocity feedback controller was applied by Amer and Agwa [9] on two degrees of freedom nonlinear system. They illustrated that the negative linear velocity feedback controller is more effective than the cubic for suppressing the amplitude of the vibration at the sub-harmonic resonance case $\omega \cong 2\omega_2$ which was the worst resonance case. The Harmonic Balance Method (HBM) is widely used and can handle many kinds of nonlinearities. The approximate period and the approximate solution of nonlinear jerk equations is evaluated by Saifur Rahman and Hasan [10] by employing a modified HBM. In the comparison of outcomes, it is noticeable that the second approximations are more accurate than the other methods. The results are also consistent with the exact solution. The dynamic behavior of the Zener model subject to base excitation has been identified in L. de Haro Silva [11] with the inclusion of a nonlinear stiffness factor to improve high frequency system transmissibility. Two degrees of freedom and two equations of motion were initially described in the method. Then, these equations were combined in such a way that a third order ordinary differential equation described the system. This representation allowed them to write the motion equations, including a nonlinear element of stiffness, so that the results of their paper and the system's frequency response could then be obtained using the harmonic balance method. In combination with the harmonic balance method, L. Guillot et al. [12] produced an expansion to the progression of periodic orbits of delay differential equations of the asymptotic numerical method. The equations may be mandatory or independent and may be of a neutral form. The established method needs that the equation system be written in a detailed quadratic expression. The technique is applied to several systems, from Van der Pol and Duffing oscillators to clarinet and saxophone toy versions. A comparison with standard time-integration solvers achieved the harmonic balance method. The control effectiveness of a nonlinear positive position feedback (NPPF) controller for vibration attenuation of a Duffing oscillator is examined by G. Zhao et al. [13]. The proposed NPPF controller is based on the classical linear positive position feedback (LPPF) controller, but according to the theory of similarity, a cubic term is included. The analytical solutions is approximated using the harmonic balance method. In order to illustrate the proposed control technique, both numerical simulations and experimental validations will be carried out. Harmonic balance method is used by F. Qian et al. [14] to obtain the approximate analytical solutions of the piezoelectric energy harvester (PEH) with the coupled higher-order nonlinear terms. An approach to solving the Jacobian matrix entries is proposed to determine the stability of the approximate solutions. This method provides a guideline for the study of solution stability of congeneric nonlinear systems with higher-order terms coupled together. Y. M. Chen et al. [15] presented a review on the multi-frequency pattern (MFP) vibration of nonlinear network systems based on the incremental harmonic balance (IHB) method. The IHB method is proposed with a time delay to solve a single-degree-of freedom system instead of directly solving the highly-dimensional systems discussed. The proposed method can provide both stable and unstable limit phases and can therefore be managed by multiple solutions successfully. For solving nonlinear forced vibration problems, M. W. Ullah et al. [16] introduced a modified harmonic balance method. A series of nonlinear algebraic equations arise from the unknown coefficients of harmonic terms and the frequency of the forcing principle. A numerical approach is typically used to solve them. A set of algebraic linear equations, along with a nonlinear one, is solved. The solution obtained by the suggested method was compared to those obtained by finite variation difference and numerical methods. Also, there are many numerical methods for solving differential equations which reported and examined in [17]-[28]. In the present paper, we study the equation governing the magnetic levitation system Eq. (2) which reported in [5] under the effect of negative velocity feedback controller. HBM is applied instead of multiple scale perturbation technique (MSPT) because the use of MSPT resulted in the absence of quadratic and cubic force coefficients in studying response system. Whereas, the application of HBM resulted in preserving the existence of all parameters, so it is possible to study their effect altogether. Response system is obtained and the stability conditions is applied the same way as reported in [14]. The present paper is organized as follows: In section 2, we obtain an approximate solution and the stability condition of the response system using Harmonic Balance Method. In section 3, we discuss the obtained approximate solution and compare it with the numerical solution obtained by applying Runge-Kutta fourth-order method. Also, we illustrate the effect of system parameters on the response system. Finally, we summarize our results in section 4. The equation of

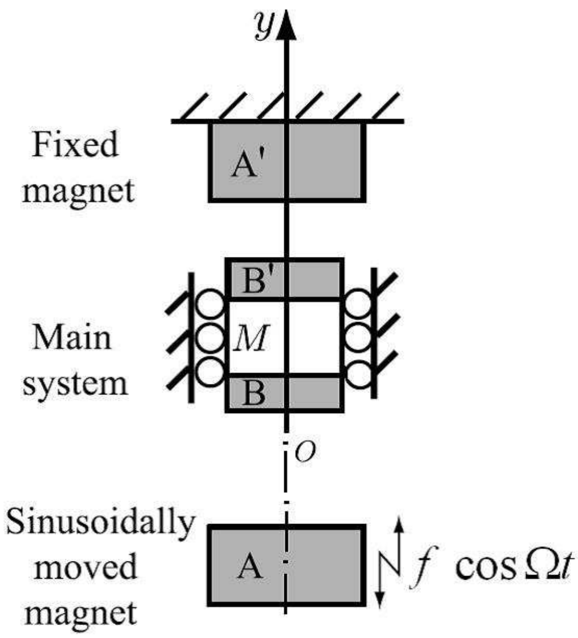

Fig. 1: Magnetic Levitation system

magnetic Levitation system can be written as:

$$\ddot{y}+2\mu\dot{y}=\sum_{n=1}^{3} l_n y^n-\sum_{n=1}^{3} k_n(y-f\cos\Omega t)^n. \quad (1)$$

where, $k_1-l_1=1$,$k_2-l_2=0$ and $k_3-l_3=\alpha$. So the equation of motion as reported in [5] written as,

$$\begin{aligned}&\ddot{y}+2\mu\dot{y}+y+\alpha y^3=k_1 f\cos\Omega t-k_2 f^2\cos^2\Omega t\\&+k_3 f^3\cos^3\Omega t+(2k_2-3k_3 f)fy\cos^2\Omega t\\&+3k_3 fy^2\cos\Omega t.\end{aligned} \quad (2)$$

We used the negative linear velocity feedback controller to control the vibrating system. Hence, Eq. (2) becomes:

$$\begin{aligned}&\ddot{y}+2\mu\dot{y}+y+\alpha y^3=k_1 f\cos\Omega t-k_2 f^2\cos^2\Omega t\\&+k_3 f^3\cos^3\Omega t+(2k_2-3k_3 f)fy\cos^2\Omega t\\&+3k_3 fy^2\cos\Omega t-G\dot{y}.\end{aligned} \quad (3)$$

where G is the gain.

2 Harmonic Balance Method

The standard harmonic balance will be presented and the fundamental harmonic only is used. The fundamental frequency of oscillation is Ω. The solution is expressed as follows:

$$y=y_1(t)\cos\Omega t+y_2(t)\sin\Omega t. \quad (4)$$

Substituting from Eq. (4) into Eq. (3), we get the following:

$$\begin{aligned}&\ddot{y}_1\cos\Omega t+\ddot{y}_2\sin\Omega t-2\Omega^2\dot{y}_1\sin\Omega t+2\Omega^2\dot{y}_2\cos\Omega t\\&-\Omega^2 y_1\cos\Omega t-\Omega^2 y_2\sin\Omega t+(2\mu+G)\\&(\dot{y}_1\cos\Omega t+\dot{y}_2\sin\Omega t-\Omega y_1\sin\Omega t+\Omega y_2\cos\Omega t)\\&+y_1\cos\Omega t+y_2\sin\Omega t-k_1 f\cos\Omega t\\&+\alpha y_1^2\cos^2\Omega t(y_1\cos\Omega t+3y_2\sin\Omega t)\\&+\alpha y_2^2\sin^2\Omega t(3y_1\cos\Omega t+y_2\sin\Omega t)\\&+k_2 f(f-2y_1)\cos^2\Omega t-k_2 fy_2\sin 2\Omega t\\&-k_3 f^2(f-3y_1)\cos^3\Omega t\\&+3k_3 f\cos^2\Omega t(fy_2\sin\Omega t-y_1^2\cos\Omega t)\\&-k_3 fy_2\sin 2\Omega t(3y_1\cos\Omega t+\frac{3}{2}y_2\sin\Omega t)=0.\end{aligned} \quad (5)$$

Converting any power trigonometric function to its equivalent expression of the linear trigonometric functions with multiple angle and balance irrelevant Eq. (5) the following equation is obtained:

$$\begin{aligned}&\ddot{y}_1+(G+2\mu)\dot{y}_1+2\Omega\dot{y}_2+\frac{1}{4}(4+9k_3 f^2-4\Omega^2)y_1\\&+(G+2\mu)\Omega y_2+\frac{3}{4}\alpha y_1(y_1^2+y_2^2)\\&-\frac{3}{4}k_3 f(3y_1^2+y_2^2)-k_1 f-\frac{3}{4}k_3 f^3=0\end{aligned} \quad (6)$$

$$\begin{aligned}&\ddot{y}_2+(G+2\mu)\dot{y}_2-2\Omega\dot{y}_1+\frac{1}{4}(4+9k_3 f^2-4\Omega^2)y_2\\&-(G+2\mu)\Omega y_1+\frac{3}{4}\alpha y_2(y_1^2+y_2^2)\\&-\frac{3}{2}k_3 fy_1y_2=0\end{aligned} \quad (7)$$

The system of the response curve is obtained by neglecting first and second derivatives in Eqs. (6) and (7):

$$\frac{1}{4}(4+9k_3f^2-4\Omega^2)y_1+(G+2\mu)\Omega y_2+\frac{3}{4}\alpha y_1(y_1^2+y_2^2)-\frac{3}{4}k_3f(3y_1^2+y_2^2)-k_1f-\frac{3}{4}k_3f^3=0 \quad (8)$$

$$\frac{1}{4}(4+9k_3f^2-4\Omega^2)y_2-(G+2\mu)\Omega y_1+\frac{3}{4}\alpha y_2(y_1^2+y_2^2)-\frac{3}{2}k_3fy_1y_2=0 \quad (9)$$

Neglecting the terms of second derivatives in Eqs. (6) and (7) then solving the two equations for the first derivative; we get:

$$\dot{y_1}=\frac{1}{(2\mu+G)^2+(2\Omega)^2}\left((2\mu+G)M_1-2\Omega M_2\right). \quad (10)$$

$$\dot{y_2}=\frac{1}{(2\mu+G)^2+(2\Omega)^2}\left((2\mu+G)M_2+2\Omega M_1\right). \quad (11)$$

where,
$M_1=-\frac{1}{4}(4+9k_3f^2-4\Omega^2)y_1-(G+2\mu)\Omega y_2-\frac{3}{4}\alpha y_1(y_1^2+y_2^2)+\frac{3}{4}k_3f(3y_1^2+y_2^2)+k_1f+\frac{3}{4}k_3f^3$,
and
$M_2=-\frac{1}{4}(4+9k_3f^2-4\Omega^2)y_2+(G+2\mu)\Omega y_1-\frac{3}{4}\alpha y_2(y_1^2+y_2^2)+\frac{3}{2}k_3fy_1y_2$.
The Jacobian of the stability is defined as:

$$J=\begin{vmatrix}\frac{\partial\dot{y_1}}{\partial y_1} & \frac{\partial\dot{y_2}}{\partial y_1}\\ \frac{\partial\dot{y_1}}{\partial y_2} & \frac{\partial\dot{y_2}}{\partial y_2}\end{vmatrix} \quad (12)$$

Relation between the first harmonic amplitude a which is defined by $a=\sqrt{y_1^2+y_2^2}$ and the detuning parameter σ which is defined by $\Omega=1+\sigma$ can be drawn with studying the response stability using Eqs. (8-12).

3 Results and Discussion

To determine the numerical solution of the Eq.(2), the Runge-Kutta fourth-order method was applied. The selected values for the given system parameters are given by: $\mu=0.02,\Omega=1,\alpha=0.894,k_1=0.5,k_3=0.447,f=0.05,G=2$. The effect of the linear velocity feedback control for reduction the vibration of the system at the primary resonance case $\Omega=1$ is illustrated by Figure 2. It shows the time history of the system without control which has an amplitude about 0.35. After applying NVF controller on the system, the amplitude suppressed to about 0.012. Thus, the controller effectiveness $E_a=29$. Figure 3 illustrated that the effect of the linear velocity feedback control for reduction the vibration of the system is better than the effect of cubic velocity feedback control by showing the comparison between them. Figure 4 illustrated the consistency between the approximate solution and the numerical result of equation of the system for active absorber at the primary resonance case. Figure 5 shows consistency between the approximate solution obtained by HBM and the numerical solution determined by applying Runge-Kutta fourth-order method. In the case of applying HBM, the effect of system parameters on the response curve is studied and illustrated by Figures (6 and 9). The behavior of the response curve is stable overall when applying HBM which illustrated that it is a very suitable method. It is found that the response curve has a skewness to the left. Figure 6 indicates that with the decrease of the damping parameter μ , the frequency response curves bent away from the linear curves, resulting in multi-valued regions

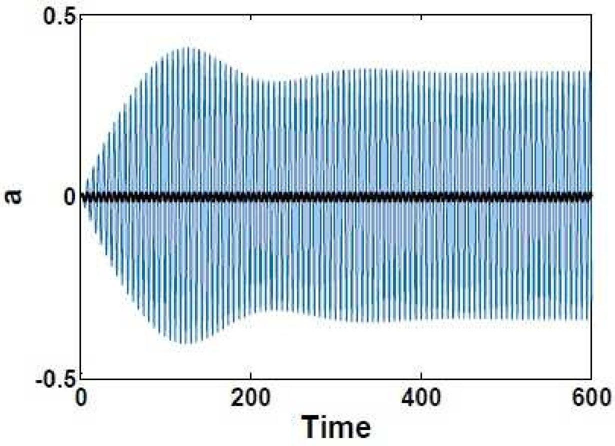

Fig. 2: Effect of NVF Controller on the time history at the primary resonance case.

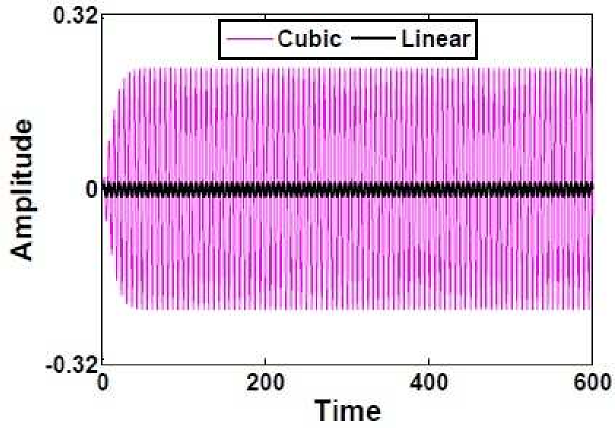

Fig. 3: Comparison between linear and cubic velocity feedback control.

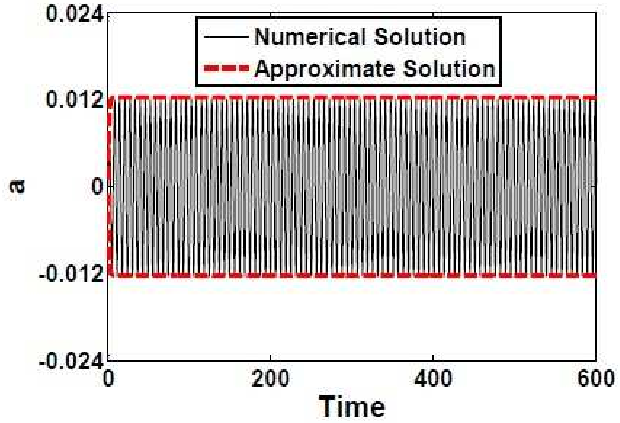

Fig. 4: Comparison between the numerical solution and the approximate solution at the primary resonance case with applying NVF control.

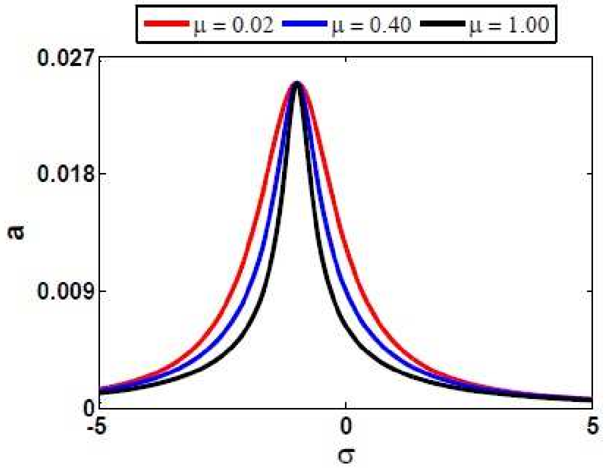

Fig. 6: Response Curve of the controller; Comparison between the numerical solution and the approximate solution.

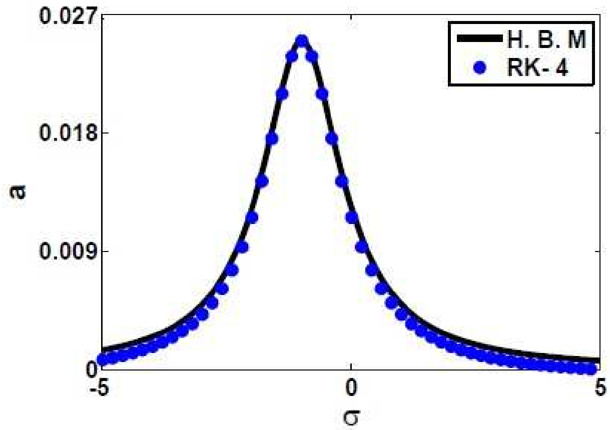

Fig. 5: Response Curve of the controller; Comparison between the numerical solution and the approximate solution.

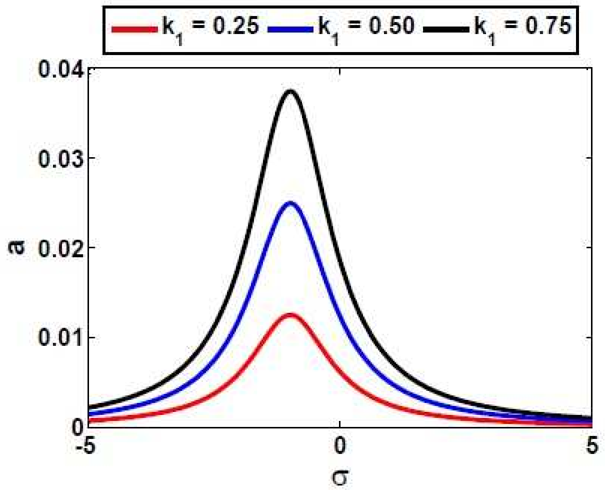

Fig. 7: Frequency response curves at different values of the coefficient of the first power k_1.

and jump phenomenon. Figures 7 and 8 show that the steady state amplitude is a monotonic increasing with the increase of the first power coefficient k_1 and the excitation force. Also, the kurtosis of the curve increases. Figure 9 reveals that the frequency response curves bent away from the linear curves, resulting in multi-valued regions and jump phenomenon with the decrease of the gain parameter G. Nonlinear coefficient α and coefficients quadratic and cubic force k_2 and k_3 have no effect on the response curve. The relation between the system amplitude and the excitation force without and with the control is demonstrated in Figure 10. In the absence of control the system amplitude increases nonlinearly for a slight increase in the excitation force. After applying NVF controller, the system amplitude leads to a saturation case that the relation became horizontal, so the system amplitude slightly changes for the large increase in the excitation force.

4 Conclusion

The effect of applying a negative linear velocity feedback control on the magnetic levitation system which produces a primary resonance vibration was addressed. The system is subjected to external and parametric linear as well as quadratic and cubic forces. Response system was investigated using HBM and illustrated that it is very suitable that the approximate and the numerical solutions are consistent with each other. Relation between the

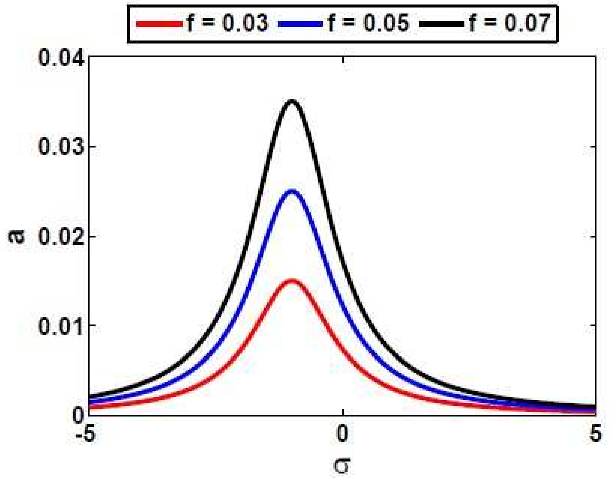

Fig. 8: Frequency response curves at different values of the force f.

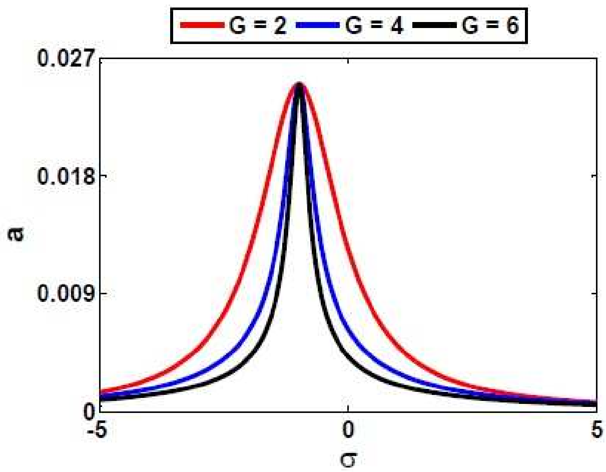

Fig. 9: Frequency response curves at different values of the gain G.

amplitude and the detuning parameter can be explained and figured by defining the first amplitude $a = \sqrt{y_1^2 + y_2^2}$ and the detuning parameter as $\Omega = 1 + \sigma$ in the obtained response system by applying HBM. The results of this paper can be summarized as follows:
1) The worst resonance case is the primary resonance case.
2) Using negative linear velocity feedback control reduces about 98% of the system vibration amplitude.
3) The effectiveness of the controller is about $E_a = 29$ for the main system.
4) The frequency response curve is effected by varying only the coefficient of the linear force k_1, while coefficients k_2 and k_3 of the quadratic and the cubic forces

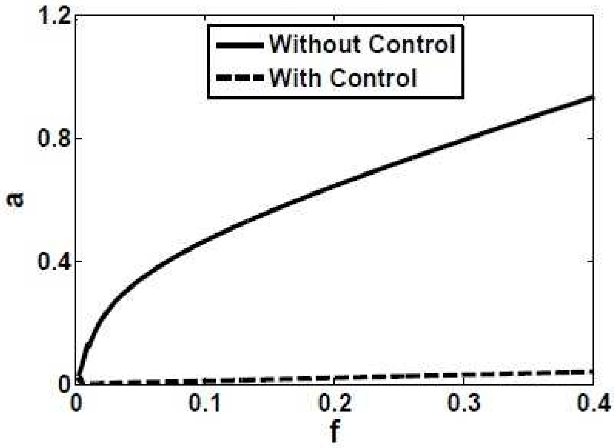

Fig. 10: Force response curves of the main system and the controller.

have no effect.
5) The frequency response curve is stable over the studied domain and has a skewness to the left.
6) The Harmonic Balance Method is very suitable for studying the response system.

Acknowledgement

The authors are grateful to the reviewers for their constructive comments.

Conflict of Interest

The authors declare that they have no conflict of interest.

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