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An EOQ Model for Items having Fuzzy Amelioration and Deterioration

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Abstract: Due to globalization and modernization, the population of cities is increasing day by day. Also, due to lack of time and work pressure, most of the people living in cities have to depend on different types of food services. To fulfill the requirement of people, the number of hotels, restaurants, and many types of food courts are increasing day by day. All these food-service points mostly depend on chicken, mutton, fish, prawns, and other different types of items that are ameliorating and deteriorating in nature. Keeping this in view, we develop here an inventory model for items that are both ameliorating and deteriorating in a crisp environment. Moreover, the technological development and uncertainty of climate have a great impact on amelioration and deterioration. As such the rate of amelioration and deterioration are of imprecision in nature. Thus, to overcome this situation we also simultaneously discuss a fuzzy model having ameliorating and deteriorating parameters as trapezoidal fuzzy numbers, and the Signed Distance Method is used for defuzzification. Different numerical examples are presented to validate the model. Finally, the sensitivity of important parameters is also studied to draw managerial insight.

Keywords: Inventory model, EOQ Model, amelioration and deterioration, Defuzzification, Signed Distance Method.

1 Introduction

The role of inventory managers is to minimize the total inventory cost by considering all the constraints. Among all the constraints, deterioration has a high priority. Since by the effect of deterioration the inventory level decreases and thereby the total cost increases, which leads inventory managers to pay attention to find the optimal replenishment time and quantity to minimize the cost. In a real scenario, most of the products are deteriorating in nature, and the other inventory constraints differ from inventory to inventory based on the circumstances and time. Hence, different types of inventory models are developed by the researchers to find the optimal results for retailers. At first, Ghare and Schrader [1] developed the EOQ model for deteriorating items. Recently, Mishra [2], Viji and Karthikeyan [3], Shin *et al.* [4], Chen *et al.* [5], Braglia *et al.* [6], Barik *et al.* [7,8], Routray *et al.* [9, 10], and Mishra *et al.* [11,12] developed the inventory models for the deteriorating items.

Industrial development, economic growth, social changes, and many other factors cause population growth in cities. Also, the number of visitors to the cities increases due to tourism, entertainment hubs, etc. Accordingly, the foodservice points like hotels, restaurants, and mobile food courts are increasing. Also, many food services are available to the customers at their doorstep. For example, many companies are providing online food services with attractive offers. All these food services mostly depend on non-vegetarian recipes. The raw materials for the non-vegetarian recipes are both ameliorating and deteriorating items like chickens, sheep, rabbits, pigs, cattle, broilers, fish, prawns, ducks, and seafood, etc. Due to the amelioration and deterioration nature, the amount of stock in the inventory increases and decreases simultaneously. Also, the storage period of these items significantly affects the inventory cost. Thus, many researchers obtained optimal results for their inventory models of ameliorating and deteriorating items under various constraints. In the past decade, Mishra *et al.*

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[13], Gwanda *et al.* [14], Mondal *et al.* [15], Nodoust *et al.* [16], Mishra *et al.* [17,18] and Barik *et al.* [19] considered the inventory models for ameliorating and deteriorating items with different constraints.

In classical inventory models, different inventory constraints are considered deterministic. These constraints are taken into account from the existing past data. But, mostly these data are imprecise inherent due to many reasons. That is, the past data is inadequate for the present scenario. Also, the change in lifestyle of people and the introduction of substitute products into the market causes the demand constraint to be fuzzy. On the other hand, both deterioration cost and holding cost lead to fuzzy due to the advancement of preservation technology. Similarly, various inventory constraints lead to fuzzy due to several socio-economic factors. Thus, to deal with this fuzziness of inventory constraints, many researchers considered fuzzy set theory and its application to find the optimal results for their inventory problems. Recently, Saha [20], Rani *et al.* [21], Shaik *et al.* [22], Srinivasan and Vijayan [23], Kumar *et al.* [24,25,26], Nayak *et al.* [27], Routray *et al.* [28], Indrajitsingha *et al.* [29] and Mishra *et al.* [30] developed their inventory models in fuzzy environment.

Motivated essentially by the aforesaid facts, here we develop an inventory model (for ameliorating and deteriorating items) having exponential demand without any shortages for both crisp and fuzzy environments. We used the trapezoidal fuzzy numbers to characterize the fuzziness of amelioration and deterioration. Also, we used the Signed Distance method for defuzzification.

The overall objective is to find the optimal ordering quantity for the business cycle to minimize the total inventory cost. Subsequently, we have presented the mathematical approach to achieve the objective in both crisp and fuzzy environments. Finally, numerical illustration and sensitivity analysis are studied to enrich the current investigation. More specifically, our contribution to this manuscript is summarized as follows:

(i) We developed an inventory model in both the crisp and fuzzy environment for a retailer's inventory having items that ameliorate and deteriorate simultaneously. The rate of amelioration and deterioration in the crisp model is deterministic. However, it is imprecise in the fuzzy environment.
(ii) We have presented the solution procedure for obtaining an optimal strategy that minimizes the total inventory cost. However, the results of the crisp model are more appropriate for the inventory items that ameliorate and deteriorate at a known rate with certainty, and the results of the fuzzy model are more appropriate for inventories under uncertainty in the rate of amelioration and deterioration of items.
(iii) To deal with the effect of impreciseness the rate of amelioration and deterioration on the total inventory cost in the fuzzy environment, we used the concepts of fuzzy set theory.
(iv) Several sets of inventory constraints are used to validate the proposed work in both the scenarios.
(v) Managerial insights are drawn via sensitivity analysis to deal with the situations that may arise during the inventory cycle.

2 Assumptions and Notations

Inventory model considered for the homogeneous items, which are both ameliorate and deteriorate in nature over time.
Shortages are not allowed.
The instantaneous replenishment facility is available.
T : Length of the business cycle.
W : Inventory level at beginning of the cycle (Economic Ordering Quantity).
C : Replenishment cost per order.
P : Purchasing cost per unit item.
$D(t) = e^{kt}$: Exponentially increasing demand.
$y(t)$: Inventory level at any time t.
θ : The rate of amelioration.
η : The rate of deterioration.
θ_T : Total amount of ameliorating items during the cycle T.
η_T : Total amount of deteriorating items during the cycle T.
H_T : Total amount of inventory during the cycle T.
$h_c = a + bt$: Linearly increasing holding cost.
$\tilde{\theta}$: Fuzzy amelioration rate.
$\tilde{\eta}$: Fuzzy deterioration rate.
c : Unit cost per item.
FW : Economic Order Quantity in fuzzy environment.

3 Mathematical Model

In the inventory, W items are ordered initially and are received instantaneously. Following, the level of the inventory rises due to an amelioration rate θ of items and declines due to both deterioration rate η and the request rate e^{kt} of the items. Finally, the inventory level reaches zero at time $t = T$. Thus, this physical phenomenon is represented mathematically by the differential equation as follow:

$$\frac{dy(t)}{dt} = (\theta - \eta)y(t) - e^{kt} \quad (t \geq 0) \tag{1}$$

with boundary conditions $y(0) = W$, and $y(T) = 0$.

Consider

$$\frac{dy(t)}{dt} - (\theta - \eta)y(t) = -e^{kt}. \tag{2}$$

The solution of equation (2) is given by

$$y(t) = \frac{-e^{kt}}{(k - \theta + \eta)} + Ce^{(\theta-\eta)t}. \tag{3}$$

Now, applying the boundary condition $y(0) = W$, we have

$$W = \frac{-1}{(k-\theta+\eta)} + C \tag{4}$$

and C is obtained as

$$C = W + \frac{1}{(k-\theta+\eta)}. \tag{5}$$

Substituting C in equation (3), we obtain

$$\begin{aligned} y(t) &= \frac{-e^{kt}}{(k-\theta+\eta)} + \Big(W + \frac{1}{(k-\theta+\eta)}\Big)e^{(\theta-\eta)t} \\ &= \frac{-e^{kt}}{(k-\theta+\eta)} + \frac{e^{(\theta-\eta)t}}{(k-\theta+\eta)}) + We^{(\theta-\eta)t}. \end{aligned} \tag{6}$$

Apply the boundary condition $y(T) = 0$ in equation (6), we have

$$0 = \frac{-e^{kT}}{(k-\theta+\eta)} + \frac{e^{(\theta-\eta)T}}{(k-\theta+\eta)}) + We^{(\theta-\eta)T}. \tag{7}$$

Now W is obtained as follows:

$$W = \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)} - \frac{1}{(k-\theta+\eta)}. \tag{8}$$

Substituting the value of W into equation (4), we get

$$\begin{aligned} y(t) &= \frac{e^{kt}}{(k-\theta+\eta)} + \frac{e^{(\theta-\eta)t}}{(k-\theta+\eta)}) \\ &\quad + \Big(\frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)} - \frac{1}{(k-\theta+\eta)}\Big)e^{(\theta-\eta)t} \\ &= \frac{1}{(k-\theta+\eta)}(e^{(k-\theta+\eta)T+(\theta-\eta)t)-e^{kt}}). \end{aligned} \tag{9}$$

(a) **Amount of Inventory H_T**

$$\begin{aligned} H_T &= \int_0^T y(t)dt \\ &= \int_0^T \Big[\frac{1}{(k-\theta+\eta)}\big(e^{(k-\theta+\eta)T+(\theta-\eta)t)-e^{kt}}\big)\Big]dt \\ &= \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\int_0^T e^{(\theta-\eta)t}dt - \frac{1}{(k-\theta+\eta)}\int_0^T e^{kt}dt \\ &= \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\Big[\frac{e^{(\theta-\eta)T}}{\theta-\eta} - \frac{1}{\theta-\eta}\Big] \\ &\quad - \frac{1}{(k-\theta+\eta)}\Big[\frac{e^{kt}}{k} - \frac{1}{k}\Big] \\ &= \frac{k\big(e^{kT} - e^{(k-\theta+\eta)T}\big) - (\theta-\eta)(e^{kT}-1)}{k(k-\theta+\eta)(\theta-\eta)}. \end{aligned} \tag{10}$$

(b) **Total Demand D_T**

$$\begin{aligned} D_T &= \int_0^T e^{kt}y(t)dt \\ &= \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\int_0^T e^{(k+\theta-\eta)t}dt \\ &\quad - \frac{1}{(k-\theta+\eta)}\int_0^T e^{2kt}dt \\ &= \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\left(\frac{e^{(k+\theta-\eta)T}-1}{k+\theta-\eta}\right) \\ &\quad - \frac{1}{(k-\theta+\eta)}\left(\frac{e^{2kT}-1}{2k}\right) \\ &= \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)(k+\theta-\eta)}\big(e^{(k+\theta-\eta)T}-1\big) \\ &\quad - \frac{1}{2k(k-\theta+\eta)}\big(e^{2kT}-1\big). \end{aligned} \tag{11}$$

(c) **Amount of Amelioration θ_T**

$$\begin{aligned} \theta_T &= \theta H_T \\ &= \frac{\theta\big(e^{kT} - e^{(k-\theta+\eta)T}\big)}{(k-\theta+\eta)(\theta-\eta)} - \frac{\theta(e^{kT}-1)}{k(k-\theta+\eta)} \end{aligned} \tag{12}$$

(d) **Amount of Deterioration η_T**

$$\begin{aligned} \eta_T &= \eta H_T \\ &= \frac{\eta\big(e^{kT} - e^{(k-\theta+\eta)T}\big)}{(k-\theta+\eta)(\theta-\eta)} - \frac{\eta(e^{kT}-1)}{k(k-\theta+\eta)}. \end{aligned} \tag{13}$$

(e) **Holding Cost $h_c(t)$**

$$\begin{aligned} h_c(t) &= \int_0^T (a+bt)y(t)dt \\ &= \int_0^T (a+bt)\left[\frac{\big(e^{(k-\theta+\eta)T+(\theta-\eta)t)-e^{kt}}\big)}{(k-\theta+\eta)}\right]dt \\ &= \frac{a(e^{kt} - e^{(k-\theta+\eta)T})}{(k^2-(\theta-\eta)^2)} + \frac{a(e^{(k-\theta+\eta)T})}{(k^2-(\theta-\eta)^2)} \\ &\quad + \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\left[\frac{T(e^{(\theta-\eta)T})}{\theta-\eta} - \frac{e^{(\theta-\eta)T}}{(\theta-\eta)^2}\right. \\ &\quad \left. + \frac{1}{(\theta-\eta)^2} - \frac{T(e^{aT})}{a} + \frac{e^{aT}}{a^2} - \frac{1}{a^2}\right]. \end{aligned} \tag{14}$$

(f) **Total Variable Cost**

$$\begin{aligned} Z(T) &= \frac{1}{T}\left(C + h_c(t) - c\theta_t + c\eta_T\right) \\ &= \frac{1}{T}\Bigg[C + \frac{a(e^{kt} - e^{(k-\theta+\eta)T})}{(k^2-(\theta-\eta)^2)} + \frac{a(e^{(k-\theta+\eta)T})}{(k^2-(\theta-\eta)^2)} \\ &+ \frac{e^{(k-\theta+\eta)T}}{(k-\theta+\eta)}\Big[\frac{T(e^{(\theta-\eta)T})}{\theta-\eta} - \frac{e^{(\theta-\eta)T}}{(\theta-\eta)^2} \\ &+ \frac{1}{(\theta-\eta)^2} - \frac{T(e^{aT})}{a} + \frac{e^{aT}}{a^2} - \frac{1}{a^2}\Big] \\ &- \frac{c\left(e^{kT} - e^{(k-\theta+\eta)T}\right)}{(k-\theta+\eta)} + \frac{c(\theta-\eta)(e^{kT}-1)}{k(k-\theta+\eta)}\Bigg]. \end{aligned} \tag{15}$$

4 Solution of the Crisp Model

The objective of the model is to minimize the total average cost. To achieve it, we need to follow

(i) find T^*such that$\frac{dZ(T^*)}{dt} = 0$;

(ii) if $\frac{d^2Z(T^*)}{dt^2} > 0$, then T^* is the minimal point of $Z(T)$.

5 Fuzzy Model

Due to rapid technological advancement and drastic climate changes, the rate of deterioration of items always has an imprecision. Similarly, many instant food products have a significant impact on the rate of amelioration. Thus, we have considered the rate of amelioration and deterioration as fuzzy parameters for the proposed model. The resultant total fuzzy cost of the inventory is as below:

$$\begin{aligned} \tilde{Z}(T) &= \frac{1}{T}\Bigg[C + \frac{a(e^{kt} - e^{(k-\tilde{\theta}+\tilde{\eta})T})}{(k^2-(\tilde{\theta}-\tilde{\eta})^2)} + \frac{a(e^{(k-\tilde{\theta}+\tilde{\eta})T})}{(k^2-(\tilde{\theta}-\tilde{\eta})^2)} \\ &+ \frac{e^{(k-\tilde{\theta}+\tilde{\eta})T}}{(k-\tilde{\theta}+\tilde{\eta})}\Big[\frac{T(e^{(\tilde{\theta}-\tilde{\eta})T})}{\tilde{\theta}-\tilde{\eta}} - \frac{e^{(\tilde{\theta}-\tilde{\eta})T}}{(\tilde{\theta}-\tilde{\eta})^2} \\ &+ \frac{1}{(\tilde{\theta}-\tilde{\eta})^2} - \frac{T(e^{aT})}{a} + \frac{e^{aT}}{a^2} - \frac{1}{a^2}\Big] \\ &- \frac{c\left(e^{kT} - e^{(k-\tilde{\theta}+\tilde{\eta})T}\right)}{(k-\tilde{\theta}+\tilde{\eta})} + \frac{c(\tilde{\theta}-\tilde{\eta})(e^{kT}-1)}{k(k-\tilde{\theta}+\tilde{\eta})}\Bigg]. \end{aligned} \tag{16}$$

6 Solution Algorithm for Fuzzy Model

In order to minimize the total cost, we have to follow the following steps:

Step-1: Let us consider the fuzzy parameters as trapezoidal fuzzy numbers for amelioration and deterioration. That is, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ and $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4)$.

Step-2: Find $Z_i(T)$ by replacing $\tilde{\theta}$ by θ_i and η by η_i for $(i = 1,2,3,4)$ in equation (16).

Step-3: Use Signed Distance Method for Defuzzification. That is,

$$SZ(T) = \frac{1}{4}\left[Z_1(T) + Z_2(T) + Z_3(T) + Z_4(T)\right].$$

Step-4: Find T^* such that$\frac{dSZ(T^*)}{dt} = 0$ and $\frac{d^2SZ(T^*)}{dt^2} > 0$.

Step-5: $SZ(T^*)$ is the fuzzy optimal cost.

7 Numerical Examples

Example 1

(a)*Crisp Model*

$k = 1.2$, $\theta = 0.6$, $\eta = 0.2$, $a = 5$, $b = 50$, $C = 710$ and $c = 120$.

Solution

$T = 1.053087, W = 1.65261, Z(T) = 5693.91.$

(b) *Fuzzy Model*

$k = 1.2$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.58, 0.59, 0.6, 0.61)$, $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4) = (0.18, 0.19, 0.2, 0.21)$, $a = 5$, $b = 50$, $C = 710$ and $c = 120$.

Solution

$T = 1.0567421,\ FW = 1.661118,\ SZ(T) = 5584.72.$

Example 2

(a) *Crisp Model*

$k = 1.4$, $\theta = 0.7$, $\eta = 0.4$, $a = 158$, $b = 600$, $C = 710$ and $c = 198$.

Solution

$T = 0.741759, W = 1.14663, Z(T) = 168467.$

(b) *Fuzzy Model*

$k = 1.4$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.68, 0.69, 0.7, 0.71)$, $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4) = (0.38, 0.39, 0.4, 0.41)$, $a = 158$, $b = 600$, $C = 710$ and $c = 198$.

Solution

$T = 0.741957,\ FW = 1.14707,\ SZ(T) = 163908.$

Example 3

(a) ***Crisp Model***

$k = 1.8$, $\theta = 0.6$, $\eta = 0.8$, $a = 15$, $b = 150$, $C = 100$ and $c = 80$.

Solution

$T = 0.564491$, $W = 1.04625$, $Z(T) = 91563.6$.

(b) ***Fuzzy Model***

$k = 1.8$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.58, 0.59, 0.6, 0.61)$, $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4) = (0.78, 0.79, 0.8, 0.81)$, $a = 15$, $b = 150$, $C = 100$ and $c = 80$.

Solution

$T = 0.56452$, $FW = 1.04634$, $SZ(T) = 89600.9$.

Example 4

(a) ***Crisp Model***

$k = 1.5$, $\theta = 0.5$, $\eta = 0.675$, $a = 1000$, $b = 900$, $C = 80000$ and $c = 1600$.

Solution

$T = 0.781134$, $FW = 1.61209$, $SZ(T) = 842359$.

(b) ***Fuzzy Model***

$k = 1.5$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.48, 0.49, 0.5, 0.51)$, $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4) = (0.655, 0.665, 0.675, 0.685)$, $a = 1000$, $b = 900$, $C = 80000$ and $c = 1600$.

Solution

$T = 0.783596$, $FW = 1.62121$, $SZ(T) = 823441$.

Example 5

(a) ***Crisp Model***

$k = 1.53$, $\theta = 1.6$, $\eta = 0.675$, $a = 158$, $b = 80$, $C = 70000$ and $c = 300$.

Solution

$T = 3.33102$, $W = 10.7483$, $Z(T) = 7051.47$.

(b) ***Fuzzy Model***

$k = 1.53$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4) = (1.58, 1.59, 1.6, 1.61)$, $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4) = (0.655, 0.665, 0.675, 0.685)$, $a = 158$, $b = 80$, $C = 70000$ and $c = 300$.

Solution

$T = 3.30835$, $FW = 10.5794$, $SZ(T) = 7670.6$.

Remark

From the above examples, it is clear that the total cycle time, economic ordering quantity in the fuzzy model is greater than the crisp model, and the total inventory cost in the fuzzy model is less than the crisp model in Examples 1 to 4. Whereas in Example 5, the cycle time, economic ordering quantity in the fuzzy model is less than the crisp model and the total inventory cost in the fuzzy model is greater than the crisp model.

8 Sensitivity Analysis

Here, we consider Example 1 to study the effect of changes in different parameters on the optimized results.

Table-1: Sensitivity of different parameters

parameter	Value	T	FW	$SZ(T)$
k	1.2	1.0567	1.66112	5584.72
	1.21	1.05058	1.65669	5513.69
	1.22	1.04449	1.65227	5446.25
	1.23	1.03847	1.64787	5382.16
	1.24	1.03253	1.64349	5321.16
C	710	1.05674	1.66112	5584.72
	810	1.07684	1.70834	5678.46
	910	1.09664	1.75554	5770.48
	1010	1.11614	1.80279	5860.86
	1110	1.1354	1.85018	5949.69
c	120	1.05674	1.66112	5584.72
	130	1.05843	1.66505	5580.29
	140	1.06013	1.66902	5575.85
	150	1.06185	1.67303	5571.4
	160	1.06358	1.67709	5566.93

From above Table 1 and Figures 1 to 9, the following managerial insights are drawn:

(i) The increase in value of demand parameter results the decrease in total cycle time, economic ordering quantity as well as the total inventory cost.
(ii) The increase in ordering cost result the increase in total cycle time, economic ordering quantity as well as the total inventory cost.
(iii) The increase in unit cost of the item results the increase in total cycle time, economic ordering quantity and decrease in the total inventory cost.

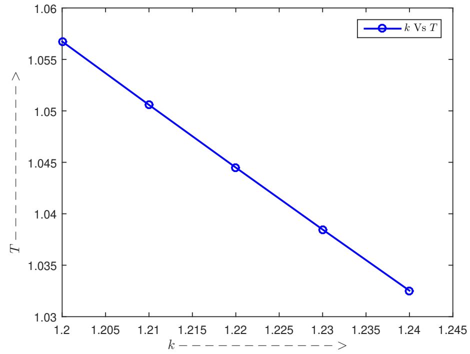

Fig. 1: k vs T

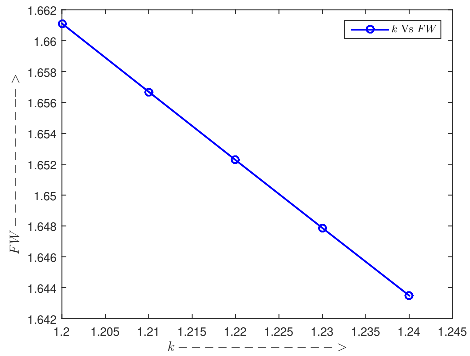

Fig. 2: k vs FW

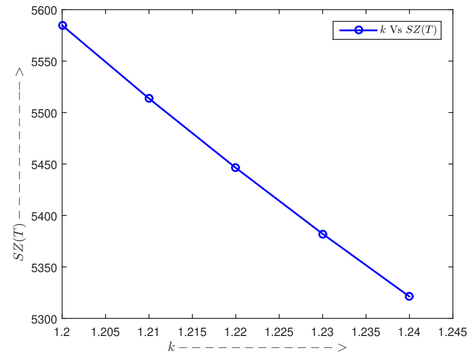

Fig. 3: k vs $SZ(T)$

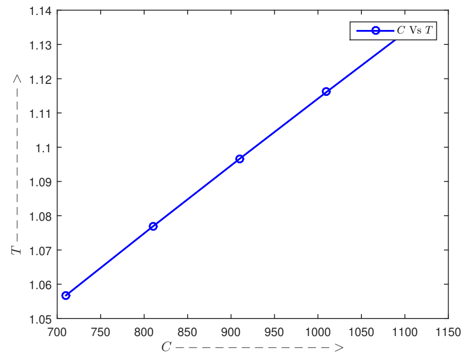

Fig. 4: C vs T

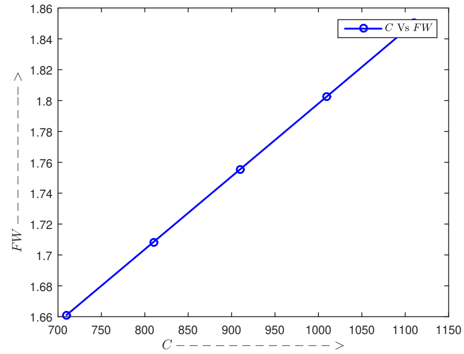

Fig. 5: C vs FW

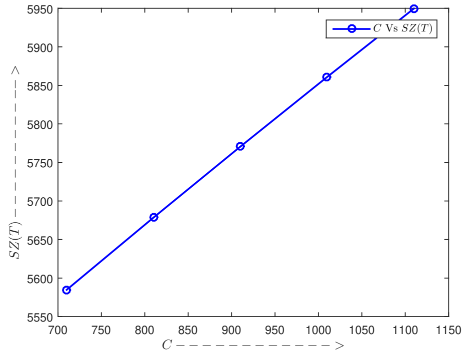

Fig. 6: C vs $SZ(T)$

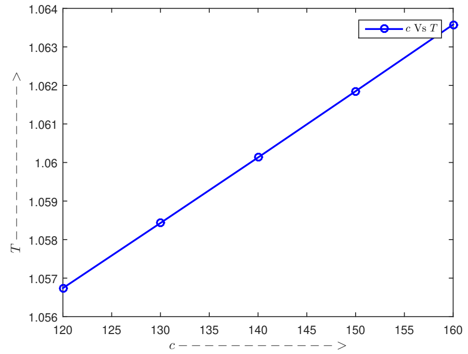

Fig. 7: c vs T

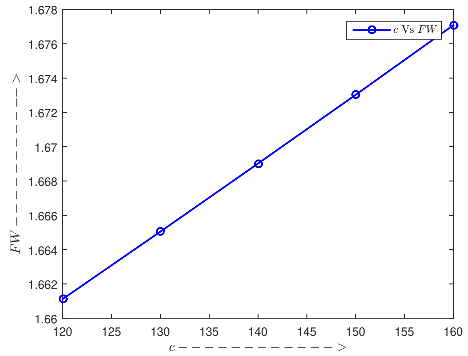

Fig. 8: c vs FW

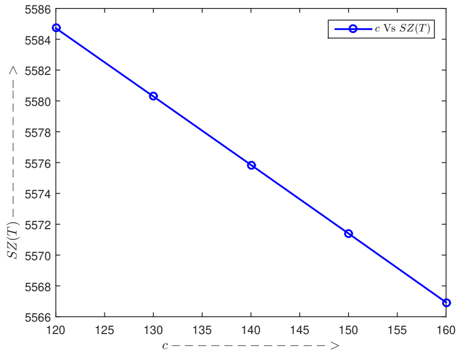

Fig. 9: c vs $SZ(T)$

9 Conclusion

The population density in cities increases due to various facilities, and many other reasons. Also, different types of food services are increasing day by day. Mostly, all these foodservice points have livestock as the raw material. Consequently, the inventory management of livestock items (ameliorating and deteriorating) is quite relevant in the present scenario. Thus, we developed the inventory model for both ameliorating and deteriorating items without any shortages. Moreover, we considered the model in a fuzzy environment to overcome the uncertainty of amelioration and deterioration. Here, we have taken the imprecise parameters as the trapezoidal fuzzy numbers, and the Signed Distance method is used for the defuzzification. Furthermore, we have considered different numerical examples to specify the effect of fuzzy amelioration and deterioration on the optimal results. Finally, the effect of changes in important parameters on the optimal results is tabulated and shown graphically.

This article can be extended by incorporating the shortages with backlogging, trade credit financing, inflation, various demands, and deteriorations under fuzzy environment.

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