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Predicting the Behavior of Solar Energy in Tulkarm City Using Markov Chains and Fuzzy Markov Chains

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Abstract: In this paper, we predict the behavior of solar energy in Tulkarm city, Palestine, using Markov chains and fuzzy Markov chains. Relying on solar radiation data in Tulkarm city during 2013–2016, two models are built that correspond to each season. A year is divided into four seasons, each of which consists of three months. Data from the period 2013–2015 are used to build the models, whereas data from 2016 is compared with the results. The accuracy of the models was better in the summer and winter.

Keywords: Solar energy, Markov chain, fuzzy numbers, fuzzy Markov chain.

1 Introduction

All related research has shown a steady growth in the use of renewable energy globally. Many of these studies focus on predicting renewable energy due to the global importance of solar radiation [1].

Fuzzy logic was introduced by Zadeh in 1965, when he created fuzzy sets to mathematically describe unclear situations [2]. Fuzzy matrices were introduced for the first time by Thomason in 1977, who discussed the convergence of powers of a fuzzy matrix [3]. In 1987, Kruse et al. introduced the fuzzy Markov chain as a classical Markov chain based on fuzzy probabilities, where they used a fuzzy set to denote the transition matrix with the uncertain data in the Markov chains [4].

Since 1965, many findings about fuzzy logic, fuzzy numbers, and fuzzy Markov chains have been obtained, for example [5] and [6–13]. Many authors used fuzzy logic and fuzzy Markov chains to predict certain phenomena, in particular solar energy [7–9].

In this paper, we apply the classical and fuzzy Markov chains to the prediction of the behavior of solar energy in Tulkarm city, Palestine. We deal with solar radiation data from 2012 to 2016 according to the months of the four seasons (winter, spring, summer, and autumn). Then, each day is compared with the previous day (10 a.m.) for each season, and Markov chain matrices are built to predict solar radiation behavior for each season. The Kingdom of Saudi Arabia has devoted a lot of money to strategic and large-scale infrastructure projects in different cities [6]. For example, the goal of the Saudi government, as stated in the Saudi Vision 2030, is to increase its capacity and enable 30 million people to visit Makkah for Umrah every year [8]. Therefore, to accomplish the vision of the Kingdom 2030, Makkah Roads Initiative played a main role in ensuring the success of the deal. Therefore, delayed government projects cost the government a lot of money and restrain people's lives, mainly local residents'.

The Ministry of Economy and Planning has established a program entitled Mashroat to support project management by applying the latest level of quality standards (international and regulations) and the top work practices in this field [10]. The government monitors the large financial assets expenditure, leadership directions, and insightful follow-ups of officials for such projects. However, the outcomes do not reflect the lawmakers' target levels, which results in failure to meet the citizens' requirements. The reasons behind delaying and suspending these projects are unclear. Furthermore, there are no work solutions or approaches to monitor these projects. The absence of project coordination among parties is a challenge that must be dealt with.

2 Mathematical Concepts

2.1 Markov chains

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Definition 1. A stochastic process is a family of random variables $\{X_t\}$ indexed by a parameter t, where t belongs to the index set T representing time. If t represents specific time points, we have a discrete time stochastic process, and we can replace the general subscript t by n. Hence, we talk about the discrete time process $\{X_n\}$. In general, for a discrete time process, the random variable X_n depends on earlier values of the process, X_{n-1},X_{n-2}, etc. Similarly, in continuous time, X_t generally depends on values X_u for $u < t$.

Therefore, we are often interested in conditional distributions of the form $P_r(X_{t_k} | X_{t_{k-1}}, X_{t_{k-2}}, \ldots, X_{t_1})$ for some sets of times $t_k > t_{k-1} > \cdots > t_1$.
In general, this conditional distribution depends on the values of $X_{t_{k-1}}, X_{t_{k-2}}, \ldots, X_{t_1}$.

Definition 2. A Markov chain is a sequence of random variables with the Markov property; which is the present state depends only on the last state and does not depend on the states before the last state.
Let S be a countable set. Suppose that to each i and j in S, there is assigned a nonnegative number p_{ij} and that these numbers satisfy the constraint $\sum_{j \in S} p_{ij} = 1, \forall\ , j \in S$. (1)
Let X_t denote a random variable representing the state of a system at time t, where t = 0, 1, 2, etc. If X_{t+1} only depends on the state X_t and does not depend on the states before X_t, then formally, the following is true:

$$P(X_{n+1} = x_{n+1} \mid X_1 = x_1, X_2 = x_2, \ldots\ldots X_n = x_n) = P(X_{n+1} = x_{n+1} \mid X_n = x_n) \quad (2)$$

We say X_t with Markov property, and the sequence $\{X_t\}$ is a Markov chain. If

$P(X_{n+1} = x_{n+1} \mid X_n = x_n) = P(X_n = x_n \mid X_{n-1} = x_{n-1})$, (3)

then $\{X_t\}$ is called homogeneous, otherwise it is called inhomogeneous.

Let p_{ij} denote the probability that the system is in a state j at time t +1, given the system is in state i at time t. If the system has a finite number of states, 1, 2……n, the homogeneous Markov chain can be defined by the following transition probability matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{2n} & \cdots & p_{nn} \end{bmatrix}$$

The transition probability matrix of a homogeneous Markov chain can be generated from the observations of the previous system state.
When we have the observations of the system states $X_0, X_1, X_2,\ldots X_n$ with respect to time, we can get the following transition probability matrix:

$$P_{ij} = \frac{N_{ij}}{N_i}, \quad (4)$$

where N_{ij} is the number of observation pairs X_t and X_{t+1} with X_t in state i and X_{t+1} in state j; and N_i is the number of observation pairs X_t and X_{t+1} with X_t in state i and X_{t+1} in any state.

Definition 3. If S is a finite state space, then:

1. A state $j \in S$ is *persistent (or recurrent)* if, upon entering this state, the process will definitely return to this state again. Otherwise, it is transient.
2. A state $j \in S$ is periodic with period t if $p^n_{ij} > 0$ implies that t divides $\boldsymbol{n}$ and $\boldsymbol{t}$ is the largest integer with this property. In other words, the period of j is the greatest common divisor of the set of integers $\{n: n \geq 1, p^n_{ij} > 0\}$. If t = 1, then the state is aperiodic (or non-periodic).
3. A Markov chain is called irreducible if $\exists n \in \mathbb{N}$, such that $p^n_{ij} > 0,\ \forall i,\ j \in S$, otherwise it is called reducible. That is, a Markov chain is irreducible if and only if every state can be reached from every other state in a finite number of steps.
4. An aperiodic persistent state $j \in S$ is called ergodic. Therefore, a Markov chain is ergodic if all its states are ergodic.

Let $P = [p_{ij}]$ the transition matrix of a Markov chain X_n, $n \geq 0$, the n^{th} power of P is $P^n = [p^n_{ij}]$, where p^n_{ij} represents the probability of a transition from state i to state j in n steps, and p^n_{ij} is called the n step transition probability for the Markov chain.
For all $n > 1$ and i, j in the state space S, $p^n_{ij} = P[X_n = j \mid X_0 = i]$.
A set of probabilities (π_j), $j \in S$ satisfying the following:
$\sum_{i \in S} \pi_i\, p_{ij} = \pi_j, i, j \in S$ (5)
is called a stationary distribution. For an irreducible aperiodic finite Markov chain, there exists a unique stationary distribution satisfying the following:
$\pi_j > 0$ and $\sum_{j \in S} \pi_j = 1$.
In matrix language, (5) means $\pi P = \pi$, and $\lim_{n\to\infty} {p_{ij}}^n = \pi_j$ (6).

That is, the limit does not depend on the initial state i. The limit of the power of P consists of identical rows each of which is π, that is:

$$\lim_{n\to\infty} P^n = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_n \\ \pi_1 & \pi_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_n \end{bmatrix}$$

and for any n, $\pi P^n = \pi$.

2.2 Fuzzy Numbers

One of the important concepts of fuzzy logic is fuzzy numbers. There are several types of fuzzy numbers, including Gaussian, trapezoidal, triangular, sigmoid, and bell-shaped [10], [11], and [12].

In this paper, triangular fuzzy numbers are used to fuzzify the crisp values. A triangular membership function is:

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

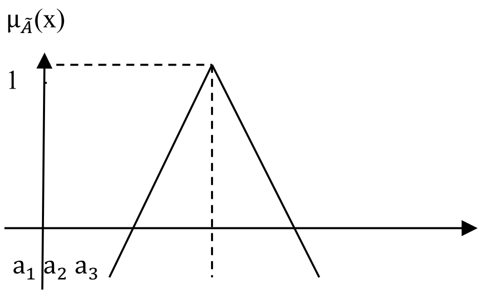

Fig.1: A triangular fuzzy number $\tilde{N} = (a_1 \,/\, a_2 \,/ a_3)$.

A triangular fuzzy number $\tilde{N}$ is defined by three numbers: $a_1 < a_2 < a_3$, where $\tilde{N}\,(x) = 1$ at x = a_2 .The graph of $\tilde{N}\,(x)$ on $[a_1\,, a_2]$ is a straight line from $(a_1\;,\;0\;)$ to $(a_2\,, 1)$. Further, on $[a_2\,, a_3]$, the graph is a straight line from $(a_2, 1)$ to $(a_3, 0)$. $\tilde{N}$ = 0 for x ≤ $a_1$or x ≤ a_3. We write $\tilde{N}$ = $(a_1 \,/\, a_2 \,/ a_3)$ for a triangular fuzzy number $\tilde{N}$.

2.3 Fuzzy Markov Chains

Many authors applied the fuzzy theory to Markov chains to generate the fuzzy Markov chains [5], [6], [9], and [13]. A fuzzy array is used to refer to the transition matrix with uncertain data. In fuzzy Markov chains, fuzzy matrix multiplication is used to calculate the fuzzy transition matrix from n steps. The transition probability may change to meet the uncertainty in Markov chains and uses a fuzzy number to indicate the possibility of a mysterious transmission.

The fuzzy set package is aimed at computing a vector of fuzzy stationary probabilities $\tilde{\pi}$= ($\tilde{\pi}_1$,………, $\tilde{\pi}_n$) departing from an uncertain (fuzzy) transition matrix $\tilde{P}$.

In most applications of Markov chains, we get the data using estimation or experience, which makes the data imprecise. It is necessary to take the fuzziness into consideration when dealing with these problems. The fuzzy transition probability is used to replace the traditional crisp transition probability, which is the foot of the fuzzy Markov chain model. Making the transition probability fuzzy is the most scientific method when precise values cannot be obtained without enough information.

Though the values are fuzzy, it is closer to the essence of the decision-making problems.

Our data consist of a finite sequence of observations drawn from the system at consecutive time points. Each observation is the state of the system at that time point and is represented by an integer belonging to the state space of the chain. In such sequences, an estimation of the transition probability p_{ij} can be computed as the number of times N_{ij} that state i was followed by j in our data, divided by the number of transitions N_i observed from i to another state (including i itself). However, this would just be a point estimate and does not capture the uncertainty associated with it. Interval estimation would be the next step, as it also considers the sample size and variance of the data; however, it only calculates an interval for a significance level that is fixed beforehand.

Our aim is to model each uncertain transition probability as a fuzzy number that properly captures all the uncertainty of the data, regardless of external parameters, such as the confidence level. For this purpose, the fuzzy number is obtained by the superposition of confidence intervals for the true transition probabilities at different confidence levels, one on top of another. Although the significance level and membership degree are numerically the same in this case, it should be noted that they are completely different concepts for which the letter is commonly used. Such intervals are simultaneous confidence intervals for multinomial proportions, since the problem of estimating the transition probabilities from a given state can be reduced to that of estimating the parameters (proportions) of a multinomial distribution.

In this paper, a triangle fuzzy number is used to represent the transition. Three values are easily obtained to estimate the transition probability, which is the maximum, minimum, and most probable values, respectively. When the traditional Markov chain model deals with the decision-making problems, the most probable value is considered, namely p_{ij}, while information about the maximum and minimum values is neglected. Therefore, the traditional method cannot reflect the essence of the decision-making problems. Let the peak point of the triangle fuzzy number be (V, 1), namely $\tilde{p}_{ij}[1]$ = V, and the support interval of the triangle fuzzy number satisfies $\tilde{p}_{ij}[0] = [V_{min}, V_{max}]$. Using the triangle fuzzy number to represent the transition probability means that the fuzzy Markov chain model contains uncertain information, which can describe the decision-making problem more precisely.

It can be concluded through $p_{ij} = \tilde{p}_{ij}\,[1]$ that the traditional Markov chain model is a special case of the fuzzy Markov chain model, whereas the fuzzy Markov chain model is an extension of the traditional Markov chain model at the fuzziness aspect.

Fuzzy numbers are used for those entries of the transition matrix where there is uncertainty. The rest of the elements could be crisp, since a crisp number is a special fuzzy number. However, the uncertainty is due to the probabilities rather than the fact that every row must add to 1.

Now, assume a Markov chain with r states and fuzzy transition matrix $\tilde{P}_{ij}$; then, for a given α ∈ [0; 1], $\tilde{P}$ (α) = $(\tilde{p}_{ij}(\alpha))$ represents a matrix of intervals. Such a matrix can also be thought of as the set of all $r \times r$ matrices M, such that their elements fall inside their corresponding interval, $m_{ij} \in \tilde{p}_{ij}(\alpha)$, and every row adds to 1.The domain of row i for a given α value is defined as the set of r-dimensional vectors that simultaneously fulfill those two constraints for row i, i.e., the set of probability distributions that row i could take in our uncertain transition matrix when we take its α-cuts for that α.

Let $Q = [q_{ij}]$ be an $r \times r$ transition matrix of a Markov chain. If a q_{ij}= 0 or q_{ij} = 1, then we assume that there is no uncertainty in this value, otherwise we assume there is uncertainty in the transition probability q_{ij} i.e., when $0 < q_{ij} < 1$. In the last case, we replace each of q_{ij} with a fuzzy number $\tilde{p}_{ij}$, where $0 \tilde{<} p_{ij} < 1$, with the restriction that there are $p_{ij} \in \tilde{p}_{ij}[1]$, such that $P = [p_{ij}]$ is a transition matrix. We then define the fuzzy transition matrix $P = [\tilde{p}_{ij}]$, with the understanding that $\tilde{p}_{ij} = 0$ when q_{ij}= 0 and $\tilde{p}_{ij}$= 1 when q_{ij} = 1.

The restriction that there are $p_{ij} \in \tilde{p}_{ij}[\alpha]$, such that $P = [p_{ij}]$ is a transition matrix, guarantees that $p_{ij} \in \tilde{p}_{ij}[\alpha]$ for all $0 \le \alpha \le 1$. Since $\tilde{p}_{ij}$ is a fuzzy number, then $\tilde{p}_{ij}[\alpha]$ is a closed and bounded interval for all $0 \le \alpha \le 1$, so we let $\tilde{p}_{ij}[\alpha] = [\tilde{p}_{ij1}(\alpha), \tilde{p}_{ij2}(\alpha)]$.

3 Solar Radiation Data for the Seasons

We studied every value we had in our data and formatted it to investigate the relationship between the date and the solar radiation for all years and seasons (spring, autumn, summer, and winter) to understand the behavior.

We studied all the values to investigate the relationship between history and solar radiation throughout 2013–2016 and all seasons (spring, autumn, summer, and winter) to study the behavior of solar radiation.

The data for solar radiation for the winter season in the three months (December, January, and February) in three years from December 1, 2012, to February 2, 2015, have a highest value of 0.386 and lowest value 0.007 at 10 a.m.
In the spring months (March, April, and May), the maximum is 0.622, and the minimum is 0.169 at 10 a.m.
In the summer months (June, July, and August), the maximum is 0.644, and the minimum is 0.31 at 10 a.m.
Finally, in the autumn months (September, October, and November), the maximum is 0.386, and the minimum is 0.064 at 10 a.m.

The states of the Markov chain are all possible differences between each day and the day before it (rounding to the nearest decimal). The difference in weather and its fluctuation between moderate to cold or hot lead to differences.
The graphs below show the data for the solar energy for the seasons (at 10 a.m.).

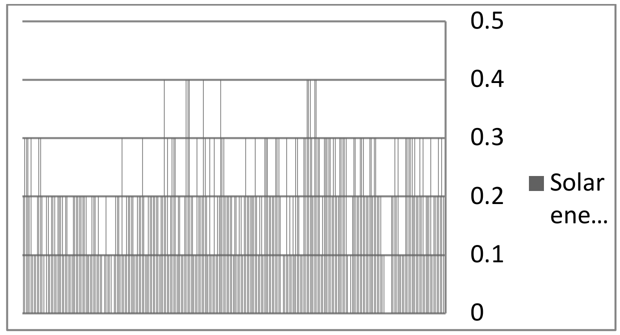

Fig.2: Winter season.

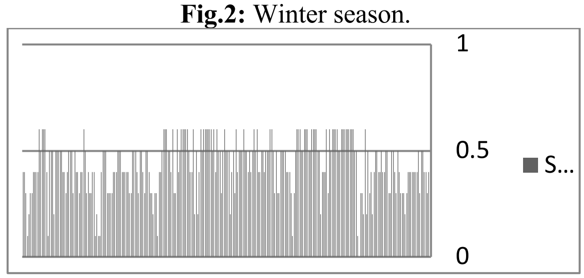

Fig.3: Spring season.

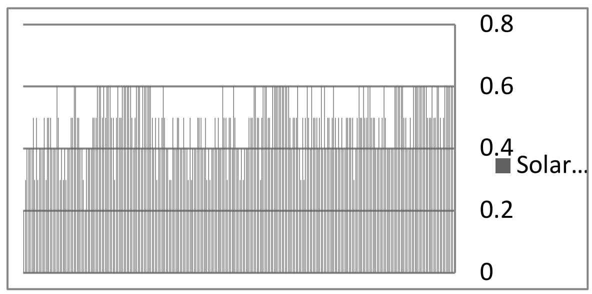

Fig.4: Summer season.

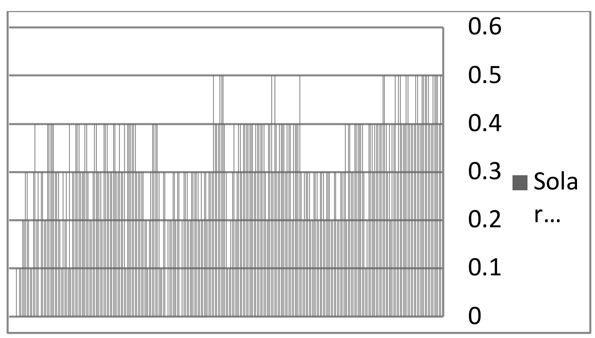

Fig. 5: Autumn season.

4 The Models: Markov Chains and Fuzzy Markov Chains

4.1 Winter Season

4.1.1 The Markov Chain Model

We calculate the change between the past and the present. We have five states, namely 0, 0.1, 0.2, -0.1, and -0,2.

Table 1: From one state to another.

State	0	0.1	0.2	-0.1	-0.2	Total
0	58	20	6	28	9	121
0.1	33	8	1	13	1	56
0.2	4	1	-	5	6	16
-0.1	17	20	9	8	-	54
-0.2	10	6	-	1	-	17

The probability transition matrix is as follows:

$$P = \begin{bmatrix} \frac{58}{121} & \frac{20}{121} & \frac{6}{121} & \frac{28}{121} & \frac{9}{121} \\ \frac{33}{56} & \frac{8}{56} & \frac{1}{56} & \frac{13}{56} & \frac{1}{56} \\ \frac{4}{16} & \frac{1}{16} & 0 & \frac{5}{16} & \frac{6}{16} \\ \frac{17}{54} & \frac{20}{54} & \frac{9}{54} & \frac{8}{54} & 0 \\ \frac{10}{17} & \frac{6}{17} & 0 & \frac{1}{17} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.48 & 0.17 & 0.05 & 0.23 & 0.07 \\ 0.59 & 0.14 & 0.02 & 0.23 & 0.02 \\ 0.25 & 0.06 & 0 & 0.31 & 0.38 \\ 0.31 & 0.37 & 0.17 & 0.15 & 0 \\ 0.58 & 0.35 & 0 & 0.06 & 0 \end{bmatrix}$$

The Markov chain is ergodic, so we can find the steady state by applying Equation **(6)**, which is:

$$\lim_{n\to\infty} P^n = \begin{bmatrix} 0.46 & 0.21 & 0.06 & 0.21 & 0.06 \\ 0.46 & 0.21 & 0.06 & 0.21 & 0.06 \\ 0.46 & 0.21 & 0.06 & 0.21 & 0.06 \\ 0.46 & 0.21 & 0.06 & 0.21 & 0.06 \\ 0.46 & 0.21 & 0.06 & 0.21 & 0.06 \end{bmatrix}$$

P^7 consists of identical rows, and each row is π, which means the effect of the initial state disappears after seven steps.

The Markov chain method is a special method of predicting behavior over a longer period. The limit of the matrix means the following: The probability that the solar radiation at a specific day at 10:00 a.m. will remain unchanged from the previous day is 0.46. The probability that the solar radiation on a specific day at 10:00 a.m. will increase by 0.1 from the previous day is 0.21. The probability that the solar radiation on a specific day at 10:00 a.m. will increase by 0.2 from the previous day is 0.06. The probability that the solar radiation on a specific day at 10:00 a.m. will decrease by 0.1 from the previous day is 0.21. Finally, the probability that the solar radiation on a specific day at 10:00 a.m. will decrease by 0.1 from the previous day is 0.06.

4.1.2 The Fuzzy Markov Chain Model

Using triangular fuzzy numbers (Figure 6), the fuzzy Markov Chain $\widetilde{P}$ is:

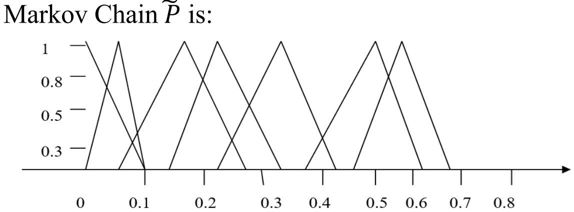

Fig.6: Triangular Fuzzy number, winter season.

The fuzzy Markov Chain $\widetilde{P}$ is:

$$\begin{bmatrix} (0.38/0.48/0.58) & (0.07/0.17/0.27) & (0/0.05/0.10) & (0.13/0.23/0.33) & (0.02/0.07/0.12) \\ (0.49/0.59/0.69) & (0.04/0.14/0.24) & (0/0.01/0.02) & (0.13/0.23/0.33) & (0/0.01/0.02) \\ (0.15/0.25/0.35) & (0.01/0.06/0.11) & (-0.1/0/0.1) & (0.21/0.31/0.41) & (0/0.01/0.02) \\ (0.21/0.31/0.41) & (0.27/0.37/0.47) & (0.07/0.17/0.27) & (0.05/0.15/0.25) & (-0,1/0/0.1) \\ (0.48/0.58/0.68) & (0.25/0.35/0.45) & (-0.10/0/0.10) & (0/0.06/0.12) & (-.10/0/0.10) \end{bmatrix}$$

If $\tilde{A}$ [α] = $[a_1(\alpha), a_2(\alpha)]$, then $a_1(\alpha)$ and $a_2(\alpha)$ can be obtained by solving $\tilde{A}$ [α] = $[(a_1 + (a_3 - a_2)\alpha, a_3 + (a_1 - a_2)\alpha]$.

Choosing different values of α, we get:

Table 2: From α–cut.

$\tilde{\pi}_j$	α [0.3]	α [0.5]	α [0.8]	α [0]	α [1]
$\tilde{\pi}_0$	(0.39,0.53)	(0.41,0.51)	(0.44,0.48)	(0.36,0.56)	(0.46)
$\tilde{\pi}_{0.1}$	(0.14,0.23)	(0.16,0.26)	(0.19,0.23)	(0.11,0.31)	(0.21)
$\tilde{\pi}_{0.2}$	(0.02,0.09)	(0.03,0.8)	(0.04,0.07)	(0.01,0.11)	(0.06)
$\tilde{\pi}_{-0.1}$	(0.14,0.28)	0.16,0.26)(	(0.19,0.23)	(0.11,0.310	(0.21)
$\tilde{\pi}_{-0.2}$	(0.02,0.09)	(0.03,0.08)	(0.04,0.07)	(0.01,0.11)	(0.06)

Notice that the length of the interval is getting smaller and closer to the top until it reaches α =1, which returns to the normal value (crisp). Further, α = 0 is the longest period and repeats $[a_1, a_3]$.

In the next sections, we do the same for the other seasons.

4.2 Spring Season

4.2.1 Markov Chain

We have nine stats (0,0.1,0.2,0.3,0.4, -0,1, -0,2, -0.3, -0.4).

Table 3: From one state to another.

Stat.	0	0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4	Total
0	53	22	4	1	-	25	7	3	-	115
0.1	29	6	-	-	1	8	6	1	-	51
0.2	6	3	1	-	-	5	1	2	-	18
0.3	4	-	1	-	-	-	2	1	-	8
0.4	1	-	-	-	-	-	-	-	2	3
-0.1	13	14	5	2	-	7	2	-	1	44
-0.2	7	4	6	2	-	-	2	1	-	22
-0.3	1	1	2	2	2	-	-	-	-	8
-0.4	1	1	-	1	-	-	1	-	-	4

The transition matrix is:

$$Q=\begin{bmatrix}0.46 & 0.19 & 0.03 & 0.01 & 0 & 0.22 & 0.06 & 0.03 & 0\\ 0.57 & 0.12 & 0 & 0 & 0.02 & 0.16 & 0.12 & 0.01 & 0\\ 0.33 & 0.19 & 0.05 & 0 & 0 & 0.27 & 0.05 & 0.11 & 0\\ 0.5 & 0 & 0.12 & 0 & 0 & 0 & 0.25 & 0.13 & 0\\ 0.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.67\\ 0.29 & 0.32 & 0.11 & 0.05 & 0 & 0.16 & 0.05 & 0 & 0.02\\ 0.32 & 0.18 & 0.27 & 0.09 & 0 & 0 & 0.09 & 0.05 & 0\\ 0.12 & 0.13 & 0.25 & 0.25 & 0.25 & 0 & 0 & 0 & 0\\ 0.25 & 0.25 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0\end{bmatrix}$$

For n ≥ 9, Q^n consists of identical rows, and each row is π, where π is:

π_0 = 0.42, $\pi_{0.1}$ = 0.13, $\pi_{0.2}$ = 0.06, $\pi_{0.3}$ = 0.02, $\pi_{0.4}$ = 0.06

$\pi_{-0.1}$ = 0.16, $\pi_{-0.2}$ = 0.07 $\pi_{-0.3}$= 0.02, $\pi_{-0.4}$ = 0.11

4.2.2 Fuzzy Markov Chain

Table 4: From α–cut.

π_j	$\alpha[0.3]$	$\alpha[0.5]$	$\alpha[0.8]$	$\alpha[0]$	$\alpha[1]$
π_0	(0.35,0.49)	(0.37,0.47)	(0.40,0.44)	(0.32,0.52)	(0.42)
$\pi_{0.1}$	(0.06,0.200	(0.08,0.18)	(0.11,0.15)	(0.03,0.23)	0.13
$\pi_{0.2}$	(0.02,0.09)	(0.03,0.08)	(0.05,0.07)	(0.01,0.11)	0.06
$\pi_{0.3}$	(0.013,0.027)	(0.015,0.025)	(0.018,0.022)	(0.01,0.03)	0.02
$\pi_{0.4}$	(0.003,0.017)	(0.005,0.015)	(0.008,0.012)	(0,0.02)	0.01
$\pi_{-0.1}$	(0.09,0.23)	(0.11,0.21)	(0.14,0.18)	(0.06,0.26)	0.16
$\pi_{-0.2}$	(0.03,0.09)	(0.04,0.095)	(0.06,0.08)	(0.02,0.12)	0.07
$\pi_{-0.3}$	(0.013,0.027)	(0.015,0.025)	(0.018,0.022)	(0.01,0.03)	0.02
$\pi_{-0.4}$	(0.04,0.18)	(0.06,0.16)	(0.09,0.13)	(0.01,0.21)	0.11

4.3 Summer Season

4.3.1 Markov Chain

We have five stats (0, 0.1, 0.2, -0,1, -0,2).

Table 5: From one state to another.

Stat.	0	0.1	0.2	-0.1	-0.2	Total
0	59	25	5	22	4	115
0.1	25	9	1	20	5	60
0.2	5	1	-	6	5	17
-0.1	22	17	7	10	3	59
-0.2	4	6	9	-	-	19

$$R=\begin{bmatrix}\frac{59}{115} & \frac{25}{115} & \frac{5}{115} & \frac{22}{115} & \frac{4}{115}\\ \frac{25}{60} & \frac{9}{60} & \frac{1}{60} & \frac{20}{60} & \frac{5}{60}\\ \frac{5}{17} & \frac{1}{17} & 0 & \frac{6}{17} & \frac{5}{17}\\ \frac{22}{59} & \frac{17}{59} & \frac{7}{59} & \frac{10}{59} & \frac{3}{59}\\ \frac{4}{19} & \frac{6}{19} & \frac{9}{19} & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0.51 & 0.22 & 0.04 & 0.19 & 0.04\\ 0.42 & 0.15 & 0.02 & 0.33 & 0.08\\ 0.29 & 0.06 & 0 & 0.36 & 0.29\\ 0.37 & 0.28 & 0.13 & 0.17 & 0.05\\ 0.21 & 0.32 & 0.47 & 0 & 0\end{bmatrix}$$

$$R^{13}=\begin{bmatrix}0.42 & 0.21 & 0.08 & 0.22 & 0.07\\ 0.42 & 0.21 & 0.08 & 0.22 & 0.07\\ 0.42 & 0.21 & 0.08 & 0.22 & 0.07\\ 0.42 & 0.21 & 0.08 & 0.22 & 0.07\\ 0.42 & 0.21 & 0.08 & 0.22 & 0.07\end{bmatrix}$$

This means for n ≥ 13, R^n consists of identical rows, and each row is π.

4.3.2 Fuzzy Markov Chain

Table 6: From α–cut.

$\tilde{\pi}_j$	α[0.3]	α[0.5]	α[0.8]	α[0]	α[1]
$\tilde{\pi}_0$	(0.35,0.49)	(0.37,0.47)	(0.40,0.44)	(0.32,0.52)	0.42
$\tilde{\pi}_{0.1}$	(0.14,0.28)	(0.16,0.26)	(0.19,0.23)	(0.11,0.31)	0.21
$\tilde{\pi}_{0.2}$	(0.04,0.11)	(0.05,0.10)	(0.07,0.09)	(0.03,0.13)	0.08
$\tilde{\pi}_{-0.1}$	(0.15,0.29)	(0.17,0.27)	(0.20,0.24)	(0.12,0.32)	0.22
$\tilde{\pi}_{-0.2}$	(0.03,0.10)	(0.04,0.09)	(0.06,0.08)	(0.2,0.12)	0.07

4.4 Autumn Season

4.4.1 Markov Chain

We have seven stats (0,0.1,0.2,0.3, -0,1, -0,2, -0.3).

Table 7: From one state to another.

state	0	0.1	0.2	0.3	-0.1	-0.2	-0.3	Total
0	36	21	2	-	34	10	1	104
0.1	35	7	1	-	3	2	1	49
0.2	1	3	-	-	8	-	-	12
0.3	2	-	-	-	2	-	1	5
-0.1	26	16	3	-	6	3	-	54
-0.2	4	3	5	3	1	-	-	16
-0.3	-	-	1	2	-	-	-	3

$$S=\begin{bmatrix} \frac{36}{131} & \frac{21}{131} & \frac{2}{131} & 0 & \frac{34}{131} & \frac{10}{131} & \frac{1}{131} \\ \frac{35}{49} & \frac{7}{49} & \frac{1}{49} & 0 & \frac{3}{49} & \frac{2}{49} & \frac{1}{49} \\ \frac{1}{12} & \frac{3}{12} & 0 & 0 & \frac{8}{12} & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & \frac{1}{5} \\ \frac{26}{54} & \frac{16}{54} & \frac{3}{54} & 0 & \frac{6}{54} & \frac{3}{54} & 0 \\ \frac{4}{16} & \frac{3}{16} & \frac{5}{16} & \frac{3}{16} & \frac{1}{16} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0.35 & 0.20 & 0.02 & 0 & 0.33 & 0.09 & 0.01 \\ 0.72 & 0.14 & 0.02 & 0 & 0{,}06 & 0.04 & 0.02 \\ 0.08 & 0.25 & 0 & 0 & 0.67 & 0 & 0 \\ 0.40 & 0 & 0 & 0 & 0.40 & 0 & 0.20 \\ 0.48 & 0.29 & 0.06 & 0 & 0.11 & 0.06 & 0 \\ 0.25 & 0.19 & 0.31 & 0.19 & 0.06 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 & 0 & 0 \end{bmatrix}$$

For n ≥ 13, S^nconsists of identical rows, and each row is π, where:

π_0= 0.49, $\pi_{0.1}$ = 0.19, $\pi_{0.2}$ = 0.04, $\pi_{0.3}$= 0.02, $\pi_{-0.1}$ = 0.19, $\pi_{-0.2}$ = 0.06, $\pi_{-0.3}$ = 0.01.

4.4.2 Fuzzy Markov Chain

Table 8: From α–cut.

π_j	α [0.3]	α [0.5]	α [0.8]	α [0]	α [1]
π_0	(0.42,0.52)	(0.44,0.54)	(0.47,0.51)	(0.39.0.59)	0.49
$\pi_{0.1}$	(0.12,0.26)	(0.14,0.24)	(0.17,0.21)	(0.09,0.29)	0.19
$\pi_{0.2}$	(0.033, 0.047)	(0.035,0.045)	(0.038,0.042)	(0.03,0.05)	0.04
$\pi_{0.3}$	(0.013, 0.027)	(0.015,0.025)	(0.018,0.022)	(0.01,0.03)	0.02
$\pi_{-0.1}$	(0.12,0.26)	(0.14,0.24)	(0.17,0.21)	(0.09,0.29)	0.19
$\pi_{-0.2}$	(0.053, 0.067)	(0.055,0.065)	(0.058,0.062)	(0.05,0.07)	0.06
$\pi_{-0.3}$	(0.003, 0.017)	(0.005,0.015)	(0.008,0.012)	(0,0.02)	0.01

5 Comparison

We consider the data of the year 2016 and compare them with the results. For example, for the winter season, the transition matrix is:

$$Z=\begin{bmatrix} 0.48 & 0.13 & 0 & 0.3 & 0.08 \\ 0.58 & 0.18 & 0 & 0.24 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.38 & 0.38 & 0 & 0.19 & 0.05 \\ 0.5 & 0 & 0.25 & 0.25 & 0 \end{bmatrix}$$

$$\lim_{n\to\infty} Z^n=\begin{bmatrix} 0.48 & 0.19 & 0.03 & 0.25 & 0.05 \\ 0.48 & 0.19 & 0.03 & 0.25 & 0.05 \\ 0.48 & 0.19 & 0.03 & 0.25 & 0.05 \\ 0.48 & 0.19 & 0.03 & 0.25 & 0.05 \\ 0.48 & 0.19 & 0.03 & 0.25 & 0.05 \end{bmatrix}$$

In the Markov chain model, the probability that it will not change is 0.46, which is close to the real data of 0.48. The probability that it increases by 0.1 is 0.21, which is close to the real data of 0.19. The probability that it increases by 0.2 is 0.06, which is close to the real data of 0.03. The probability that it decreases by 0.1 is 0.21, which is close to the real data of 0.25. Finally, the probability that it decreases by 0.2 is 0.06, which is close to the real data of 0.05.

In fuzzy Markov chain, it suffices to be a ratio of the steady state π_n and is located in the triangular fuzzy number of the Markov chain. In the unchanging case in the Markov chain method, the probability is 0.48 in 2016. Further, there is a triangular fuzzy number (0.36/0.46/0.56), and the probability that it increases by 0.1 is 0.19 in 2016. For this ratio, there is a triangular fuzzy number (0.11/0.21/0.31).
By testing the four-season fuzzy models, the accuracy of the models is better in the summer and winter seasons than in the spring and autumn.

The Markov chain for each season is ergodic, which enables us to find the steady state for each season. The method in this paper is generalizable to many areas.
The fuzzy logic approach is a strong alternative for accurate prediction. Allowing intermediate values between 0 and 1, the fuzzy sets theory uses mathematical formulas to capture the uncertainties associated with natural phenomena.

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