Applied Mathematics & Information Sciences

Volume 10
Issue 5 *Sep. 2016*

Article 29

9-1-2016

A Study on Recovery Rate for Counter - Current Imbibition Phenomenon with Corey's Model Arising during Oil Recovery Process

Hardik S. Patel
Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat 395007, India, meher_ramakanta@yahoo.com

Ramakanta Meher
Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat 395007, India, meher_ramakanta@yahoo.com

Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100529

A Study on Recovery Rate for Counter - Current Imbibition Phenomenon with Corey's Model Arising during Oil Recovery Process

*Hardik S. Patel and Ramakanta Meher**

Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat 395007, India

Received: 20 Apr. 2016, Revised: 19 Jun. 2016, Accepted: 20 Jun. 2016
Published online: 1 Sep. 2016

Abstract: This paper presents a new numerical formulation of Counter – Current Imbibition Phenomenon for the simulation of immiscible and incompressible phase flow in heterogeneous porous media for Corey's model and Scheidegger-Johnson model. Adomian Decomposition Method is used numerically to investigate the saturation rate as well as the recovery rate for both models with its convergence analysis. A detailed discussion of the saturation rate as well as recovery rate with dimensionless time is studied for both models with its comparison study and its physical interpretation.

Keywords: Counter – current imbibition, Capillary pressure, Heterogeneous porous media, Adomian Decomposition Method

1 Introduction

Counter-current imbibition is a process whereby a wetting phase spontaneously imbibes into a porous media displacing the non-wetting phase and a counter flow of the resident fluid from the medium towards the wetting phase. This type of phenomenon is called Imbibition. Imbibition generally happens due to the difference in capillary pressure between oil and water and it depends on the fracture network and water injection rates, which occurs in a reservoir in both counter-current and co – current flow models [17]. This process is an important recovery mechanism during water flooding in fractured reservoirs whose porous media is heterogeneous in nature. Imbibition has been investigated by several other authors either for co-current or counter – current or both of them together. Ries and Cil [18] introduced a one – dimensional model for expulsion by counter – current water imbibition in rock. Analytical study of oil recovery during the counter – current imbibition in strongly water – wet system was given by Tavassoli et al.[23]. Ruth et al. provided an approximate analytical solution for counter – current spontaneous imbibition [19]. Barenblatt et al.[4] proposed a theory of multiphase flow with a relaxation time to explain the counter-current imbibition. Parikh, Mehta and Pradhan [15] used generalized separable method and Meher [11] used Adomian decomposition method to obtain the solution of counter-current imbibition phenomenon with Scheidegger – Johnson [22] Model in homogeneous porous media. Patel and Mehta [16] discussed counter – current imbibition phenomenon in heterogeneous porous media by using power series solution method with the consideration of Scheidegger – Johnson [22] Model and compared there solution with homogenous porous media solution. Here in this work the work of Patel and Mehta [16] has been extended with the help of Corey's model [13] as well as Scheidegger – Johnson [22] Model. Patel and Mehta [16] considered capillary pressure as linear function of saturation i.e. $P_c = -\beta S_w$ but here the work has been extended by taking the relation of capillary pressure with saturation form the work of Meher and Meher[12]. Adomian decomposition method is used to obtain an analytical solution of the problem with its convergence study and interpretation of the results has been done with maple software. Finally the comparison has been made between Corey's model [13] and Scheidegger – Johnson[22] model for saturation of water to study the recovery rate for different values of porosity ϕ and initial conditions.

* Corresponding author e-mail: meher_ramakanta@yahoo.com

2 Problem Formulation

Here it is considered a cylindrical piece of heterogeneous porous matrix of length L having its three sides are surrounded by impermeable surface; one open end of the cylinder is labelled as imbibition face $X = 0$. Wetting phase imbibes inwards in a one dimensional porous media of length L at the imbibition face $X = 0$ having capillary pressure P_c and flow is happens at all level of the porous media. Schematic diagram of this phenomenon be shown in Fig. 1.

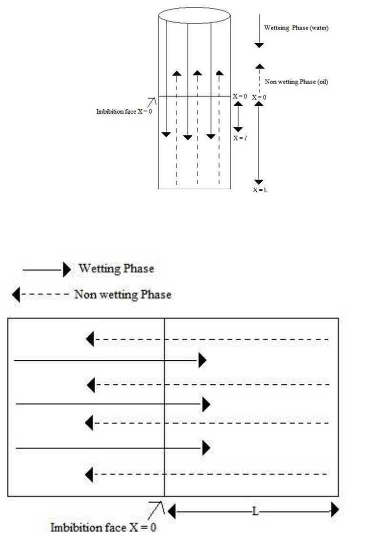

Figure 1: schematic diagram of the problem under consideration

During imbibition, when water is injected into oil saturated porous matrix at imbibition face $X = 0$. The oil is displaced through small distance $X = l$ due to the difference in phase viscosities as shown in Fig. 1. Hence equation of conservation of water volume in one dimensional with over all flow can be expressed as

$$\phi(x)\frac{\partial S_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0 \tag{1}$$

Where ϕ is the porosity of heterogeneous porous media and seepage velocity of wetting phase (water) and non-wetting phase (oil) can be represented due to Darcys law as,

$$v_w = -K(x)\frac{k_w}{\mu_w}\left(\frac{\partial p_w}{\partial x}\right) \tag{2}$$

$$v_o = -K(x)\frac{k_o}{\mu_o}\left(\frac{\partial p_o}{\partial x}\right) \tag{3}$$

Where v_w and v_o are represents velocity of water and oil respectively, K is the variable permeability of the heterogeneous porous medium, k_w and k_o are relative permabilities of the water and oil which are a function of their saturations S_w and S_o respectively, p_w and p_o are pressures of water and oil, μ_w and μ_o are the constant kinematic viscosities of water and oil respectively.
The imbibition condition for counter – current imbibitions and capillary pressure can be expressed due to Scheidegger [21] as

$$v_w = -v_o \tag{4}$$

$$P_c(S_w) = p_o - p_w \tag{5}$$

The analytical linear relationship between capillary pressure and phase saturation [12] can be written as

$$p_c = \beta\left(C_0 + S_w^{-2}\right) \tag{6}$$

Where β and C_0 are constant.
Since the problem is dealing with heterogeneous porous media so, the porosity and permeability of heterogeneous porous media. According to Oroveanu [14] as

$$\phi(x) = \frac{1}{a_1 - b_1 x}$$
$$K(x) = K_c\phi(x) \tag{7}$$

where $a_1 - b_1 x \geq 1$.

case 1: Corey's Model[13]

Using Corey's model [13], the analytical relationship between the relative permeability and phase saturation can be written as

$$k_w = S_w^4 \tag{8}$$

The counter – current imbibition eq. (4) together with capillary pressure eq. (5) gives

$$v_w = K(x)\frac{k_o k_w}{k_w\mu_o + k_o\mu_w}\left[\frac{\partial p_c}{\partial x}\right] \tag{9}$$

Substituting v_w in eq. (1), it obtains

$$\phi(x)\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K(x)\frac{k_o k_w}{k_w\mu_o + k_o\mu_w}\left[\frac{\partial p_c}{\partial S_w}\frac{\partial S_w}{\partial x}\right]\right] = 0 \tag{10}$$

Using $\frac{k_o k_w}{k_w\mu_o + k_o\mu_w} \approx \frac{k_w}{\mu_w} = \frac{S_w^4}{\mu_w}$ Scheidegger [21] and $p_c = \beta\left(C_0 + S_w^{-2}\right)$ Meher [12] and simplifying eq. (10) becomes

$$\frac{\partial S_w}{\partial t} = \frac{\beta K_c}{\mu_w}\left[\frac{\partial}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right) + \frac{1}{\phi}\frac{\partial\phi}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right)\right] \tag{11}$$

This is the desired non-linear partial differential equation that describes the linear counter – current imbibition phenomenon in a heterogeneous porous media.
By choosing the dimensionless variables $X = \frac{x}{L}$ and

$T=\frac{K_c\beta}{2L^2\mu_w}t$.
The dimensionless form of eq. (11) can be written as

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left(S_w\frac{\partial S_w}{\partial X}\right)+\frac{1}{\phi}\frac{\partial \phi}{\partial X}\left(S_w\frac{\partial S_w}{\partial X}\right) \quad (12)$$

Simplification of $\frac{1}{\phi}\frac{\partial\phi}{\partial X}$ as
$\frac{1}{\phi}\frac{\partial\phi}{\partial X}=\frac{\partial}{\partial X}(\log\phi)=\frac{\partial}{\partial X}\left[\frac{b_1}{a_1}LX-\log a_1\right]$ (Neglecting higher order term of X) = $\frac{b_1L}{a_1}$
It leads eq. (12), in to the form as

$$\frac{\partial S_w}{\partial T}=\frac{\partial^2 S_w^2}{\partial X^2}+\frac{b_1L^2}{a_1}\frac{\partial S_w^2}{\partial X} \quad (13)$$

With suitable initial condition $S_w(X,0)=\left(1+X^2\right)^{-\frac{1}{2}}$ as discussed in Meher [12].

case 2: Scheidegger – Johnson[22] model

Using Scheidegger – Johnson [22] model, the analytical relationship between the relative permeability and phase saturation can be written as

$$k_w=S_w \quad (14)$$

and capillary pressure $p_c=\beta\left(C_0+S_w^{-2}\right)$.
Using eq.(14), the conservation eq.(1) in one dimensional form, becomes

$$\frac{\partial S_w}{\partial t}=\frac{2\beta K_c}{\mu_w}\left[\frac{\partial}{\partial x}\left(\frac{1}{S_w^2}\frac{\partial S_w}{\partial x}\right)+\frac{1}{\phi}\frac{\partial\phi}{\partial x}\left(\frac{1}{S_w^2}\frac{\partial S_w}{\partial x}\right)\right] \quad (15)$$

This is the desired non-linear partial differential equation that describes the linear counter–current imbibition phenomenon in a heterogeneous porous media.
By choosing the dimensionless variable $X=\frac{x}{L}$ and $T=\frac{2K_c\beta}{L^2\mu_w}t$.
The dimensionless form of eq. (15) together with the Simplifying form of $\frac{1}{\phi}\frac{\partial\phi}{\partial X}$ gives

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left(\frac{1}{S_w^2}\frac{\partial S_w}{\partial X}\right)+\frac{b_1L^2}{a_1}\left(\frac{1}{S_w^2}\frac{\partial S_w}{\partial X}\right) \quad (16)$$

With suitable initial condition $S_w(X,0)=\left(1+X^2\right)^{-\frac{1}{2}}$ Meher [12].
In [3], Aronofsky first proposed an empirical function to study the recovery rate during oil recovery process as $R=R_\infty\left(1-e^{-\alpha T}\right)$.
Where R is the recovery, R_∞ is the ultimate recovery and α is a constant that best matches the data with a value of approximately 0.05.
Later on imbibition data experimentally, Mattax and Kyte [9] used this result for Alundum samples and wailer sandstones, Hamon and Vidals [8] used this result for synthetic materials and in [24], Zhang used results for Berea sandstones with different boundary conditions.

3 Convergence Analysis of the Adomian decomposition method

We recall the following theorem from [10] which guarantees the convergence of Adomians method for the general operator equation given by $LS_w+RS_w+NS_w=g$. Consider the Hilbert space $H=L^2\left((\alpha,\beta)\times[0,T]\right)$ defined by the set of applications:

$$S_w:(\alpha,\beta)\times[0,T]\to R$$

with

$$\int_{(\alpha,\beta)\times[0,T]} S_W^2(\eta,\xi)\,d\eta\,d\xi<+\infty \quad (17)$$

Let us denote

$$LS_w=\frac{\partial S_w}{\partial T},\quad NS_w=S_w^2$$
$$TS_w=RS_w+NS_w=\frac{\partial^2 S_w^2}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial S_w^2}{\partial X} \quad (18)$$

Theorem 1

Let $TS_w=-RS_w-NS_w$ be a hemi continuous operator in a Hilbert space H and satisfy the following hypothesis:
(H_1) : $(TS_w-TS_w^*,S_w-S_w^*)\geq k\|S_w-S_w^*\|^2$, $k>0$, $\forall S_w,S_w^*\in H$
(H_2) : Whatever may be $M>0$, there exist constant $D(M)>0$ such that for $S_w,S_w^*\in H$ with $\|S_w\|\leq M$, $\|S_w^*\|\leq M$, we have $(TS_w-TS_w^*,w)\leq D(M)\|S_w-S_w^*\|\,\|w\|$ for every $w\in H$.
Then, for every $g\in H$, the nonlinear functional equation $LS_w+RS_w+NS_w=g$ admits a unique solution $S_w\in H$. Furthermore, if the solution S_w can be represented in a series form given by $S_w=\sum_{n=0}^{\infty}S_{wn}\lambda^n$, then the Adomian decomposition scheme corresponding to the functional equation under consideration converges strongly to $S_w\in H$, which is the unique solution to the functional equation.

Proof

Verification of hypothesis (H_1)

$$TS_w-TS_w^*=-\left[\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X}\right]$$
$$(TS_w-TS_w^*,S_w-S_w^*)=$$
$$\left[-\left(\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X}\right),S_w-S_w^*\right] \quad (19)$$

Since ${\partial^2}/{\partial X^2}$ and ${\partial}/{\partial X}$ are differential operator in H, if there exist constants "δ_1" and "δ_2" then according to Schwartz inequality, it can be written as

$$\left(\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X},S_w-S_w^*\right) \leq \left(\delta_1\left\|S_w^2-S_w^{2^*}\right\|+\frac{b_1L}{a_1}\delta_2\left\|S_w^2-S_w^{2^*}\right\|\right).(S_w-S_w^*) \quad (20)$$

Now, by using mean value theorem, it can be written as

$$\left(\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X},S_w-S_w^*\right) \leq \left(\delta_1\left\|S_w^2-S_w^{2^*}\right\|+\frac{b_1L}{a_1}\delta_2\left\|S_w^2-S_w^{2^*}\right\|\right).(S_w-S_w^*) \leq \left(\delta_1M+\frac{b_1L}{a_1}\delta_2M\right)\|S_w-S_w^*\|^2 \quad (21)$$

For $\|S_w\|\leq M$ and $\|S_w^*\|\leq M$.
It implies

$$\left[-\left(\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X}\right),S_w-S_w^*\right] \geq \left(\delta_1M+\frac{b_1L}{a_1}\delta_2M\right)\|S_w-S_w^*\|^2 \quad (22)$$

Now by substituting eq. (27) in eq. (24), it obtains

$$(TS_w-TS_w^*,S_w-S_w^*)\geq k\|S_w-S_w^*\|^2 \quad (23)$$

Where $k=\left(\delta_1M+\frac{b_1L}{a_1}\delta_2M\right)$.
Hence hypothesis (H_1) holds true.
For hypothesis (H_2) ,

$$(TS_w-TS_w^*,V)=\left[-\left(\frac{\partial^2(S_w^2-S_w^{2^*})}{\partial X^2}+\frac{b_1L}{a_1}\frac{\partial(S_w^2-S_w^{2^*})}{\partial X}\right),V\right] = \left(M+\frac{b_1L}{a_1}M\right)\|S_w-S_w^*\|\,\|V\| = D(M)\|S_w-S_w^*\|\,\|V\| \quad (24)$$

Where $D(M)=\left(M+\frac{b_1L}{a_1}M\right)$ and therefore (H_2) holds. The proof is complete.
Remark 2: We note that the constant $D(M)$ is function of M, and the linearity of T allows us to prove (H_2). Furthermore, since every linear continuous operator is hemi continuous, the operator T is hemi continuous.

4 Analysis of the problem

Case 1

For the purposes of illustration of the ADM, in this study we shall consider eq.(13), in an operator form as

$$L_TS_w(X,T)=L_{XX}(NS_W)+\frac{b_1L^2}{a1}L_X(NS_W) \quad (25)$$

With the initial condition $S_w(X,0)=\left(1+X^2\right)^{-\frac{1}{2}}$.
Following [1,2] we define the linear operator $L_T=\frac{\partial}{\partial T}$, $L_{XX}=\frac{\partial^2}{\partial X^2}$ and the definite integration inverse operator L_T^{-1} and the nonlinear term as NS_w.
Therefore, the solution of eq.(13) in T - direction, can be written as

$$\sum_{n=0}^{\infty}S_{wn}(X,T)=\left(1+X^2\right)^{-\frac{1}{2}}+L_T^{-1}\left[L_{XX}\left(\sum_{n=0}^{\infty}A_n\right)\right] +\frac{b_1L^2}{a_1}L_T^{-1}\left[L_X\left(\sum_{n=0}^{\infty}A_n\right)\right] \quad (26)$$

Where $S_{w0},S_{w1},S_{w2},\ldots$ are the saturation of the different fingers at any distance X and any time $T>0$ and $A_n's$ are the Adomians special polynomials to be determined by using the formula

$$A_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^kS_{wk}\right)\right]\right]_{\lambda=0}, \quad n\geq 0$$

Now from eq.(26), $S_{w0}(X,0)=\left(1+X^2\right)^{-\frac{1}{2}}$ and the recurrence relation

$$S_{w,k+1}=L_T^{-1}\left[(A_k)_{XX}\right]+\frac{b_1L^2}{a_1}L_T^{-1}\left[(A_k)_X\right],k\geq 1,2,3,\ldots$$

gives the approximate analytical solution of problem eq.(13), and it can be written in the series form up to three terms as

$$S_w(X,T)=S_{w0}+S_{w1}+S_{w2}+\ldots$$

$$S_w(X,T)=\frac{1}{\sqrt{X^2+1}} -\frac{\left(L^2b_1X^3+L^2b_1X-3a_1X^2+a_1\right)T}{a_1\left(X^2+1\right)^3} -\frac{1}{2}\frac{\left(20L^2b_1X^5+5L^2b_1X^3-90a_1X^3\right)T^2}{a_1\left(X^2+1\right)^{\frac{11}{2}}} -\frac{1}{2}\frac{\left(-15L^2b_1X+149a_1X^2+13a_1\right)T^2}{a_1\left(X^2+1\right)^{\frac{11}{2}}}+\cdots \quad (27)$$

Case 2

The operator form of eq.(16), becomes

$$L_T S_w(X,T) = L_X(NS_W) + \frac{b_1L^2}{a_1}(NS_W) \quad (28)$$

With the initial condition $S_w(X,0) = (1+X^2)^{-\frac{1}{2}}$.
Following the procedure for Adomian decomposition method and using the initial condition $S_w(X,0) = (1+X^2)^{-\frac{1}{2}}$.
Therefore, the solution of eq.(16) in T - direction, can be written as

$$\sum_{n=0}^{\infty} S_{wn}(X,T) = (1+X^2)^{-\frac{1}{2}} + L_T^{-1}\left[L_X\left(\sum_{n=0}^{\infty} A_n\right)\right] + \frac{b_1L^2}{a_1}L_T^{-1}\left[\left(\sum_{n=0}^{\infty} A_n\right)\right] \quad (29)$$

Where $S_{w0}, S_{w1}, S_{w2}, \ldots$ are the saturation of the different fingers at any distance X and any time $T > 0$ and $A_n{}'s$ are the Adomians special polynomials are to be determined.
Now from eq.(26), $S_{w0}(X,0) = (1+X^2)^{-\frac{1}{2}}$ and the recurrence relation

$$S_{w,k+1} = L_T^{-1}[(A_k)_X] + \frac{b_1L^2}{a_1}L_T^{-1}[(A_k)], k \geq 1,2,3,\ldots \quad (30)$$

gives the approximate analytical solution of problem eq.(16), and it can be written in the series form up to three terms as

$$S_w(X,T) = S_{w0} + S_{w1} + S_{w2} + \ldots$$

$$\begin{aligned} S_w(X,T) = &\frac{1}{\sqrt{X^2+1}} \\ &- \frac{(L^2b_1X^3 + L^2b_1X + a_1)\,T}{a_1(X^2+1)^{\frac{3}{2}}} \\ &- \frac{1}{2}\frac{(2L^4b_1^2X^6 + 5L^4b_1^2X^4 + 2L^2a_1b_1X^5)}{a_1^2(X^2+1)^{\frac{5}{2}}} \\ &- \frac{1}{2}\frac{(4L^4b_1^2X^2 + 4L^2a_1b_1X^3 + L^4b_1^2)}{a_1^2(X^2+1)^{\frac{5}{2}}} \\ &- \frac{1}{2}\frac{(2L^2a_1b_1X + 2a_1^2X^2 - a_1^2)\,T^2}{a_1^2(X^2+1)^{\frac{5}{2}}} + \cdots \end{aligned} \quad (31)$$

Eq.(27) and Eq.(31) represents saturation of water during counter – current imbibition phenomenon for Corey's model and Scheidegger – Johnson model corrected up to three terms.

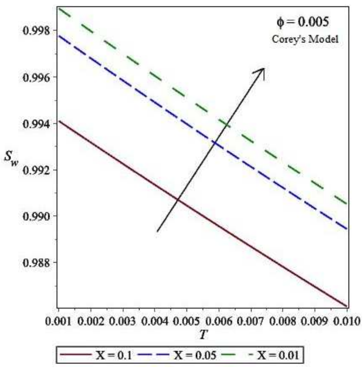

Figure 2: Effect of initial condition on saturation

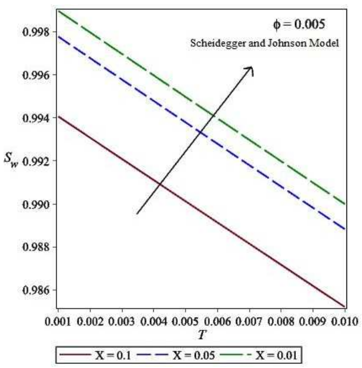

Figure 3: Effect of initial condition on saturation

Fig. 2 and Fig.3 represent the variation of saturation with dimensionless time T for different initial values. Which study the effect of initial condition on saturation rate for different values of X with $\phi = 0.005$ which shows saturation rate is maximum as it is closer to the imbibition face and with Corey's model as compared to Scheidegger – Johnson model.

Table 1: Saturation vs. Time for different Initial Conditions for $X = 0.1$, $X = 0.05$, $X = 0.01$.

$\phi = 0.005$		
	X = 0.1	
	Case 1	**Case 2**
T = 0.001	0.9941006333	0.9940519851
T = 0.002	0.9931746601	0.9930677346
T = 0.003	0.9922589392	0.9920844390
T = 0.004	0.9913531375	0.9911020983
T = 0.005	0.9904569218	0.9901207125
T = 0.006	0.9895699590	0.9891402816
T = 0.007	0.9886919157	0.9881608056
T = 0.008	0.9878224590	0.9871822845
T = 0.009	0.9869612554	0.9862047183
T = 0.010	0.9861079719	0.9852281070

$\phi = 0.005$		
	X = 0.05	
	Case 1	**Case 2**
T = 0.001	0.9977731502	0.9977563203
T = 0.002	0.9968059894	0.9967612913
T = 0.003	0.9958504121	0.9957672507
T = 0.004	0.9949059789	0.9947741983
T = 0.005	0.9939722502	0.9937821342
T = 0.006	0.9930487863	0.9927910584
T = 0.007	0.9921351477	0.9918009709
T = 0.008	0.9912308950	0.9908118716
T = 0.009	0.9903355884	0.9898237607
T = 0.010	0.9894487885	0.9888366380

$\phi = 0.005$		
	X = 0.01	
	Case 1	**Case 2**
T = 0.001	0.9989569597	0.9989506022
T = 0.002	0.9979764226	0.9979522013
T = 0.003	0.9970079124	0.9969547999
T = 0.004	0.9960509509	0.9959583978
T = 0.005	0.9951050633	0.9949629952
T = 0.006	0.9941697734	0.9939685921
T = 0.007	0.9932446024	0.9929751883
T = 0.008	0.9923290761	0.9919827840
T = 0.009	0.9914227178	0.9909913791
T = 0.010	0.9905250490	0.9900009736

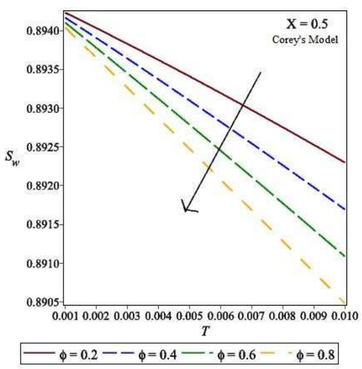

Figure 4: Effect of initial condition on saturation

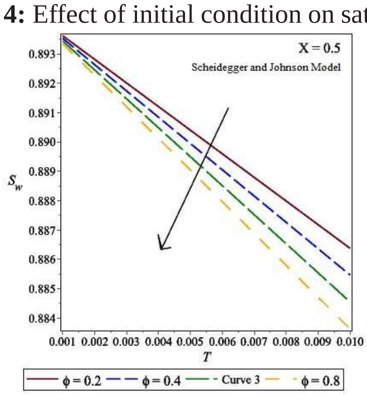

Figure 5: Effect of initial condition on saturation

Fig.4 and Fig.5 represent the variation of saturation rate with dimensionless time for different values of porosity keeping dimensionless X fixed. It shows that as the porosity of the medium increases saturation rate decreases with time and comparison shows, saturation rate be maximum for Corey's model as compared to Scheidegger – Johnson model.

Table 2: Saturation vs. Time for different Porosity.

X = 0.5 and L = 1		
	$S(X,T)$	
	$\phi = 0.2$	$\phi = 0.2$
T = 0.001	0.8942328401	0.8936222324
T = 0.002	0.8940339150	0.8928173284
T = 0.003	0.8938305883	0.8920124781
T = 0.004	0.8936230356	0.8912076814
T = 0.005	0.8934114331	0.8904029385
T = 0.006	0.8931959564	0.8895982492
T = 0.007	0.8929767815	0.8887936135
T = 0.008	0.8927540843	0.8879890315
T = 0.009	0.8925280406	0.8871845032
T = 0.010	0.8922988263	0.8863800286

X = 0.5 and L = 1		
	$S(X,T)$	
	$\phi = 0.4$	$\phi = 0.4$
T = 0.001	0.8941691921	0.8935326197
T = 0.002	0.8939073205	0.8926377632
T = 0.003	0.8936417449	0.8917426205
T = 0.004	0.8933726374	0.8908471915
T = 0.005	0.8931001700	0.8899514763
T = 0.006	0.8928245150	0.8890554750
T = 0.007	0.8925458444	0.8881591874
T = 0.008	0.8922643303	0.8872626135
T = 0.009	0.8919801449	0.8863657535
T = 0.010	0.8916934602	0.8854686073

X = 0.5 and L = 1		
	$S(X,T)$	
	$\phi = 0.6$	$\phi = 0.6$
T = 0.001	0.8941055437	0.8934429534
T = 0.002	0.8937807238	0.8924579833
T = 0.003	0.8934528944	0.8914722798
T = 0.004	0.8931222223	0.8904858429
T = 0.005	0.8927888745	0.8894986726
T = 0.006	0.8924530174	0.8885107688
T = 0.007	0.8921148181	0.8875221316
T = 0.008	0.8917744432	0.8865327610
T = 0.009	0.8914320597	0.8855426569
T = 0.010	0.8910878341	0.8845518194

X = 0.5 and L = 1		
	$S(X,T)$	
	$\phi = 0.8$	$\phi = 0.8$
T = 0.001	0.8940418953	0.8933532334
T = 0.002	0.8936541253	0.8922779888
T = 0.003	0.8932640374	0.8912014562
T = 0.004	0.8928717918	0.8901236357
T = 0.005	0.8924775483	0.8890445272
T = 0.006	0.8920814669	0.8879641307
T = 0.007	0.8916837077	0.8868824462
T = 0.008	0.8912844305	0.8857994738
T = 0.009	0.8908837954	0.8847152134
T = 0.010	0.8904819624	0.8836296650

Recovery rate

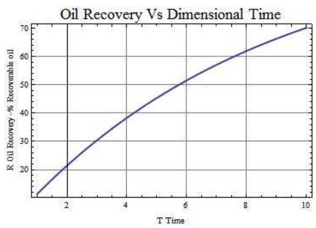

Figure 6: Oil recovery rate vs. Dimensionless time for Corey's model

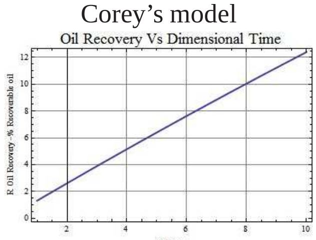

Figure 7: Oil recovery rate vs. Dimensionless time for Scheidegger - Johnson model

Fig. 6 and Fig. 7 discuss the recovery rate vs. Dimensionless time for Coreys model and Scheidegger - Johnson model. It is observed from fig 6, 7 and table 3. That saturation rate is faster as well as more in case of Coreys model as compared to Scheidegger – Johnson Model.

Table 3: Comparison of recovery rate for Coreys model and Scheidegger - Johnson model.

$K_c = 0.5,\ \beta = 0.4,\ \mu_w = 0.697 \times 10^{-2},\ L = 25$	
Corey's Model	**S.J. Model**
11.3855	1.3150
21.4748	2.6126
30.4153	3.8932
38.3379	5.1570
45.3584	6.4041
51.5796	7.6349
57.0926	8.8495
61.9778	10.0480
66.3068	11.2309
70.1430	12.3982

5 Conclusion

Here saturation rate of wetting phase for counter – current imbibition phenomenon in heterogeneous porous media for Corey's model and Scheidegger – Johnson model has been derived and compared the values for both model graphically. The recovery rate for both models has been calculated for some parametric values and compared the results graphically. It is observed that as the saturation rate increases, recovery rate increases with time and recovery rate be maximum in Corey's model as compared to Scheidegger – Johnson model. So finally, it can be concluded that Corey's model in counter – current imbibition phenomenon be more appropriate as compared to Scheidegger – Johnson model to study the saturation as well as recovery rate.

References

[1] Adomian, G. "Solving Frontier Problems of Physics: The Decomposition Method. 1994." Klumer, Boston.
[2] Abbaoui, K., and Y. Cherruault. "Convergence of Adomian's method applied to differential equations."Computers & Mathematics with Applications, 28.5 (1994): 103-109.
[3] Aronofsky, J.S., Masse, L., Natanson, S.G., 1958. "A model for the mechanism of oil recovery from the porous matrix due to water invasion in fractured reservoirs". Trans. AIME 213, 17– 19.
[4] Barenblatt, G. I., T. W. Patzek, and D. B. Silin. "The mathematical Model of non-equilibrium effects in water-oil displacement, SPE 75169." Proceedings of the SPE/DOE Thirteenth Symposium on Improved Oil Recovery held in Tulsa, Oklahoma. 2002.
[5] Behbahani, Hassan, and Martin J. Blunt. "Analysis of imbibition in mixed-wet rocks using pore-scale modelling." SPE Journal 10.04 (2005): 466-474.
[6] Booth, R.J., "Miscible flow through porous media" .PhD thesis, University of Oxford, (2008) 5-10.
[7] Bourbiaux, B. J., and Kalaydjian, F. J. "Experimental study of Cocurrent and Counter-Current flows in natural porous media." SPE Reservoir Engineering 5.03 (1990): 361-368.
[8] Hamon, G., Vidal, J., 1986. "Scaling-up the capillary imbibition process from laboratory experiments on homogeneous and heterogeneous samples". SPE 15852, European Petroleum Conference. London, UK.
[9] Mattax, C.C., Kyte, J.R., 1962. "Imbibition oil recovery from fractured water drives reservoirs". SPEJ, 177–184 (June 1962).
[10] Mavoungou, Thophile, and Yves Cherruault. "Convergence of Adomian's method and applications to non-linear partial differential equations."Kybernetes, 21.6 (1992): 13-25.
[11] Meher, R. K., and Meher S. K. "Imbibition phenomenon arising in double phase flow through porous medium with capillary pressure. "Proceedings of the World Congress on engineering, London. Vol. 1. 2011.
[12] Meher, R. K., and Meher S. K. "Analytical Treatment and Convergence of the Adomian Decomposition Method for Instability Phenomena Arising during Oil Recovery Process. "International Journal of Engineering Mathematics 2013 (2013).

[13] Miller, Cass T., et al. “Multiphase flow and transport modelling in heterogeneous porous media: challenges and approaches.” Advances in Water Resources,21.2 (1998): 77-120.

[14] Oroveanu, Teodor. “Scurgerea fluidelor prin medii poroase neomogene”. Editura Academiei Republicii Populare Romne, 1963.

[15] Parikh, A. K., Mehta, M. N., and Pradhan, V. H. “Generalised separable solution of counter-current imbibition phenomenon in homogeneous porous medium in horizontal direction.” IJES 2.1 (2013): 220-226.

[16] Patel, K. R., Mehta, M. N., and Patel, T. R. “A mathematical model of imbibition phenomenon in heterogeneous porous media during secondary oil recovery process.” Applied Mathematical Modelling 37.5 (2013): 2933-2942.

[17] Pooladi-Darvish, Mehran, and Abbas Firoozabadi. “Cocurrent and counter-current imbibition in a water-wet matrix block.” *SPE Journal* 5.01 (2000): 3-11.

[18] Reis, J. C., and M. Cil. “A model for oil expulsion by counter-current water imbibition in rocks: one-dimensional geometry.” Journal of Petroleum Science and Engineering 10.2 (1993): 97-107.

[19] Ruth, Douglas W., et al. “An approximate analytical solution for counter-current spontaneous imbibition.” Transport in porous media 66.3 (2007): 373-390.

[20] Scheidegger, Adrian E. “The Physics of Flow through Porous Media.” *Soil Science* 86.6 (1958): 355.

[21] Scheidegger, A.E, “The Physics of Flow through Porous Media”, University of Toronto press, (1960).

[22] Scheidegger, Adrian E., and Edward F. Johnson. “The statistical behaviour of instabilities in displacement processes in porous media.” Canadian Journal of physics 39.2 (1961): 326-334.

[23] Tavassoli, Zohreh, Robert W. Zimmerman, and Martin J. Blunt. “Analysis of counter-current imbibition with gravity in weakly water-wet systems.” Journal of Petroleum Science and Engineering 48.1 (2005): 94-104.

[24] Zhang, X., Morrow, N.R., Ma, S., 1996. “Experimental verification of a modified scaling group for spontaneous imbibition”. SPE Reserv. Eng. 11, 280– 285.

Ramakanta Meher is working as an Assistant Professor in the Department of Applied Mathematics in S.V National Institute of Technology, Surat, India. He has a teaching and research experience of above 10 years. His research interests are centered in Fluid flow through porous media, Decomposition Method and partial differential equation. He has authored over 25 refereed publications in the area of Fluid flow through porous media, Fluid Dynamics and Decomposition Method. He received his PhD in Mathematics from S.V National Institute of Technology, Surat, India, MSc. and BSc. in Mathematics from Sambalpur University, Orissa, India.

Hardik Patel is Research Scholar in the Department of Applied Mathematics in S.V National Institute of Technology, Surat, India. His research interests are centred in Fluid flow through porous media, Adomian Decomposition Method and Differential Equation. He has authored over 08 refereed publications in the area of Fluid flow through porous media, Fluid Dynamics and Decomposition Method.