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Generalized Connectedness in Fuzzy Bitopological Spaces

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Abstract: The primary objective of this research is to study generalized connectedness ideas in the domain of fuzzy bitopological spaces. Additionally, it presents basic theorems to figuring out how they relate to one another, and to explore some of the primary characteristics of connectedness structures. Finally, we looked at the idea of disconnection as well as all of its fundamental theories and characteristics.

Keywords: bitopological fuzzy spaces (fbts), fuzzy generalized connected (g−conn), fuzzy generalized closed groups (g−closed), fuzzy generalized disconnected (g−disconn).

MSC: 54A40; 03E72; 47S40; 94D05; 03B52; 28E10.

1 Introduction

Our research of fuzzy bitopology was inspired by fuzzy topological spaces, which was begun in 1965 by researcher Zadeh [1], has been given priority in this project. Following this, other academics improved the idea of fuzzy topology by applying basic concepts from general topology to fuzzy environments. For instance, Chang (1968), developed certain fuzzy ideas [2]. After that, Kandil (1989) presented fuzzy bitopological spaces [3]. Then, in 1997 Balasubramanian and Sundaram constructed generalized fuzzy closed groups in the domain of fuzzy topology [4]. In addition, some scientists presented several research papers on the generalized closed group of fuzzy space, as an example [5,6]. In 2011, scientists Zahran and Al-Maghribi presented a circular on some operations in fuzzy space [7]. As are some scholars also presented different types of studies on connectedness in fuzzy topology space, such as [8,9].

The research is organized as follows: Section 1 (introduction), in this section we review the history of the topic, its importance, and related research. In section 2 (preliminaries), we mention a few key antecedent concepts that are important in this study. In section 3 (Generalized Connectedness Concepts in Bitopological Fuzzy Spaces) this part presents the idea of generalized connectedness notions and describes them in relation to significant theorems and specific features. Finally, we summarize our results in section 4 (Conclusion).

2 Preliminaries

In the section that follows, we cover a few antecedent concepts that are important in this study.

Definition 1.[11] Considering that I represent the closed interval [0,1] and X is nonempty, the following is known as:

1) a fuzzy set H is known as a function with an X domain and I range, H(t) ∈ [0,1] if t ∈ H, but H(t) = 0 if t /∈ H.
2) R contains H as referred to H ⊆ R when H(t) ≤ R(t), wherever t ∈ X.
3) H ∨ R is the combination of groups that defined as (H ∨ R)(t) = upper{H(t), R(t)} ∀ t ∈ X.

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Definition 2. [11] The pair \((X, \delta)\) is considered fuzzy topology if the next three conditions hold:

1. \(0, 1 \in \delta\), since \(0(t) = 0, 1(t) = 1\), as \(t \in X\).
2. \(H \wedge R \in \delta\), \(\forall H, R \in \delta\).
3. \(\vee_{i \in \mathbb{Z}} H_i \in \delta\), \(\forall (H_i)_{i \in \mathbb{Z}} \in \delta\).

As \((X, \delta)\) is referred to as "fuzzy topology space," or "fuzzy" shortly. Also, the elements of \(\delta\) are known as fuzzy open groups. When \(F \in \delta\), hence \(F^c\) is fuzzy closed, and the set of all fuzzy closed groups denoted by \(\mathbb{F}\).

Definition 3. A bitopological fuzzy spaces, often known as fbts, \((X, \delta_1, \delta_2)\) as \(\delta_1, \delta_2\) are fuzzy topology on \(X\) that is nonempty. In this essay \(X\) take on \((X, \delta_1, \delta_2), Y\) of \((Y, \delta_1, \delta_2)\), such that \(i, j \in \{1, 2\}\) with \(i \neq j\).

Definition 4. [12]. A fuzzy group \(\omega\) of \(X\) is referred to a fuzzy point (singleton) if \(\omega(x) = r, (0 < r \leq 1)\) with a specific \(x \in X\), \(\omega(y) = 0\) with each elements \(y \in X\), excluding \(x\), and it is indicated by \(x_s\).

Definition 5. Every fuzzy class \(H\) of \(X\) is known as a generalized fuzzy closed if closure \(H\) is a subgroup of \(R\), while \(H\) is a subgroup of \(R\), which is open. i.e., \(H\) is generalised fuzzy closed when \(cl(H) \leq R\), as \(H \leq R\), and \(R\) is fuzzy open.

3 Generalized Connectedness Concepts in Bitopological Fuzzy Spaces

This part presents the notion of generalized connectedness notions by applying some forms of generalized closed groups in bitopological fuzzy spaces with characterizing them in relation to significant theorems and certain features. The concept of generalized closed groups is the basis for constructing the definition of connectedness and we introduced it as follows:

Definition 6. Each fuzzy subgroup \(H\) of fbts \((X, \delta_1, \delta_2)\) is named as:

1. Fuzzy \((i, j)\) generalized \(\psi\)-closed (shortly, \((i, j) \rightarrow g\psi\) - cl\(d\)) if \(\delta_1 \rightarrow \psi - cld\) or \(\delta_2 \rightarrow \psi - cld\).
2. (-\(\psi\)-open of \(X\) is a supplementary of fuzzy \((i, j) \rightarrow g\psi\) - cl\(d\).

Remark. (1) The overall class of each fuzzy \((i, j) \rightarrow g\psi\) - cl\(d\) also \((i, j) \rightarrow g\psi\) - cl\(d\) groups of \((X, R, \delta_1, \delta_2)\) showed by \(\mathbb{F}_{i, R, \delta_1, \delta_2}\), and so forth.

The following theorem explains the relationships between all varieties of generalized closed groups in fuzzy bitopology.

Theorem 1. When \((X, \delta_1, \delta_2)\) is fbts. Consequently, the following statements are true:

1. \(\forall (i, j) \rightarrow g\alpha - cl\(d\) is \((i, j) \rightarrow g\alpha - cl\(d\)
2. \(\forall (i, j) \rightarrow g\alpha - cl\(d\) is \((i, j) \rightarrow g\beta - cl\(d\)
3. \(\forall (i, j) \rightarrow g\beta - cl\(d\) or \((i, j) \rightarrow g\beta - cl\(d\)

Proof. It is evident from Definition 6 and the connections between various fuzzy set types, where \(\text{open} \rightarrow (\alpha \text{open}) \rightarrow (\text{open})
\)

Next, we provide an introduction to the most important concepts and theories of the term of generalized connectedness:

Definition 7. If \(H, R\) are fuzzy \((i, j) \rightarrow g\psi\) - cl\(d\) sets, then they are separated if they are disjoint and \(H \cap \{(i, j) \rightarrow g\psi - cl\(d\) = R\cap \{(i, j) \rightarrow g\psi - cl\(d\)\}

In other words, neither contains an accumulation point of the other, called Hausdorff Lennes separation condition of fuzzy \((i, j) \rightarrow g\psi\) - cl\(d\) groups.

Definition 8. A fuzzy subgroup \(H\) of \((X, \delta_1, \delta_2)\) is named \((i, j) \rightarrow g\psi\) - disconnected (in sum, \((i, j) \rightarrow g\psi\) - disconnect) if \(\exists G, H\) \((i, j) \rightarrow g\psi\) - open sets of \((X, \delta_1, \delta_2)\) as \((H \cap G, H \cap H)\) are disjoint nonempty whose combination is \(H\). In this instance \(G \cup H\) is known as \((i, j) \rightarrow g\psi\) - disconnection of \(H\).

So, we can infer the following from the above definition:

Corollary 1. A fuzzy group \(R\) is fuzzy \((i, j) \rightarrow g\psi\) - connected (in sum, \((i, j) \rightarrow g\psi\) - conn) if it is a group that invalid as being a fuzzy \((i, j) \rightarrow g\psi\) - disconnect.

Example 1. In any fuzzy bitopological space we notice that 0, and any fuzzy singleton group are always fuzzy \((i, j) \rightarrow g\psi\) - conn groups.

Theorem 2. A fuzzy group \(R\) is fuzzy \((i, j) \rightarrow g\psi\) - conn \(\Leftrightarrow\) it is not a merger of two fuzzy nonempty \((i, j) \rightarrow g\psi\) - open separated groups.
Proof. We prove equivalently, that $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn} \iff R$ is the union of two non empty $(i, j) - g_{\psi} - \text{open}$ separated groups. Assume $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn} \subseteq G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $R$. Then $R$ is the union of non empty sets $(R \cap G), (R \cap H).$ So, we need to show that each of $(R \cap G)$ and $(R \cap H)$ contains no accumulation point of the other. Thus, let $p$ be an accumulation point of $R \cap G,$ $p \in (R \cap H).$ Then $H$ is $(i, j) - g_{\psi} - \text{open}$ including $p$ and so $H$ contains a point of $(R \cap G)$ other than $p.$ So, $(R \cap G) \cap H \neq \emptyset$ but $(R \cap G) \cap H = \emptyset,$ according $p \notin (R \cap H).$ Similarly, where $p$ is an accumulation point of $(R \cap H), p \notin (R \cap G).$ Thus $(R \cap G)$ and $(R \cap H)$ are fuzzy $(i, j) - g_{\psi} - \text{open}$ separated sets.

Proposition 1. If $G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $R$ with $C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ subset of $R.$ Then $C \cap H = 0$ or $C \cap G = 0$ and so either $C \subseteq G$ or $C \subseteq H.$

Proof. Since $C \subseteq R,$ and $R \subseteq (G \cup H),$ then $C \subseteq (G \cup H), \text{and } G \cap H \subseteq C.$ Hence both $C \cap G$ and $C \cap H$ are non empty, then $G \cup H$ form $(i, j) - g_{\psi} - \text{disconn}$ of $C.$ But $C$ is fuzzy $(i, j) - g_{\psi} - \text{conn},$ hence the following condition is met $C \cap H = 0$ and $C \cap G = 0.$

Theorem 3. If $R$ and $C$ are fuzzy $(i, j) - g_{\psi} - \text{conn}$ which are not separated, then $R \cup C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group.

Proof. Suppose $R \cup C$ is $(i, j) - g_{\psi} - \text{disconn},$ and $G \cup H$ is $(i, j) - g_{\psi} - \text{disconn}$ of $R \cup C.$ Since $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn} \subseteq R \cup C,$ either $R \subseteq G$ or $R \subseteq H$ by Proposition 1. Similarly for $C$ either $C \subseteq G$ or $C \subseteq H.$ Now, if $R \subseteq G$ and $C \subseteq H$ or $(C \subseteq G, R \subseteq H),$ hence $(R \cup C) \cap G = R, (R \cup C) \cap H = C,$ but this contradicts the hypothesis $R \subseteq G, C \subseteq H.$ Thus $R \cup C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group. Hence either $R \cup C \subseteq G$ or $R \cup C \subseteq H.$ Thus, $R \cup H$ is not fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $R \cup C.$ It means, $R \cup C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group.

Corollary 2. If $\Lambda = \{R_i\}$ is a class of fuzzy $(i, j) - g_{\psi} - \text{conn}$ groups as no two members of $\Lambda$ are separated, then $C = \bigcup_i R_i$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group.

Proof. Suppose $C$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ and $G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $C.$ Since $\forall R_i \subseteq \Lambda$ is fuzzy $(i, j) - g_{\psi} - \text{conn},$ as by Proposition 1, contained either $G$ or $H$ and disjoint from the other. Furthermore, any $R_1, R_2 \subseteq \Lambda$ are not separated sets and so by Theorem 3, $R_1 \cup R_2$ is fuzzy $(i, j) - g_{\psi} - \text{conn}.$ Thus $R_1 \cup R_2$ is contained in $G$ or $H$ and disjoint from the other. Accordingly, all members of $\Lambda$, so $C = \bigcup_i R_i$ must be contained in either $G$ or $H$ and disjoint from the other. But this goes against the idea that $G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $C.$ Therefore $C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group.

Corollary 3. If $\Lambda = \{R_i\}$ is the class of fuzzy $(i, j) - g_{\psi} - \text{conn}$ groups with a non empty intersection, then $C = \bigcup_i R_i$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ group.

Proof. As $\bigcap_i R_i \neq \emptyset,$ thus any two members of $\Lambda$ are not disjoint and so no separated, thus by Corollary 2, $C = \bigcup_i R_i$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ set.

Theorem 4. If $R$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ subset of $X$ and $R \subseteq \{i, j\} - g_{\psi} - \text{cl}(R).$ Then $C$ is fuzzy $(i, j) - g_{\psi} - \text{conn},$ due to this in particular $(i, j) - g_{\psi} - \text{cl}(R)$ is $(i, j) - g_{\psi} - \text{conn}.$

Proof. Assume $C$ is fuzzy $(i, j) - g_{\psi} - \text{disconn},$ with $G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $C.$ While $R$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ subset of $C,$ then by Proposition 1 either $R \cap H = 0$ or $R \cap G = 0,$ if we choose $R \cap H = 0.$ Hence $R \subseteq H^c$ which is fuzzy $(i, j) - g_{\psi} - \text{cl}d.$ So $R \subseteq C \subseteq (i, j) - g_{\psi} - \text{cl}(R) \subseteq H^c.$ Consequently, $C \cap H = 0.$ But this contradicts the idea that $G \cup H$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $C.$ Thus $C$ is fuzzy $(i, j) - g_{\psi} - \text{conn}.$

Theorem 5. If $R$ is fuzzy subgroup of $(X, \delta_1, \delta_2),$ after that $R$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ of $X \iff R$ is fuzzy $(i, j) - g_{\psi} - \text{conn}$ in terms of the relative topology on $R.$

Proof. Suppose $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $X$ with $G \cup H$ is $(i, j) - g_{\psi} - \text{disconn}$ of $R.$ Now, $G, H$ are fuzzy $(i, j) - g_{\psi} - \text{open}$ groups of $X$ with $G \cap H = 0,$ so $R \subseteq G$ and $R \cap H$ are disjoint $(i, j) - g_{\psi} - \text{open}$ groups regarded to fuzzy topology on $R.$ Since $(R \cap G)$ and $(R \cap H)$ form a $(i, j) - g_{\psi} - \text{disconn}$ of $R$ by fuzzy $(i, j) - g_{\psi} - \text{open}$ of $R,$ thus $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ with respect to the relative topology on $R.$

In contrast, consider $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ regarding to fuzzy topology on $R,$ and let $G^*, H^*$ form $(i, j) - g_{\psi} - \text{disconn}$ of $R.$ Then $G^* = G \cap R,$ $H^* = H \cap R.$ But $R \subseteq G^* = R \cap (R \cap G) = R \cap G,$ and $R \cap H^* = R \cap (R \cap H) = R \cap H,$ hence $G \cup H$ is $(i, j) - g_{\psi} - \text{disconn}$ of $R$ by fuzzy $(i, j) - g_{\psi} - \text{open}$ groups of $X.$ This means $R$ is fuzzy $(i, j) - g_{\psi} - \text{disconn}$ of $X.$

Definition 9. An fbls $(X, \delta_1, \delta_2)$ is known as fuzzy $(i, j) - g_{\psi} - \text{conn}$ iff $X$ is not combination of two nonempty fuzzy $(i, j) - g_{\psi} - \text{open}$ groups.

In different terms, iff 3 non empty fuzzy $(i, j) - g_{\psi} - \text{open}$ sets $R, C$ as $R \cup C = X = 1,$ and $R \cap C = \emptyset = 0.$

Theorem 6. Consider $X$ is fbls. Then the next instances are equivalent:

1. $X$ is fuzzy $(i, j) - g_{\psi} - \text{conn}.$
2. $0, 1$ are the only fuzzy subset of $X$ which are both $(i, j) - g_{\psi} - \text{open}$ also $(i, j) - g_{\psi} - \text{cl}d.$
Proof. Suppose $X$ is fuzzy $(i, j) - g\psi - \text{conn}$, $R$ is non empty $(i, j) - g\psi - \text{open}$ also $(i, j) - g\psi - \text{cld}$ group of $X$ as $R \cap R^c = 0, R \cup R^c = 1$. Now let $C_1 = R, C_2 = R^c$. Thus $C_1, C_2$ are complement of each other, so they are both $(i, j) - g\psi - \text{open}$ also $(i, j) - g\psi - \text{cld}$. For $C_1, C_2$ are nonempty $(i, j) - g\psi - \text{open}$ groups of $X$, we have $C_1 \cap C_2 = R \cap R^c = 0$. Also, $C_1 \cup C_2 = R \cup R^c = 1$. So, they form a separation of $X$, that contradicts the fact $X$ is fuzzy $(i, j) - g\psi - \text{conn}$.

Inversely, suppose $X$ is fuzzy $(i, j) - g\psi - \text{disconn}$. So, $\exists$ fuzzy $(i, j) - g\psi - \text{open}$ groups $R, C$ as $R \cup C = 1, R \cap C = 0$. Here $R^c = C$, so $R$ is both $(i, j) - g\psi - \text{open}$ also $(i, j) - g\psi - \text{cld}$ which contradicts that $X$ has only 0 and 1 which are both $(i, j) - g\psi - \text{open}$ also $(i, j) - g\psi - \text{cld}$. So, $X$ is fuzzy $(i, j) - g\psi - \text{conn}$.

Corollary 4. Suppose $X$ is fbts. Hence the coming instances are equivalent:

1) $X$ is fuzzy $(i, j) - g\psi - \text{disconn}$.
2) $\exists$ fuzzy subset of $X$ which both $(i, j) - g\psi - \text{open}$ also $(i, j) - g\psi - \text{cld}$.

Corollary 5. $X$ is fuzzy $(i, j) - g\psi - \text{conn} \iff \exists$ fuzzy point sets of $X$ which their union equal 1 and their intersection equal 0 except 0 and 1.

Theorem 7. When $(X, \delta_1, \delta_2)$ is fbts. Hence the coming instances are equivalent:

(i) $X$ is fuzzy $(i, j) - g\psi - \text{conn}$.
(ii) When $R, H$ are nonempty fuzzy $(i, j) - g\psi - \text{open}$ groups of $X$ with $R \cup H = 1$ and $R \cap H = 0$, then $R, H$ are complementary to each other.
(iii) If $R, H$ are fuzzy $(i, j) - g\psi - \text{cld}$ groups of $X$ with $R \cup H = 1$ and $R \cap H = 0$, then $R, H$ are complementary to each other.

Proof. (i) $\Rightarrow$ (ii). Assume that (ii) is not true and $R \neq H^c, R \cup H = 1, R \cap H = 0$. Then by De Morgan’s rule $(R \cap H)^c = 1 \Rightarrow R^c \cap H = 0$ and $(R \cap H)^c = 0 \Rightarrow R \cup H = 1$. Now $(i, j) - g\psi - cl(R^c) \cap H^c = 0$, also $R^c \cap (i, j) - g\psi - cl(H^c) = 0$. Since $R, H \neq 0$ and their union equal 1 and their intersection equal 0, hence they form a separation of $X$, thus $X$ is not fuzzy $(i, j) - g\psi - \text{conn}$, which is a contradiction. So, $R = H^c$.
(ii) $\iff$ (iii). It is clear by applying De Morgan’s law.
(iii) $\iff (i)$. Suppose that (i) is not true. Then $\exists$ two nonzero $(i, j) - g\psi - \text{open}$ groups $R, H$ as $(i, j) - g\psi - cl(R) \cap H = 0 \Rightarrow R \cap (i, j) - g\psi - cl(H)$. This implies that (iii) is not true as they are complementary to each other.

Theorem 8. Suppose $(X, \delta_1, \delta_2)$ is fbts. Thus, the following claims are true:

1) $\exists \delta_1 - \text{conn}, (i, j) - g - \text{conn}$.
2) $\exists (i, j) - g - \text{conn} (i, j) - g\alpha - \text{conn}$.
3) $\exists (i, j) - g\alpha - \text{conn}, (i, j) - g\alpha - \text{conn}$ and $(i, j) - gp - \text{conn}$.
4) $\forall (i, j) - gs - \text{conn}$, or $(i, j) - gp - \text{conn}$ is $(i, j) - g\beta - \text{conn}$.

Proof. It is clear from the concept of connectedness in Definition 9, Theorem 2, and the connections between different forms of $(i, j) - g\psi - \text{cld}$ sets in Theorem 1. Also, based on the reference [13] which discussed the relationships between these groups in detail and explained the inverse relationships as mentioned in remark 3 in the reference.

Remark. The next diagram shows the connections between different forms of fuzzy $(i, j) - g\psi - \text{conn}$.

Fig. 1: Presents the relationship through all varieties of fuzzy $(i, j) - g\psi - \text{conn}$.

Theorem 9. When $(X, \delta_1, \delta_2)$ is fbts and $R, H$ form a separation of $X$. Then for any fuzzy set $G$ in $X$, $G \cap R$ and $G \cap H$ are fuzzy $(i, j) - g\psi - \text{disconn}$.

Proof. Since $R, H$ are separated. Then $R, H$ are non empty $(i, j) - g\psi - \text{open}$ sets, as $R \cap (i, j) - g\psi - cl(H) = H \land ((i, j) - g\psi - cl(R)) = R$. Now $G \cap R \subseteq R \Rightarrow (i, j) - g\psi - cl(G \cap R) \subseteq (i, j) - g\psi - cl(R)$. Also, $G \cap H \subseteq H \Rightarrow (i, j) - g\psi - cl(G \cap H) \subseteq (i, j) - g\psi - cl(R)$. Now, $(i, j) - g\psi - cl(G \cap H) \cap (G \cap H) \subseteq (i, j) - g\psi - cl(R) \cap H = 0$. Hence $((i, j) - g\psi - cl(G \cap R)) \cap (G \cap H) = 0$. In a similar way, we get $(G \cap R) \cap (i, j) - g\psi - cl(G \cap H) = 0$. Hence $G \cap R, G \cap H$ form a separation. So $G \cap R, G \cap H$ are fuzzy $(i, j) - g\psi - \text{disconn}$. 

Definition 10. A mapping $f : (X, \delta_1, \delta_2) \to (Y, \sigma_1, \sigma_2)$ termed a fuzzy $(i, j) - g\psi - \text{continuous}$ (briefly, $(i, j) - g\psi - \text{conts}$) when each corresponding image of all fuzzy open of $(Y, \sigma_1)$ is fuzzy $(i, j) - g\psi - \text{open}$ of $X$.

Theorem 10. The image of fuzzy $(i, j) - g\psi - \text{conn}$ under $(i, j) - g\psi - \text{continuous}$ mapping is fuzzy $(i, j) - g\psi - \text{conn}$.

Proof. Let $f : (X, \delta_1, \delta_2) \to (Y, \sigma_1, \sigma_2)$ be fuzzy $(i, j) - g\psi - \text{conts}$ onto function, and $(X, \delta_1, \delta_2)$ be
Theorem 11. If f : (X, σ_i, σ_j) → (Y, σ_1, σ_2) is a mapping (i, j) → gψ conn space. Also, assume R, H are fuzzy σ_i - open groups of X, and Y is fuzzy σ_j - disc conn. Then Y = R ∪ H. Since all fuzzy σ_j - open sets are open, and f is fuzzy (i, j) → gψ conn, then X = f_1^{-1}(R) ∪ f_1^{-1}(H), which is contradiction that X is fuzzy (i, j) → gψ conn space. Therefore Y is fuzzy σ_i - conn.

Definition 11A mapping f : (X, δ_1, δ_2) → (Y, σ_1, σ_2) termed a fuzzy (i, j) = generalized ψ - irresolute mapping (briefly, (i, j) = gψ - irres) when each corresponding image of all fuzzy (i, j) = gψ - open group of X is fuzzy (i, j) = gψ - open of Y.

Theorem 11If f : (X, δ_1, δ_2) → (Y, σ_1, σ_2) is fuzzy (i, j) = gψ - irresolute with surjective mapping, then Y is fuzzy (i, j) = gψ - conn, where X is fuzzy (i, j) = gψ - conn space.

Proof. Suppose f : (X, δ_1, δ_2) → (Y, σ_1, σ_2) is fuzzy (i, j) = gψ - irresolute, surjective function, with X is (i, j) = gψ - conn space. In addition, assume R, H are fuzzy (i, j) = gψ - open groups of Y, also Y is fuzzy (i, j) = gψ - disc conn. After that Y = R ∪ H. As f is fuzzy (i, j) = gψ - irresolute, thus X = f_1^{-1}(R) ∪ f_1^{-1}(H), this runs counter to X is fuzzy (i, j) = gψ - conn space. Therefore Y is (i, j) = gψ - conn.

4 Conclusion

In the above research, we defined also examined the terms of different varieties of generalized connectedness ideas in fuzzy bitopology space, as well as some relationships between them. Then we examined some fundamental theorems and characteristics of these concepts.

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References


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